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Study of dynamical behaviour and fermionization of a bosonic gas in funnel potential*

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This paper presents a funnel external potential model to investigate dynamic properties of ultracold Bose gas. By using variational method, we obtain the ground-state energy and density properties of ultracold Bose atoms. The results show that the ultracold Bose gas confined in a funnel potential experiences the transition from three-dimensional regime to quasi-one-dimensional regime in a small aspect ratio, and undergoes fermionization process as the aspect ratio increases.

Keywords: Bose gas, fermionization, dynamics
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1. Introduction

Since the experimental realization of Bose–Einstein condensate (BEC) in the gas of ultracold alkali-atoms confined in the magnetic traps, physics of ultracold Bose gases has attracted considerable experimental and theoretical attention.[1−4] The recent remarkable achievements in experimental and theoretical investigations have been made.[5,6] The development of trapping and cooling techniques has enabled experimental realizations of low-dimensional gases in both one and two dimensions.[7−9] Compared with three-dimensional system, we find that one dimensional (1D) system has unusual features, the more important the inter-particle interaction become, the more dilute the gas is. Particularly, at sufficiently low densities and strongly interacting limit the Bose gas enters so-called Tonks–Girardeau (TG) regime, where the boson particles behave as the properties of spinless fermions and become impenetrable bosons.[10] However, such bosons do not exhibit completely ideal fermionic quantum behaviour, for instance, this is reflected in their characteristic momentum. Recently, Paredes *et al* firstly have achieved TG regime experimentally. They measured the momentum distribution of bosons, and found that it agreed closely with the theoretical prediction.[11] In order to realize this particular regime, rather strict conditions for the temperature, gas density, interaction potential, and trapping potential must be satisfied.[12] So far, the researches of TG regime have been focused on the homogeneous Bose gas loaded in 1D optical-lattice trap or highly elongated cylindrical trap.[11,13−16]

In this paper we propose a novel way to study fermionization loading ultracold Bose gases in a funnel shaped trapping potential. Ultracold Bose gas is loaded conveniently, and realizes the crossover from three-dimensional regime to quasi-1D regime. Over the past year, various authors employed the Lieb–Liniger model and local density approximation to investigate the properties of TG gases. Recently, Das *et al* [13] have proposed and demonstrated a variational theory to study the crossover from the three-dimensional regime to the 1D regime for ultracold Bose gases. The variational technique gives reliable results even in the large condensate number limit.[13,17] We will employ this variational approach to study the properties of ultracold Bose gases confined in a funnel potential, and search for a new method of fermionizing

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ultracold Bose gas.

The rest of this paper is organized as follows. In Section 2, we describe the variational calculation for the ground-state of the ultracold Bose gas confined in the funnel shaped tapping potential. In Section 3, we give the ground-state structural properties of bosons and compare with that of bosons under highly elongated confinement. Finally, we draw some conclusions in Section 4.

2. Energy of ground state

We consider a system in which N spinless bosons (e.g. ^{87}Rb atoms with frozen spin degrees of freedom) of mass m are confined in an external trap potential $V_{\rm ext}(\boldsymbol{r})$, and interacting via the two-body potential $V_{\rm int}(\boldsymbol{r}_1, \boldsymbol{r}_2)$. The Hamiltonian for this system is

$$H = \sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\nabla_i^2 + V_{\rm ext}(\boldsymbol{r}_i)\right] + \sum_{i<j}^{N} V_{\rm int}(\boldsymbol{r}_i, \boldsymbol{r}_j), \quad (1)$$

here, we consider the funnel shaped trapping potential, satisfying

$$V_{\rm ext}(\boldsymbol{r}) = -\frac{1}{2}m\omega_\perp^2 a_\perp^3 \frac{1}{\rho'} + \frac{1}{2}m\omega_z^2 z'^2, \quad (2)$$

where $\rho' = \sqrt{x^2+y^2}$ is the radial coordinate, ω_z is corresponding axial angular frequency, $a_\perp = \sqrt{\hbar/(m\omega_\perp)}$ and $\omega_\perp$ the parameters of the radial funnel potential. Here when $\rho' = a_\perp$, we can have the same field force of funnel as one of harmonic oscillator at the same oscillating length. When $\rho' < a_\perp$, the constraint of funnel potential is stronger than that of the harmonic oscillator. The transverse confinement is much stronger towards the bottom of funnel potential. Bosons are tightly confined in the transverse direction, so that the transverse excitations are restrained, which realizes the crossover from the weak correlation system to the strong correlation system by adjusting the longitudinal confinement.

