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Soliton solution and interaction property for a coupled modified Korteweg–de Vries (mKdV) system

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Hirota's bilinear direct method is applied to constructing soliton solutions to a special coupled modified Korteweg–de Vries (mKdV) system. Some physical properties such as the spatiotemporal evolution, waveform structure, interactive phenomena of solitons are discussed, especially in the two-soliton case. It is found that different interactive behaviours of solitary waves take place under different parameter conditions of overtaking collision in this system. It is verified that the elastic interaction phenomena exist in this (1+1)-dimensional integrable coupled model.

Keywords: coupled mKdV system, Hirota's bilinear method, soliton solution, elastic interaction
PACC: 0230, 0420J, 0340, 0340K

1. Introduction

Soliton theory is an important aspect of nonlinear science. Solutions and integrability of soliton systems play such an important role in soliton theory that many mathematicians and physicists are interested in this topic.[1−8] In order to construct soliton solutions and solve nonlinear differential equations, many well-known methods such as Hirota's bilinear method,[9] the mapping and deformation approach,[10] the standard and extended truncated Painlevé analysis,[11] the Darboux transformation,[12] the formal[13] and the informal[14] variable separation methods have been proposed and used widely. Of them, the Hirota bilinear direct method is demonstrated to be one of the most powerful and the simplest techniques for constructing soliton solutions of nonlinear evolutive systems.[15]

Since the first coupled KdV system was presented by Hirota and Satsuma (HS) in 1981[16] and studied in detail,[17] some important coupled Korteweg–de Vries (KdV) models have been proposed.[18] Recently, some quite general coupled KdV equations have been derived in real physical areas, such as in two-layer fluid of atmospheric dynamical system,[19] in a two-wave model of shallow stratified liquid[20] and in a two-component Bose-Einstein condensate.[21] With the approaches mentioned above, some meaningful results for several kinds of coupled KdV system have been obtained.[22]

Lately, a quite general coupled variable coefficient modified KdV (VCmKdV) equation in a two-layer fluid system, which is used to describe the atmospheric and oceanic phenomena, has been derived by using the reductive perturbation method. This VCmKdV equation reads [23]

$$\begin{aligned}
&A_{1T} + r_1 A_{1XXX} + \left(r_2 A_1^2 + r_3 A_1 A_2 + r_4 A_1 + r_5 A_2^2 + r_6 A_2 + r_7\right) A_{1X} \\
&+ \left(r_8 A_1^2 + r_9 A_1 A_2 + r_{10} A_1 + r_{11} A_2 + r_{12}\right) A_{2X} + r_{13} A_1 = 0, \\
&A_{2T} + e_1 A_{2XXX} + \left(e_2 A_2^2 + e_3 A_1 A_2 + e_4 A_2 + e_5 A_1^2 + e_6 A_1 + e_7\right) A_{2X} \\
&+ \left(e_8 A_2^2 + e_9 A_1 A_2 + e_{10} A_2 + e_{11} A_1 + e_{12}\right) A_{1X} + e_{13} A_2 = 0,
\end{aligned} \tag{1}$$

where A_1 and A_2 are the functions of X and T, r_i and e_i $(i = 1, 2, ..., 13)$ are the arbitrary functions of T.

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In the present paper, we concentrate on a new special coupled mKdV system

$$\begin{aligned} &u_t - 6u^2u_x + 12uvv_x + 6v^2u_x + u_{xxx} = 0, \\ &v_t - 6u^2v_x - 12uvu_x + 6v^2v_x + v_{xxx} = 0, \end{aligned} \tag{2}$$

where u and v are corresponding to A_1 and A_2 in system (1), respectively. Obviously, system (2) is a special case of physical system (1) when the variable coefficients are taken as follows: $r_1 = 1,\ r_2 = -6,\ r_5 = 6,\ r_9 = 12,\ r_3 = r_4 = r_6 = r_7 = r_8 = r_{10} = r_{11} = r_{12} = r_{13} = 0$; and $e_1 = 1,\ e_2 = 6,\ e_5 = -6,\ e_9 = -12,\ e_3 = e_4 = e_6 = e_7 = e_8 = e_{10} = e_{11} = e_{12} = e_{13} = 0$. According to Ref.[23], system (2) may represent one group of the vortex solutions of two-layer fluid model in fluid dynamics.

