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Generation of cluster states for cavity fields*

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This paper proposes an alternative scheme for generating cluster-type of entangled coherent states. This scheme is based on resonant interaction of a two-mode cavity with a two-level atom driven by strong classical fields. Thus the required interaction time is greatly shortened, which is very important in view of decoherence.

Keywords: cluster state, coherent state, bimodal cavity
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Quantum entanglement, one of the intriguing features of quantum mechanics, plays a fundamental role in the demonstration of quantum nonlocality.[1] It is also recognized nowadays as a very useful resource in quantum information and computation, such as secret key distribution,[2] teleportation,[3] and dense coding.[4] Due to its diverse and promising applications, much efforts have been devoted to the generation of entangled states both theoretically and experimentally. Most of the researches in quantum information processing is based on quantum entanglement of two qubits. In the past few years, there was much interest in quantum entanglement of many qubits. For instance, schemes have been proposed for the generation of the multi-atom Greenberg–Horne–Zeilinger (GHZ) state and W state.[5] Recently Briegel and Raussendorf introduced another class of entangled states, i.e., the cluster states.[6] The cluster states have a large persistency of entanglement and can be regarded as a resource for other multiqubit entangled states. Thus cluster states become an important resource in quantum information. Therefore a number of schemes for generating cluster states have been proposed in both cavity quantum electronic dynamics (QED) and ion trap. Zou and Mathis proposed a scheme for generating four-photon polarization-entangled cluster state,[7] and generating a four-atom cluster state via the resonant atom–cavity interaction.[8] Cho and Lee showed a method for the generation of cluster states based on the cavity input–output process and the single-photon polarization measurement.[9] In a recent paper Zheng has given two schemes for the generation of four-qubit cluster states in ion-trap systems.[10]

More recently Becerra-Castro *et al* proposed an experimental scheme for generating cluster-type entangled coherent states for four cavity modes.[11] The scheme uses two bimodal cavities, a single two-level (Rydberg) atom and a dispersive atom–field interaction. Here we propose an alternative simplified scheme to generate a cluster-type of entangled coherent states in cavity QED. Unlike the scheme of Ref.[11], our scheme requires resonant interaction between the atom and cavity field. And the four cavity modes are initially prepared in the vacuum state. The interaction time is very short, which is important in view of decoherence.

The interaction of a single mode (frequency $\omega_{\rm c}$) in a cavity of high quality factor with a two-level atom (transition frequency ω_0) driven by a classical field (frequency $\omega_{\rm L}$) is described by the Hamiltonian (assuming $\hbar = 1$)[12,13]

$$H = \omega_0 S_z + \omega_{\rm c} a^\dagger a + g(a^\dagger S^- + aS^+) + \Omega(S^+ {\rm e}^{-{\rm i}\omega_{\rm L} t} + S^- {\rm e}^{-{\rm i}\omega_L t}), \quad (1)$$

where a^+ and a are the creation and annihilation operators for the cavity field respectively. $S_z = \frac{1}{2}(|e\rangle\langle e| -$

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$|g\rangle\langle g|$), $S^+ =| e\rangle\langle g|$, and $S^- =| g\rangle\langle e|$, g is the atom–field coupling strength, and Ω is Rabi frequency of classical field. Under the condition $\omega_0 = \omega = \omega_{\rm L}$, in the interaction picture, we obtain interaction Hamiltonian

$$H_{\rm I} = g_a(a^\dagger S^- + aS^+) + \Omega(S^+ + S^-). \tag{2}$$

Define the new atomic bases

$$\begin{aligned} |+\rangle &= \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle), \\ |-\rangle &= \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle), \end{aligned} \tag{3}$$

then we can rewrite $H^{\rm L}$ as

$$\begin{aligned} H^{\rm L} = {} & \Omega(|+\rangle\langle +|-|-\rangle\langle -|) \\ & + \frac{g_a}{2}[a^\dagger(|+\rangle\langle +|-|-\rangle\langle -|+|-\rangle \\ & \times\langle +|-|+\rangle\langle -|) + \text{H.C}]. \end{aligned} \tag{4}$$