Interaction between bosons confined in the funnel shaped potential is very complex. With the pseudopotential method, the interatomic interaction is approximated by a delta potential.[18]

$$V_{\rm int}(|\boldsymbol{r}-\boldsymbol{r}'|) = g\delta(\boldsymbol{r}-\boldsymbol{r}'), \quad (3)$$

where $g = 4\pi\hbar^2 a_{\rm s}/m$ is the effective nonlinear coupling constant which characterizes the strength of interatomic interaction, $a_{\rm s}$ denotes the three-dimensional s-wave scattering length. When $a_{\rm s} > 0$, the interaction between the particles is repulsive. In the opposite case ($a_{\rm s} < 0$) the interaction is attractive. The Fermi pseudopotential gives the correct dilute gas limit.[19]

At zero temperature, the dilute Bose condensate gas confined in the funnel shaped potential is described by the wavefunction that minimizes the ground state energy functional. In order to separate a transverse and a longitudinal part, we choose the cylindrical coordinates. The ground state energy functional is

$$E'(\psi) = \int 2\pi\rho' \mathrm{d}\rho' \int \mathrm{d}z' \left[\frac{\hbar^2}{2m}|\nabla\psi(\rho', z')|^2 + V(\rho', z')|\psi(\rho', z')|^2 + \frac{g}{2}|\psi(\rho', z')|^4\right]. \quad (4)$$

Where $\psi(\rho', z')$ is the ground-state wavefunction of the Bose gas, and the normalization for $\psi(\rho', z')$ is taken as

$$\int |\psi(\rho', z')|^2 2\pi\rho' \mathrm{d}\rho' \mathrm{d}z' = N, \quad (5)$$

where N is the number of atoms confined in the trap. The ground state characteristic of the Bose gas is mainly determined by the ground-state wavefunction.

In order to facilitate the analysis and calculation, we rescale the spatial coordinate, energy and the wavefunction, introducing the following transformation:

$$\begin{aligned} &\rho' = a_z\rho, \\ &z' = a_z z, \\ &E' = \hbar\omega_z E, \\ &\psi(\rho', z') = \sqrt{\frac{N}{a_z^3}}\psi(\rho, z), \end{aligned} \quad (6)$$

where $a_z = \sqrt{\hbar/(m\omega_z)}$ is the oscillator length in the longitudinal direction as the standard length, $\psi(\rho, z)$ satisfies the normalization condition in the new coordinate. We introduce the asymmetry parameter, which is the aspect ratio, as the ratio of the radial and the axial trapping frequencies $L = \omega_\perp/\omega_z$. The interaction strength of the Bose gas is determined by the scaled scattering length $a_{\rm s}/a_z$ and by the number N of the bosons, so we define the quantity $\gamma = Na_{\rm s}/a_z$. Based on the above transformations, one can rewrite Eq.(4) in a dimensionless form.

$$\frac{E(\psi)}{N} = \int \pi\rho \mathrm{d}\rho \int \mathrm{d}z \left[|\nabla\psi(\rho, z)|^2 + \left(-\frac{\sqrt{L}}{\rho} + z^2\right) \times |\psi(\rho, z)|^2 + 4\pi\gamma|\psi(\rho, z)|^4\right]. \quad (7)$$

The variational technique based on the Gaussian trial functions leads to the reliable results.[20,21]

Next we take a variational approach with an appropriate trial wavefunction that contains some parameters. The trial wavefunction for the ground-state is taken a Gaussian form

$$\psi(\rho, z) = C\exp\left[-\left(\frac{\rho^2}{2\alpha^2} + \frac{z^2}{2\beta^2}\right)\right]. \tag{8}$$

Where α and β are variational parameters and C is the normalization constant given by

$$C = \sqrt{\frac{1}{\sqrt{\pi^3}\alpha^2\beta}}. \tag{9}$$

The variational parameters α and β will be determined by minimizing the energy functional after integration in the transverse plane and longitudinal direction.