On the other hand, if we consider a (1+1)-dimensional integrable standard mKdV equation

$$U_t - 6U^2U_x + U_{xxx} = 0, \tag{3}$$

via

$$U = u + I\,v, \quad \text{and} \quad I \equiv \sqrt{-1}, \tag{4}$$

Eq.(2) can be derived simply from the real part and imaginary part of Eq.(3). Because Eq. (3) can be deduced also from Eq.(1) in Ref.[23], systems (2) and (3) are two special cases of the system (1).

As is well known, the interaction among solitons is generally elastic in (1+1)-dimensional systems, i.e. there is no exchange between physical quantities (energy, amplitude, velocity and shape) after interaction.[24] To integrable (2+1)-dimensional KdV, mKdV and Nizhnik–Novikov–Veselov systems, the interactions between two dromions may be elastic or inelastic for different forms of solutions.[2] Recently, completely inelastic interactions such as fission and fusion are observed in some soliton models. The fission and fusion phenomena of soliton are also discovered in many real physical systems.[25] Here, we are interested in the possible localized excitations and interaction properties among solitons and/or solitary waves for an integrable coupled system. Now we come to consider system (2) and search its new soliton solutions, for the first time, by using Hirota's bilinear direct method. Using the extended truncated Painlevé method, the following Bäcklund transformation for Eq.(2) can be derived as

$$u = \frac{1}{2}\left(\ln\frac{F_1^2 + F_2^2}{G_1^2 + G_2^2}\right)_x,$$

and

$$v = \left(\arctan\frac{F_2}{F_1} - \arctan\frac{G_2}{G_1}\right)_x. \tag{5}$$

Substituting expression (5) into Eq.(2), we can deduce the following equations in the bilinear form:

$$\begin{aligned} &\left(D_t + D_x^3\right)\ (F_1\cdot G_1 - F_2\cdot G_2) = 0, \\ &\left(D_t + D_x^3\right)\ (F_1\cdot G_2 + F_2\cdot G_1) = 0, \\ &D_x^2\ (F_1\cdot G_1 - F_2\cdot G_2) = 0, \end{aligned}$$

and

$$D_x^2\ (F_1\cdot G_2 + F_2\cdot G_1) = 0, \tag{6}$$

where Hirota's bilinear D-operators are defined as[9]

$$\begin{aligned} D_t^m D_x^n (F.G) \equiv &\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^m \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^n \\ &\times F(x,t)\,G(x',t')\,|_{x'=x,\,t'=t}\ . \end{aligned} \tag{7}$$

In order to construct soliton solutions for Eq.(2), a simple way is to solve its bilinear form Eq.(6) by using exponential functions.

2. Soliton solutions and interaction property

2.1. One-soliton case

To obtain one-soliton solution (1SS) for Eq.(2), we take F and G in Eq.(6) as the following exponential ansatzes:

$$\begin{aligned} &F_{11} = 1 + \mathrm{e}^n, \quad G_{11} = 1 - \mathrm{e}^n, \\ &F_{21} = c\mathrm{e}^n, \quad G_{21} = -c\mathrm{e}^n, \end{aligned} \tag{8}$$

and

$$n = px + \Omega\, t + m, \tag{9}$$

where p and Ω are constant parameters; c and m are arbitrary real constants. Here, if one takes $c = 0$, the solutions F_{11} and G_{11} become 1SS in the bilinear form for the mKdV Eq.(3).[15] Because the parameters p and Ω must satisfy the dispersion relation

$$A\ (p,\ \Omega)\ = 0, \tag{10}$$

we can obtain the term from Eq.(6)

$$\Omega = -p^3. \tag{11}$$

According to expressions (8) and (11), the solutions of Eq.(6) become

$$\begin{aligned} &F_{11} = 1 + \mathrm{e}^{px - p^3t + m}, \quad G_{11} = 1 - \mathrm{e}^{px - p^3t + m}, \\ &F_{21} = c\,\mathrm{e}^{px - p^3t + m}, \ \text{ and } \ G_{21} = -c\mathrm{e}^{px - p^3t + m}. \end{aligned} \tag{12}$$