The Hamiltonian $H^{\rm L}$ in the new interaction picture yields

$$\begin{aligned} H_{\rm I}^1 = {} & \frac{g_a}{2}[a^\dagger(|+\rangle\langle +|-|-\rangle\langle -|+\mathrm{e}^{-2\mathrm{i}\Omega t}|-\rangle \\ & \langle +|-\mathrm{e}^{2\mathrm{i}\Omega t}|+\rangle\langle -|) + \text{H.C}], \end{aligned} \tag{5}$$

where $H_0^{\rm L} = \Omega(|+\rangle\langle +|-|-\rangle\langle -|)$. In the strong regime, i.e. $\Omega \gg g$, we can neglect the terms oscillating fast. Then the effective Hamiltonian can be written as

$$H_{\rm eff}^1 = \frac{1}{2}g(a^\dagger + a)(|+\rangle\langle +|-|-\rangle\langle -|), \tag{6}$$

thus the evolution operators for the system can be written as

$$U = \mathrm{e}^{-\mathrm{i}H_0^{\rm L}t}\mathrm{e}^{-\mathrm{i}H_{\rm eff}^1 t}. \tag{7}$$

Now let us describe our scheme. Figure 1 displays the setup for the generation of cluster-type entangled coherent state. The atom is produced in zone B by the excitation of a velocity-selected atomic beam effusing from oven O. C_1 and C_2 represent two identical bimodal cavities, F_1, F_2, F_3, F_4 are four classical fields. R is a classical field tuned to the transitions $|g\rangle \to \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)$, $|e\rangle \to \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle)$, respectively. $D_e(D_g)$ stand for atomic ionization detector for the state $|e\rangle$ ($|g\rangle$). The two bimodal cavities sustain two nondegenerate orthogonally polarized modes, A and B in C_1 (C and D in C_2).[14]

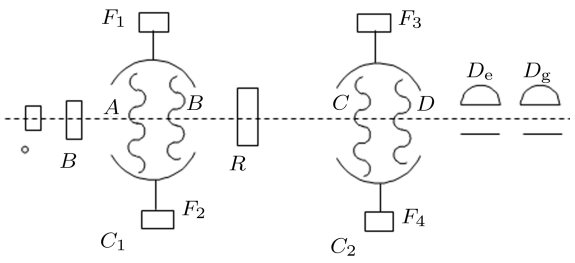


Fig.1. The setup for the generation of cluster-type entangled coherent state.

To generate the cluster states in the four modes of the two bimodal cavities, we first send the two-level atom (transition frequency ω_{eg}), in a ground state $|g\rangle$, through the cavity C_1. The atom driven by an external classical field F_1 (frequency ω_1) interacts resonantly with mode A (frequency ω_a) in cavity C_1 for an interaction time t_A. The other mode B remains unaffected.[15] Assume that the two cavities are initially in the four-mode vacuum state $|0\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D$. Under the condition $\omega_{eg} = \omega_a = \omega_1$, the system evolves as follows:

$$\begin{aligned} & |\phi^A(t_A)\rangle = U|\phi(0)\rangle \\ = {} & \mathrm{e}^{-\mathrm{i}H_0^{\rm L}t}\mathrm{e}^{-\mathrm{i}H_{\rm eff}^1 t_A}|g\rangle|0\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D \\ = {} & \frac{1}{\sqrt{2}}[\mathrm{e}^{-\mathrm{i}\Omega t_A}|+\rangle|-\alpha\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D \\ & + \mathrm{e}^{\mathrm{i}\Omega t_A}|-\rangle|\alpha\rangle_A|0\rangle_B|0\rangle_C|0\rangle_D], \end{aligned} \tag{8}$$

where $\alpha = \mathrm{i}gt_A/2$.

We then switch off the classical field F_1 and tune the atom resonantly with the mode B. Meanwhile, we drive the atom with another classical field F_2 (frequency $\omega_2 = \omega_{eg}$) in cavity C_1. After an interaction time t_B (assume $t_B = t_A = t$) the state of system is

$$\begin{aligned} & |\phi^B(t_B)\rangle = U|\phi^A(t_A)\rangle \\ = {} & \frac{1}{\sqrt{2}}[\mathrm{e}^{-2\mathrm{i}\Omega t}|+\rangle|-\alpha\rangle_A|-\alpha\rangle_B|0\rangle_C|0\rangle_D \\ & + \mathrm{e}^{2\mathrm{i}\Omega t}|-\rangle|\alpha\rangle_A|\alpha\rangle_B|0\rangle_C|0\rangle_D] \\ = {} & \frac{1}{2}[\mathrm{e}^{-2\mathrm{i}\Omega t}(|g\rangle + |e\rangle)|-\alpha\rangle_A|-\alpha\rangle_B|0\rangle_C|0\rangle_D \\ & + \mathrm{e}^{2\mathrm{i}\Omega t}(|g\rangle - |e\rangle)|\alpha\rangle_A|\alpha\rangle_B|0\rangle_C|0\rangle_D] \\ = {} & \frac{1}{2}[|g\rangle(\mathrm{e}^{-2\mathrm{i}\Omega t}|-\alpha\rangle_A|-\alpha\rangle_B|0\rangle_C|0\rangle_D \\ & + \mathrm{e}^{2\mathrm{i}\Omega t}|\alpha\rangle_A|\alpha\rangle_B|0\rangle_C|0\rangle_D) \\ & + |e\rangle(\mathrm{e}^{-2\mathrm{i}\Omega t}|-\alpha\rangle_A|-\alpha\rangle_B|0\rangle_C|0\rangle_D \\ & - \mathrm{e}^{2\mathrm{i}\Omega t}|\alpha\rangle_A|\alpha\rangle_B|0\rangle_C|0\rangle_D)]. \end{aligned} \tag{9}$$