By inserting the trial wavefunction in Eq.(7) and after spatial integration along the transverse and longitudinal direction, the energy function becomes

$$\begin{aligned}\frac{E}{N} =& \frac{1}{2\alpha^2} - \frac{1}{2\alpha}\sqrt{L\pi} + \frac{1}{4}\left(\frac{1}{\beta^2} + \beta^2\right) \\ &+ \frac{1}{\alpha^2\beta}\frac{\gamma}{\sqrt{2\pi}}.\end{aligned} \tag{10}$$

Minimizing the energy function with respect to α and β, one reads

$$\begin{aligned}\beta &= \frac{4\gamma}{\sqrt{2\pi}\left(\sqrt{L\pi}\alpha - 2\right)}, \\ \alpha^2 &= \frac{2\gamma\beta}{\sqrt{2\pi}\left(\beta^4 - 1\right)}.\end{aligned} \tag{11}$$

We obtain the coupled equations about α and β, which determine the energy spectrum.

3. Result and discussion

We consider N=3 and 10 ^{87}Rb atoms confined in a funnel shaped tapping potential, whose scattering length is a_s=109a.u.. The longitudinal frequency of the potential is ν_z=77.87 Hz, corresponding to the oscillator length a_z=23107a.u..[22] The parameter γ=0.0142 (N=3), 0.0472 (N=10) characterizes the interaction strength of the Bose gas trapped in the funnel potential. We fix the longitudinal frequency, and the variation of the aspect ratio is supposed to be realized through the variation of the transversal parameter.

In Section 2, we have obtained variational parameters α and β. Then taking the above parameters into Eq.(8) and integrating with respect to the transversal coordinate, we obtain the longitudinal density distribution of the ultracold Bose gas in funnel potential

$$n(z) = \frac{1}{\sqrt{\pi}\beta}\exp\left(-\frac{z^2}{\beta^2}\right). \tag{12}$$

For one dimensional ideal bosonic gas, density distribution is

$$n_0^{B,z}(z) = \frac{1}{\sqrt{\pi}a_z}\exp\left(-\frac{z^2}{a_z^2}\right). \tag{13}$$

The ideal fermionic gas density distribution is given by the sum of squares of the single-particle wavefunctions[22]

$$n_0^{F,z}(z) = \frac{1}{N}\frac{1}{\sqrt{\pi}a_z}\sum_{k=0}^{N-1}\frac{1}{2^k k!}H_k^2\left(\frac{z}{a_z}\right)\exp\left(-\frac{z^2}{a_z^2}\right), \tag{14}$$

where the H_k denote Hermite polynomials, for N=3 the ideal fermionic gas density distribution is

$$n_0^{F,z}(z) = \frac{1}{\sqrt{\pi}a_z}\left[\frac{1}{2} + \frac{2}{3}\left(\frac{z}{a_z}\right)^4\right]\exp\left(-\frac{z^2}{a_z^2}\right). \tag{15}$$

In Fig.1 we plot the density distribution as a function of the z coordinate for N=3 for the five different aspect ratios, L=10, 50, 100, 500 and 1000. z in the units of a_z^{-1}, where a_z=23107a.u.. We compare the longitudinal density distribution of the ultracold bosonic gas trapped in the funnel potential with that of one dimensional ideal fermionic and bosonic gas, and find some interesting phenomena. The behaviour of the ultracold Bose gas changes dramatically as a function of the aspect ratio L, the density becomes flatter and broader with L increasing. At L=1000, the density of the ultracold Bose gas approaches the density of ideal fermionic gas. However, the density of the ultracold Bose gas is close to the density of ideal bosonic gas at a lower L.

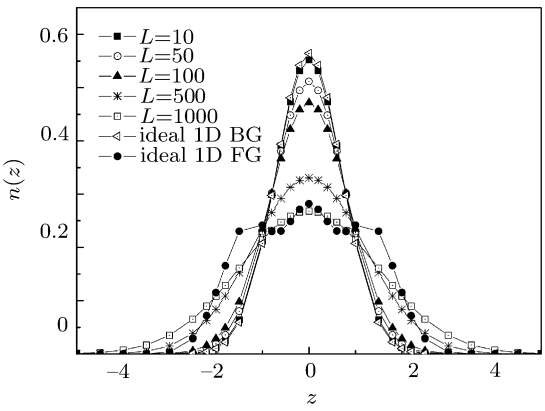


Fig.1. The density $n(z)$ for N=3 as a function of z for five different aspect ratios L=10, 50, 100, 500 and 1000. z is measured in units of a_z^{-1}, where a_z=23107a.u.