Substituting expression (12) into expression (5) yields the single-soliton solution

$$u = \frac{1}{2}\left(\ln\frac{F_{11}^2+F_{21}^2}{G_{11}^2+G_{21}^2}\right)_x \equiv \frac{L_1}{H},$$

and

$$v = \left(\arctan\frac{F_{21}}{F_{11}} - \arctan\frac{G_{21}}{G_{11}}\right)_x \equiv \frac{L_2}{H}, \qquad (13)$$

where

$$\begin{aligned} L_1 =& 2p\{4\left(1+a^2\right)\left[\left(2+a^2\right)\sinh n + a^2\cosh n\right] \\ & \times \cosh^2 n - \left(2+a^2\right)\left(4+a^4\right)\sinh n \\ & - a^2\left(4+8a^2+a^4\right)\cosh n\}, \\ L_2 =& 2pa\{4\left(1+a^2\right)\left[\left(2+a^2\right)\cosh n - a^2\sinh n\right] \\ & \times \cosh^2 n - a^2\left(4+a^4\right)\sinh n \\ & - \left(2-a^2\right)\left(2+a^2\right)^2\cosh n\}, \end{aligned}$$

and

$$\begin{aligned} H =& 16\left(1+a^2\right)^2\cosh^4 n + 8\left[a^6 - \left(2+a^2\right)^2\right] \\ & \times \cosh^2 n + \left(4+a^4\right)^2. \end{aligned}$$

2.2. Two-soliton case

We give the explicit forms of F and G in Eq.(6) for two-soliton solution (2SS):

$$\begin{aligned} F_{12} &= 1 + \mathrm{e}^{n_1} + \mathrm{e}^{n_2} + a_{12}\,\mathrm{e}^{n_1+n_2}, \\ G_{12} &= 1 - \mathrm{e}^{n_1} - \mathrm{e}^{n_2} + a_{12}\,\mathrm{e}^{n_1+n_2}, \\ F_{22} &= c_1\,\mathrm{e}^{n_1} + c_2\,\mathrm{e}^{n_2} + c_{12}\,\mathrm{e}^{n_1+n_2}, \end{aligned}$$

and

$$G_{22} = -c_1\,\mathrm{e}^{n_1} - c_2\,\mathrm{e}^{n_2} + c_{12}\,\mathrm{e}^{n_1+n_2}, \qquad (14)$$

where a_{12}, c_1, c_2 and c_{12} are real constant parameters; n_i is defined as

$$n_i = p_i x + \Omega_i\, t + m_i, \quad \text{and} \quad \Omega_i = -p_i^3, \qquad (15)$$

with $i = 1, 2$. Substituting expressions (14) and (15) into Eq.(6), we find that the coupled coefficients a_{12} and c_{12} must satisfy the following limited condition:

$$a_{12} = -\frac{(c_1c_2-1)\,(p_1-p_2)^2}{(p_1+p_2)^2},$$

and

$$c_{12} = \frac{(c_1+c_2)\,(p_1-p_2)^2}{(p_1+p_2)^2}. \qquad (16)$$

If c_1, c_2 and c_{12} are all taken to be zero, F_{12} and G_{12} become the solutions in the bilinear form for the mKdV Eq.(3).[15] Substituting expression (14) together with expressions (15) and (16) into expression (5) results in 2SS of quantities u and v for Eq.(2)

$$u = \frac{1}{2}\left(\ln\frac{F_{12}^2+F_{22}^2}{G_{12}^2+G_{22}^2}\right)_x,$$

and

$$v = \left(\arctan\frac{F_{22}}{F_{12}} - \arctan\frac{G_{22}}{G_{12}}\right)_x. \qquad (17)$$

In order to further illustrate the characteristics of two- soliton solutions, we employ evolution plot (see Fig.1) and structure plots (see Figs.2 and 3) for the physical quantities u and v. Graphic examples display that the quantity u is driven by a stagger-shape curve, and v by a ring-shape curve.