Here we have assumed $g_{\rm a} = g_{\rm b} = g$, $\Omega_1 = \Omega_2$. When the atom come out from the cavity C_1 it crosses a Ramsey zone R, tuned to transitions $|g\rangle \to \frac{1}{\sqrt{2}}(|g\rangle +$

$|e\rangle) = |+\rangle$ and $|e\rangle \to \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle) = |-\rangle$. In this way we obtain

$$\begin{aligned} &|\,\phi^B(t_B)\rangle \\ &= \frac{1}{2}[|+\rangle\,(\mathrm{e}^{-2\mathrm{i}\Omega t}\,|-\alpha\rangle_A\,|-\alpha\rangle_B\,|0\rangle_C\,|0\rangle_D \\ &\quad +\mathrm{e}^{2\mathrm{i}\Omega t}\,|\alpha\rangle_A\,|\alpha\rangle_B\,|0\rangle_C\,|0\rangle_D) + \\ &\quad |-\rangle\,(\mathrm{e}^{-2\mathrm{i}\Omega t}\,|-\alpha\rangle_A\,|-\alpha\rangle_B\,|0\rangle_C\,|0\rangle_D \\ &\quad -\mathrm{e}^{2\mathrm{i}\Omega t}\,|\alpha\rangle_A\,|\alpha\rangle_B\,|0\rangle_C\,|0\rangle_D)]. \end{aligned} \tag{10}$$

Next the atom crosses cavity C_2 and interacts resonantly with the modes C and D, and classical fields F_3 (frequency $\omega_3 = \omega_{eg}$) and F_4 (frequency $\omega_4 = \omega_{eg}$), respectively. In the same way as that occurred in C_1, then the whole state of the system is

$$\begin{aligned} &|\,\phi(t)\rangle_{\mathrm{tot}} = \frac{1}{2}[|+\rangle\,(\mathrm{e}^{-4\mathrm{i}\Omega t}\,|-\alpha\rangle_A\,|-\alpha\rangle_B\,|-\alpha\rangle_C\,|-\alpha\rangle_D + |\alpha\rangle_A\,|\alpha\rangle_B\,|-\alpha\rangle_C\,|-\alpha\rangle_D) \\ &\quad +|-\rangle\,(|-\alpha\rangle_A\,|-\alpha\rangle_B\,|\alpha\rangle_C\,|\alpha\rangle_D - \mathrm{e}^{4\mathrm{i}\Omega t}\,|\alpha\rangle_A\,|\alpha\rangle_B\,|\alpha\rangle_C\,|\alpha\rangle_D)] \\ &= \frac{1}{2\sqrt{2}}[|g\rangle\,(\mathrm{e}^{-4\mathrm{i}\Omega t}\,|-\alpha\rangle_A\,|-\alpha\rangle_B\,|-\alpha\rangle_C\,|-\alpha\rangle_D + |\alpha\rangle_A\,|\alpha\rangle_B\,|-\alpha\rangle_C\,|-\alpha\rangle_D \\ &\quad +|-\alpha\rangle_A\,|-\alpha\rangle_B\,|\alpha\rangle_C\,|\alpha\rangle_D - \mathrm{e}^{4\mathrm{i}\Omega t}\,|\alpha\rangle_A\,|\alpha\rangle_B\,|\alpha\rangle_C\,|\alpha\rangle_D) \\ &\quad +|e\rangle\,(\mathrm{e}^{-4\mathrm{i}\Omega t}\,|-\alpha\rangle_A\,|-\alpha\rangle_B\,|-\alpha\rangle_C\,|-\alpha\rangle_D + |\alpha\rangle_A\,|\alpha\rangle_B\,|-\alpha\rangle_C\,|-\alpha\rangle_D \\ &\quad +\mathrm{e}^{4\mathrm{i}\Omega t}\,|\alpha\rangle_A\,|\alpha\rangle_B\,|\alpha\rangle_C\,|\alpha\rangle_D - |-\alpha\rangle_A\,|-\alpha\rangle_B\,|\alpha\rangle_C\,|\alpha\rangle_D]. \end{aligned} \tag{11}$$