The Bose density changes significantly with the increase of L and N. Figure 2 shows the density of the ultracold Bose gas as a function of the z coordinate for five different aspect ratios, L=10, 100, 1000, 1500, 2000 when N=10. Figure 3 shows the density of the ultracold Bose gas as a function of the z coordinate five different aspect ratios, L=10, 100, 1000, 3000 and 4000 when N=30. With the increase of L, the density $n(z)$ is very close to the ideal Fermi density that corresponds to N=10, 30 respectively in Eq.(14). Contrarily, it approaches one-dimensional ideal Bose density. With the increase of N, when the Bose density approaches the ideal Fermi density, we find that for N=30 the aspect ratio is larger than that for N=10. Therefore, it needs a stronger transversal confinement for a larger N to realize fermionization of a bosonic gas.

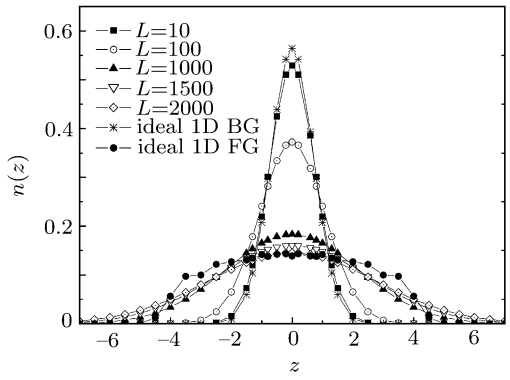


Fig.2. The density $n(z)$ for N=10 as a function of z for five different aspect ratios L=10, 100, 1000, 1500 and 2000. z in units of a_z^{-1}, where a_z=23107a.u.

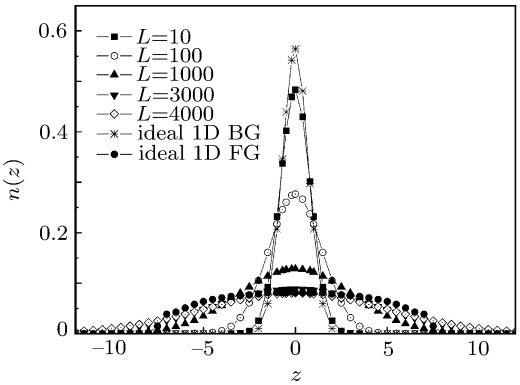


Fig.3. The density n(z) for N=30 as a function of z for five different aspect ratios L=10, 100, 1000, 3000 and 4000. z is measured in units of a_z^{-1}, where a_z=23107a.u.

Reference [22] has shown the density of bosons under highly elongated confinement. Comparing with the density of bosons under the funnel shaped confinement, we find that for N=3 the aspect ratio of the funnel shaped confinement is much less than that of highly elongated confinement, respectively, they enter the quasi-1D regime. Although experiment techniques will be further advanced, there will always be an upper limit on the aspect ratio. So we try to obtain transition of three-dimensional regime of the ultracold Bose gas to quasi-1D regime in a smaller aspect ratio.

4. Summary and conclusion

We have studied the ground-state properties of the ultracold Bose gas confined in a funnel shaped trapping potential. By using the variational approach, we have obtained the energy spectrum and longitudinal density distribution and discussed the density in detail for N=3, 10 and 30. It is found that the ultracold Bose gas undergoes fermionization as spatial ratio increases, and it needs a stronger transversal confinement for a larger N to obtain a fermionizing bosonic gas. By comparing with the Bose gas under highly elongated confinement, we found that it is easier to realize the crossover from three-dimensional regime to quasi-1D regime for the Bose gas in a funnel shaped potential. The study of Bose gas confined in the funnel potential does not show any density oscillation with a Gaussian trial wave function. It will be interesting to further study with different trial wave functions, or other approaches.

Using a current-carrying wire with a constant bias field, one can get a funnel potential experimentally,[23−26] then implement fermionization of Bose gas. The realization of TG gas has triggered extraordinary interests in the quantum information, quantum computation, quantum coherence and strongly correlated system. It is certain that TG gas will be widely applied in various fields in future.

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