Figures 1(a), 1(b), 1(c) and 1(d) display the evolutional courses for the physical quantities u and v with the constant parameter chosen as given in expressions (18), (19), (20) and (21), respectively,

$$\begin{aligned} & p_1 = 2, \quad p_2 = 4, \quad c_1 = 1, \quad c_2 = 6, \\ & m_1 = 1, \quad m_2 = -1; \qquad (18) \\ & p_1 = 2, \quad p_2 = 4, \quad c_1 = 2, \quad c_2 = 3, \\ & m_1 = 1, \quad m_2 = -1; \qquad (19) \\ & p_1 = -5, \quad p_2 = 3, \quad c_1 = 3, \quad c_2 = -8, \\ & m_1 = 1, \quad m_2 = -1; \qquad (20) \\ & p_1 = 2, \quad p_2 = -4, \quad c_1 = 1, \quad c_2 = 6, \\ & m_1 = 1, \quad m_2 = -1. \qquad (21) \end{aligned}$$

From the evolution plots it follows that four groups of solitons each move along the positive x axe; a phase shift and traversing are present. What solitons come into being is due to the overtaking collision. Figures 1(b), 1(c) and 1(d) show that different parameter conditions result in different interactive behaviours for the same quantity v. We find that the vibration direction of soliton is correlative with the sign of parameters. When $p_ic_i > 0$ ($i = 1$ or 2), the vibration direction is positive; $p_ic_i < 0$, vibration direction is negative. It is amazing that the amplitudes of two solitons occur abruptly in an interactive area when the vibration directions of two solitons are reversed as shown in Fig.1(d).