Choosing the Rabi frequency Ω approximately so that $\Omega t = 2k\pi$ is fulfilled, we have

$$\begin{aligned} &|\,\phi(t)\rangle_{\mathrm{tot}} = \frac{1}{2\sqrt{2}}[|g\rangle\,(|-\alpha\rangle_A\,|-\alpha\rangle_B\,|-\alpha\rangle_C\,|-\alpha\rangle_D + |\alpha\rangle_A\,|\alpha\rangle_B\,|-\alpha\rangle_C\,|-\alpha\rangle_D \\ &+|-\alpha\rangle_A\,|-\alpha\rangle_B\,|\alpha\rangle_C\,|\alpha\rangle_D - |\alpha\rangle_A\,|\alpha\rangle_B\,|\alpha\rangle_C\,|\alpha\rangle_D) \\ &+|e\rangle\,(|-\alpha\rangle_A\,|-\alpha\rangle_B\,|-\alpha\rangle_C\,|-\alpha\rangle_D + |\alpha\rangle_A\,|\alpha\rangle_B\,|-\alpha\rangle_C\,|-\alpha\rangle_D \\ &+|\alpha\rangle_A\,|\alpha\rangle_B\,|\alpha\rangle_C\,|\alpha\rangle_D - |-\alpha\rangle_A\,|-\alpha\rangle_B\,|\alpha\rangle_C\,|\alpha\rangle_D]. \end{aligned} \tag{12}$$

We now detect the state of the atom. If we find the atom in the state $|g\rangle$, then the field collapses onto the four-qubit coherent cluster state

$$\begin{aligned} |\,\varphi\rangle_{\mathrm{c}} = \frac{1}{2}&(|-\alpha\rangle_A\,|-\alpha\rangle_B\,|-\alpha\rangle_C\,|-\alpha\rangle_D + |\alpha\rangle_A\,|\alpha\rangle_B\,|-\alpha\rangle_C\,|-\alpha\rangle_D \\ &+|-\alpha\rangle_A\,|-\alpha\rangle_B\,|\alpha\rangle_C\,|\alpha\rangle_D - |\alpha\rangle_A\,|\alpha\rangle_B\,|\alpha\rangle_C\,|\alpha\rangle_D). \end{aligned} \tag{13}$$

Finally we give a brief discussion on the experimental realization of the proposed scheme. To generate the cluster state $|\varphi\rangle_{\mathrm{c}}$, the relations $|\alpha| = gt/2$ and $\Omega t = 2k\pi$ must be fulfilled. Setting $g \simeq 2\pi \times 51\,\mathrm{kHz}$,[16] $\Omega = 10\,g$, $k = 7$, and $|\alpha| = 2$ (when $|\alpha| = 2$, $\langle\alpha \mid -\alpha\rangle \simeq 0$), we can obtain $t \simeq 14\,\mu\mathrm{s}$. The total travelling time of the atom is about $T_{\mathrm{tot}} = 10 \times 4t = 560\,\mu\mathrm{s}$. For Rydberg atom with principle quantum numbers 51 (level $|e\rangle$) or 50 (level $|g\rangle$), the lifetime has the order of $T_{\mathrm{a}} = 30\,\mathrm{ms}$. In a recent experiment,[17] cavity decay time is $T_{\mathrm{C}} = 130\,\mathrm{ms}$. For the state $|\varphi\rangle_{\mathrm{c}}$, the decoherence time is about $T_{\mathrm{d}} = T_{\mathrm{C}}/2\,|\alpha|^2 \simeq 16\,\mathrm{ms}$. Note that the total time T_{tot} is less than the decoherence time T_{d} and the radiative time T_{a}. So the scheme maybe experimentally feasible within the realm of microwave.

In conclusion, we have proposed an alternative scheme to generate the cluster-type of entangled coherent states. It is based on resonant interaction of two two-mode cavities with a two-level atom driven by strong classical fields. In our scheme the cavity modes are initially in vacuum states. Since the atom resonantly interacts with two cavity modes, the required interaction time is very short. This is very important in view of decoherence.

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