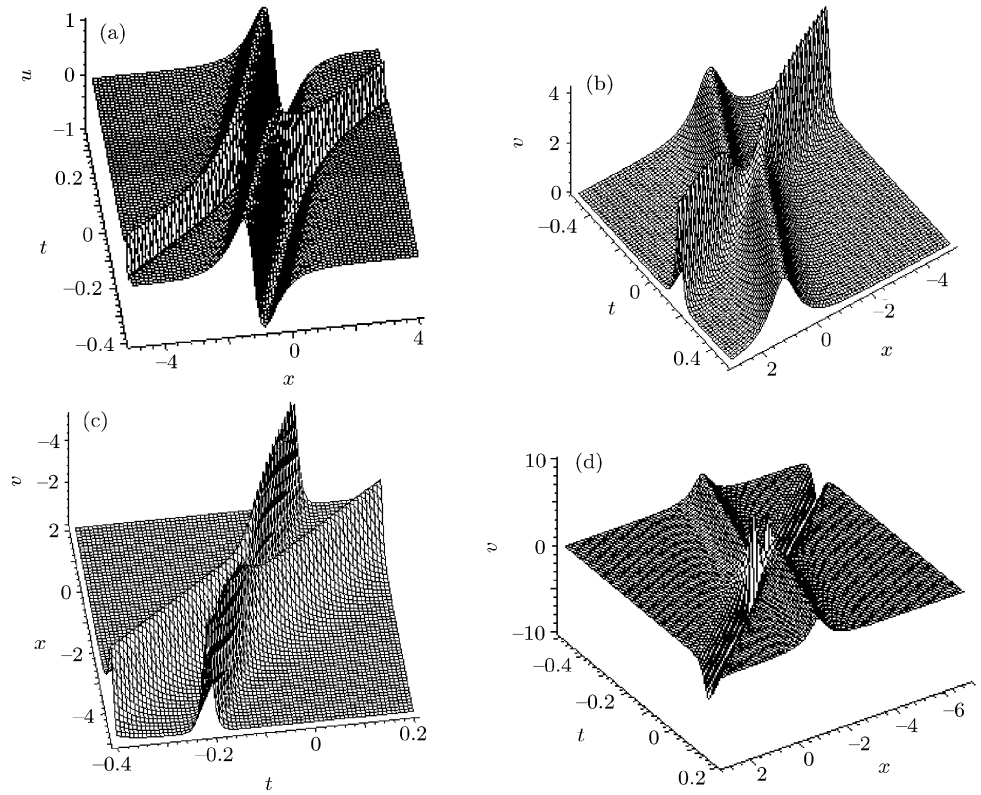


Fig.1. The spatiotemporal evolution plots of 2SS for the quantities: u determined by expressions (17), (14), and (18) (a); v by expressions (17), (14), and (19) (b); v by expressions (17), (14) and (20) (c); v by expressions (17), (14) and (21) (d).

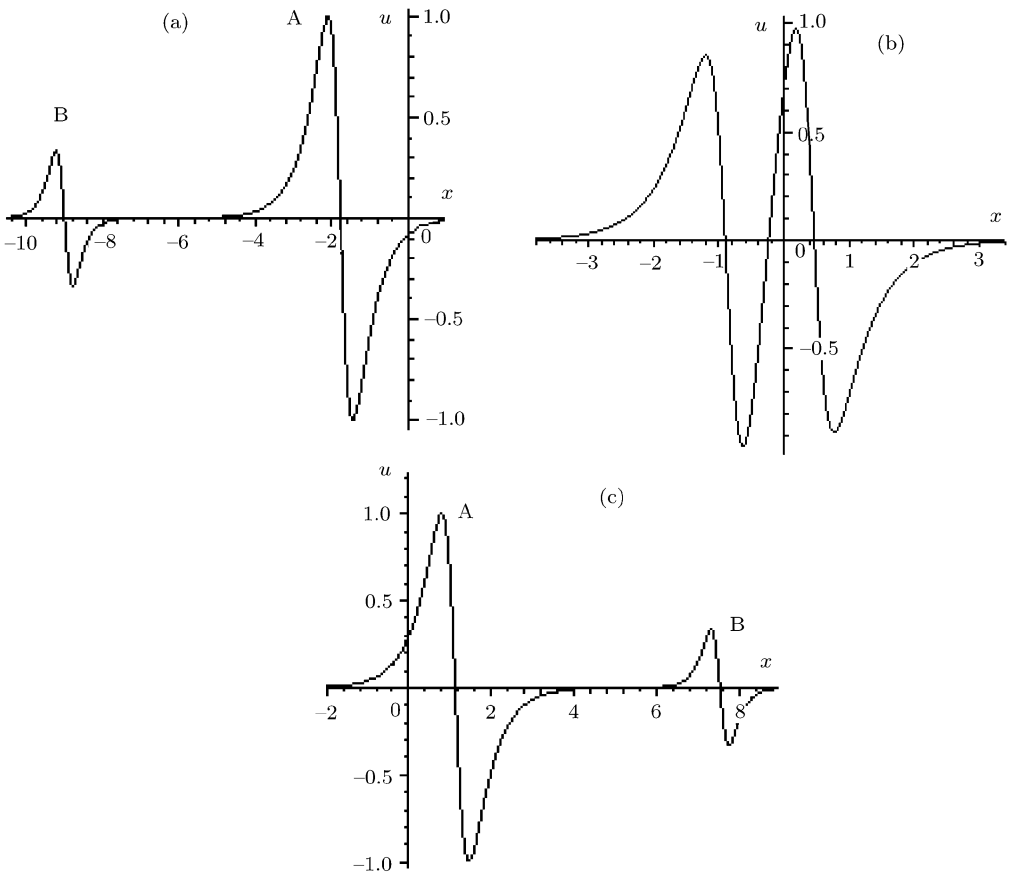


Fig.2. The waveforms of two solitons interacting for the quantity u determined by expressions (17), (14), and (18) at times: –0.55 (a), 0.02 (b) and 0.45 (c).

We use Fig.2 for the quantity u and Fig.3 for the quantity v corresponding to the parameter selections shown in expressions (18) and (19), respectively, to illustrate in detail the interactive phenomena between two solitons. In the two groups of plots, Figs.2(a) and 3(a) both display the states before two solitons meet; Figs.2(b) and 3(b) exhibit the impacted and superposed states in the interactive area; Figs.2(c) and 3(c) represent the states after collision.

In Figs.2 and 3, at first, the soliton B possesses a faster velocity. After soliton B has caught up with soliton A, they interact with each other and their solitary waves are gradually superposed on each other. A short time later, soliton B passes through and surpasses soliton A and runs ahead. Then they are separated from each other and resume their original behaviours just like those before collision as shown in Figs.2(c) and 3(c). Finally, they both leave the interactive area, and then they will not meet each other any longer. From the shape and the changes in amplitude and velocity it follows that they do not exchange their physical quantity, but undergo a phase shift. So this kind of interaction resembles the elastic collision.

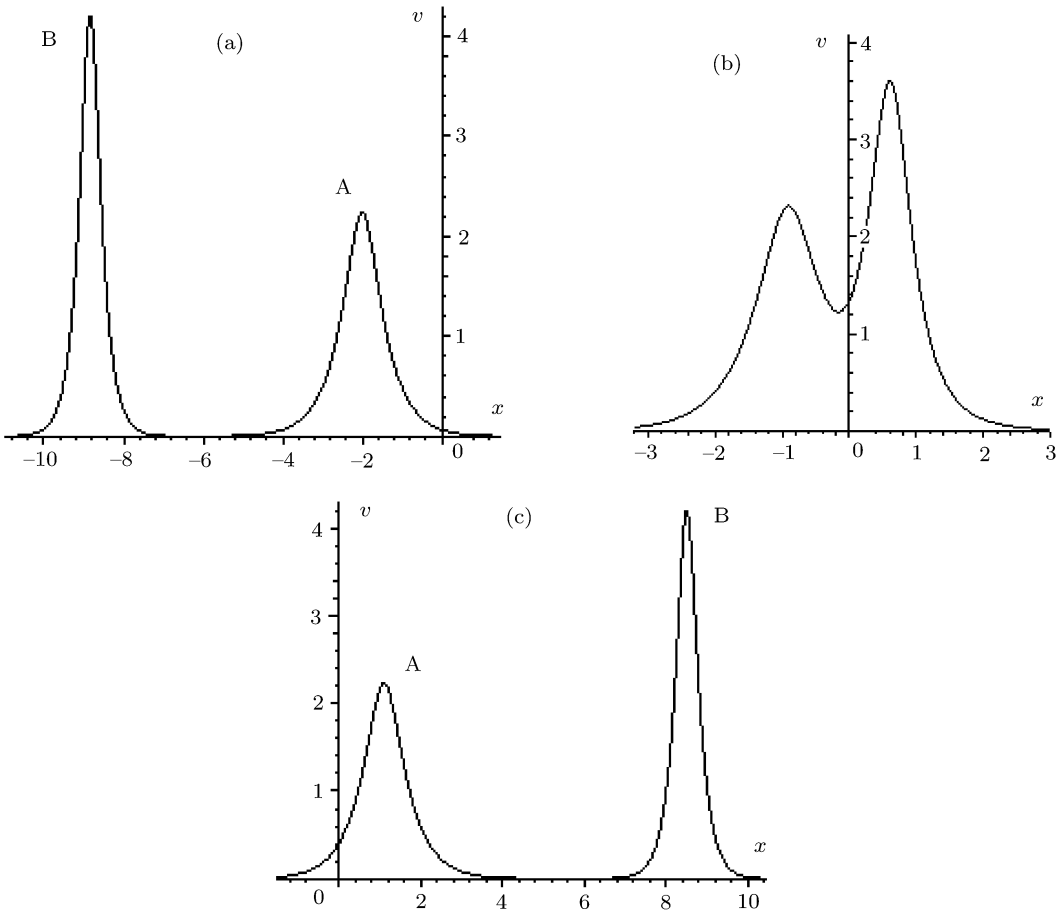


Fig.3. The waveforms of two solitons interacting for the quantity v determined by expressions (17), (14), and (19) at times: –0.55(a), 0 (b), and 0.5 (c).

2.3. Three-soliton case

Based on the foregoing soliton structure, we can construct the solutions F and G as

$$
\begin{aligned}
F_{13} &= 1 + \mathrm{e}^{n_1} + \mathrm{e}^{n_2} + \mathrm{e}^{n_3} + a_{12}\mathrm{e}^{n_1+n_2} + a_{13}\mathrm{e}^{n_1+n_3} + a_{23}\mathrm{e}^{n_2+n_3} + a_{123}\mathrm{e}^{n_1+n_2+n_3},\\
G_{13} &= 1 - \mathrm{e}^{n_1} - \mathrm{e}^{n_2} - \mathrm{e}^{n_3} + a_{12}\mathrm{e}^{n_1+n_2} + a_{13}\mathrm{e}^{n_1+n_3} + a_{23}\mathrm{e}^{n_2+n_3} - a_{123}\mathrm{e}^{n_1+n_2+n_3},\\
F_{23} &= c_1\,\mathrm{e}^{n_1} + c_2\,\mathrm{e}^{n_2} + c_3\,\mathrm{e}^{n_3} + c_{12}\mathrm{e}^{n_1+n_2} + c_{13}\mathrm{e}^{n_1+n_3} + c_{23}\mathrm{e}^{n_2+n_3} + c_{123}\mathrm{e}^{n_1+n_2+n_3},\\
G_{23} &= -c_1\,\mathrm{e}^{n_1} - c_2\,\mathrm{e}^{n_2} - c_3\,\mathrm{e}^{n_3} + c_{12}\mathrm{e}^{n_1+n_2} + c_{13}\mathrm{e}^{n_1+n_3} + c_{23}\mathrm{e}^{n_2+n_3} - c_{123}\mathrm{e}^{n_1+n_2+n_3},
\end{aligned}
\tag{22}
$$

where the coupled coefficients a_{123}, a_{ij}, c_{123} and c_{ij} $(i,j=1,2,3)$ are all real constant parameters; n_i is defined as that in expression (15). Similarly, if c_i, c_{ij} and c_{123} are all taken to be zero, the solutions F_{13} and G_{13} become 3SS in the mKdV bilinear form for Eq.(3). Substituting expressions (22) and (15) into Eq.(6), we find that the existence of multiple-soliton solutions is very restrictive and these coupled coefficients must satisfy the following additional condition:

$$a_{ij}=-\frac{(c_ic_j-1)\ (p_i-p_j)^2}{(p_i+p_j)^2},$$
$$c_{ij}=\frac{(c_i+c_j)\ (p_i-p_j)^2}{(p_i+p_j)^2},\quad (i,j=1,\,2,\,3)$$

$$a_{123}=-\frac{(c_1c_2+c_1c_3+c_2c_3-1)\,(p_1-p_2)^2\,(p_1-p_3)^2\,(p_2-p_3)^2}{(p_1+p_2)^2\,(p_1+p_3)^2\,(p_2+p_3)^2},$$

and

$$c_{123}=-\frac{(c_1c_2c_3-c_1-c_2-c_3)\,(p_1-p_2)^2\,(p_1-p_3)^2\,(p_2-p_3)^2}{(p_1+p_2)^2\,(p_1+p_3)^2\,(p_2+p_3)^2}. \tag{23}$$

Substituting expressions (22), (15) and (23) into expression (5), we obtain the quantities u and v of 3SS for Eq.(2)

$$u=\frac{1}{2}\left(\ln\frac{F_{13}^2+F_{23}^2}{G_{13}^2+G_{23}^2}\right)_x,$$

and

$$v=\left(\arctan\frac{F_{23}}{F_{13}}-\arctan\frac{G_{23}}{G_{13}}\right)_x. \tag{24}$$

According to this group of solutions, we obtain the evolution plots of 3SS as shown in Fig.4 by selecting the following same constant parameters:

$$p_1=2,\quad p_2=3,\quad p_3=4,\quad c_1=2,\quad c_2=6,\quad c_3=10,$$

$$m_1=\quad m_2=m_3=0. \tag{25}$$

From Fig.4, we see that these solitary waves all move along the positive x direction, undergo a phase shift and pass through each other. Especially, as shown in Fig.4(b), the vibration directions of three solitons are the same under those parameters chosen in the way above. The interactive phenomena are similar to those of the two solitons described in subsection 2.2, which all elastically interact with each other. Here, we do not discuss their evolution and interactive behaviour any more.

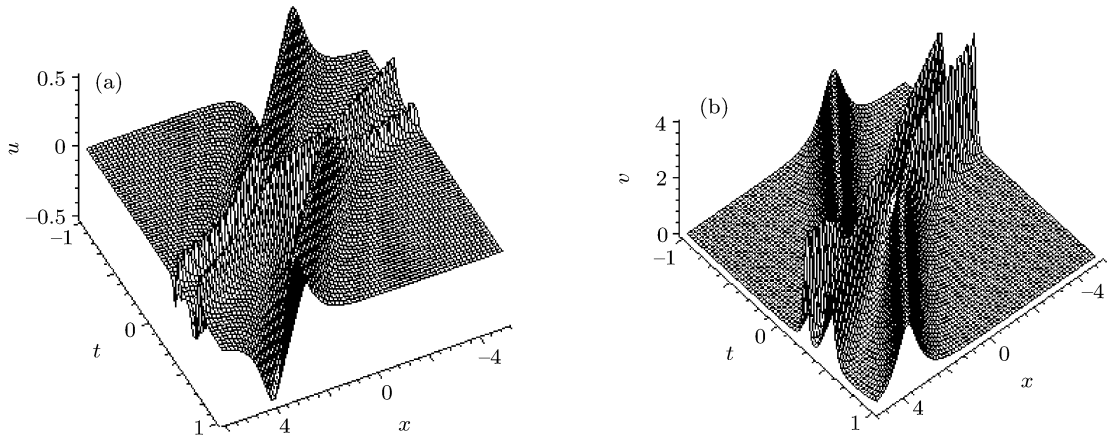


Fig.4. The spatiotemporal evolution plots of 3SS determined by expression (24) together with expressions (22), (23) and (25) for the quantities u(a) and v(b).

3. Summary and discussion

We use Hirota's bilinear method to construct the solutions for the special coupled mKdV system (2), and derive its new soliton solutions successfully. Furthermore, we study mainly the physical properties of two-soliton solutions such as spatiotemporal evolution, waveform structure, and interactive phenomena. We find some new interesting interactive phenomena and know that solitary waves have different interactive behaviours under different initial values and boundary conditions even with the same quantity. As indicated in Figs.2–4, in this special coupled mKdV system the collision between two solitons represents the elastic interaction, which corresponds to the elastic collision between classical particles to some extent. This result is similar to that in the mKdV system.[24] On the other hand, because Eq.(2) is a special case of Eq.(1), the soliton solutions and phenomena of Eq.(2) have some special action behaviours included in Eq.(1), thus they can response to certain motion phenomena appearing in a two-layer fluid system.

According to the corresponding relations of transform and bilinear solutions between Eq.(2) and Eq.(3), we can suppose that the special coupled mKdV system (2) may have all the integrable properties of the mKdV Eq.(3). As for the new phenomenon occurring in Fig.1(d), and other new soliton solutions and physical characters for systems (2) and (3), we shall discuss them in the near future.

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