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High-codimensional static bifurcations of strongly nonlinear oscillator*

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The static bifurcation of the parametrically excited strongly nonlinear oscillator is studied. We consider the averaged equations of a system subject to Duffing–van der Pol and quintic strong nonlinearity by introducing the undetermined fundamental frequency into the computation in the complex normal form. To discuss the static bifurcation, the bifurcation problem is described as a 3-codimensional unfolding with Z_2 symmetry on the basis of singularity theory. The transition set and bifurcation diagrams for the singularity are presented, while the stability of the zero solution is studied by using the eigenvalues in various parameter regions.

Keywords: bifurcation, strongly nonlinear, normal form, singularity theory
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1. Introduction

Parametrical excitation can induce rich nonlinear dynamical behaviours and occurs in a large variety of mechanical systems, such as the suspended elastic cable, rolling motion of a ship, or oscillations in supersonic conductors. Thus, in recent years, many attempts have been made to analyse the oscillations, bifurcations, and chaos in a weakly nonlinear parametrically excited system. Bajaj[1] studied a parametrically excited oscillator that included van der Pol as well as Duffing type nonlinearity on the basis of the bifurcation theory. Perkins[2] established a theoretical model that described the response of a nonlinearly suspended cable to small tangential oscillations of one support under combined parametric and external excitations. Yang and Sethna[3] used an averaging method to study the local and global bifurcations in parametrically excited, nearly square plates. Zhang and Yu[4] constructed a parametrically and externally excited mechanical system that could exhibit Homoclinic and Heteroclinic bifurcations, multiple limit cycles, and jumping phenomena in amplitude modulated oscillations.

As compared with the weakly nonlinear oscillation system (WNOS), the disturbing parameter ε may not necessarily be a small quantity for the strongly nonlinear oscillation system (SNOS).[5−7] So some methodological extensions have to be proposed first to carry out the bifurcation analysis. Through defining a new transformation parameter,[8,9] Tang *et al*[10,11] used the modified Lindstedt–Poincaré (MLP) method to transform the SNOS into the WNOS, and then studied a 2-codimensional static bifurcation from the singularity theory in a very creative way. However, there is still needed more attention paid especially to the high-codimensional and more complicated SNOS to deploy the dynamical behaviours and predict the regions of instability in the parameter space.

In the present work, we study the static bifurcation behaviours of a parametrically excited oscillator which includes quintic and Duffing–van der Pol type strong nonlinearity. The averaged equations are obtained for the principal parametric resonance by introducing the undetermined fundamental frequency to replace the natural frequency in the normal form operation.[12,13] The universal unfolding is constructed with the consideration of Z_2 symmetry to obtain a fairly complete depiction of transition set and bifurcation diagrams in various regions of the whole parameter space $\mathbb{R}^3$. In view of the transition set, the double limit points set is obtained analytically by considering the bifurcation problem with constraints,[14,15] other than geometrically.[16] We also determine the

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stable and unstable regions of the trivial solution in the (ϖ, μ) plane by using an algorithm based on the eigenvalues of Jacobi matrix.

2. Averaged equations

We consider a parametrically excited strongly nonlinear oscillator

$$\begin{aligned}&\ddot{u} - Q\sin u(\rho - M\cos u)\\ &= (\mu - \alpha u^2 - \beta u^4 - \gamma\dot{u}^2)\dot{u} + \delta u\cos(\Omega t),\end{aligned} \quad (1)$$

which is of considerable importance: it models the pendulum suspended on a rotating arm.[17] First of all, we construct a power series for the sin$u$ and cos$u$ in Eq.(1) in the vicinity of zero

$$\begin{aligned}&\ddot{u} + \omega_0^2 u\\ &= \varepsilon[\kappa u^3 + \sigma u^5 + (\mu - \alpha u^2 - \beta u^4 - \gamma\dot{u}^2)\dot{u}\\ &\quad + \delta u\cos(\Omega t)],\end{aligned} \quad (2)$$

where $\omega_0 = \sqrt{(M-\rho)Q}$ is regarded as the natural frequency ($MQ > \rho Q$), ε is the disturbing parameter of arbitrary magnitude, then $\kappa = (4M - \rho)Q/6$ and $\sigma = (\rho - 16M)Q/120$.

Using the normal form method to obtain the averaged equations[18] demands to convert Eq.(2) into a differential equation of the first order through complexification in terms of the complex unknown quantities ξ and z. Let

$$u = \xi + \bar{\xi}, \quad \dot{u} = \mathrm{i}\,\omega_{10}\,(\xi - \bar{\xi}), \quad z = \mathrm{e}^{\mathrm{i}\Omega t},$$

and

$$\dot{z} = \mathrm{i}\Omega z, \quad (3)$$

where ω_{10} is the undetermined fundamental frequency. It makes the general normal form method applicable to the analysis of the SNOS by introducing the undetermined fundamental frequency[12,13] to replace the natural frequency ω_0 in the operation. Solving Eq.(3) yields

$$\xi = \frac{1}{2}\left(u - \frac{\mathrm{i}}{\omega_{10}}\dot{u}\right)$$

and

$$\bar{\xi} = \frac{1}{2}\left(u + \frac{\mathrm{i}}{\omega_{10}}\dot{u}\right). \quad (4)$$

Differentiating Eq.(4) with respect to t and using Eqs.(2) and (3), give

$$\begin{aligned}\dot{\xi} &= \frac{\mathrm{i}}{2\omega_{10}}[(\xi + \bar{\xi})\omega_0^2 + (\xi - \bar{\xi})\omega_{10}^2]\\ &\quad - \frac{\mathrm{i}\varepsilon}{2\omega_{10}}\Big\{\kappa(\xi + \bar{\xi})^3 + \sigma(\xi + \bar{\xi})^5 + \mathrm{i}\omega_{10}(\xi - \bar{\xi})\\ &\quad \times[\mu - \alpha(\xi + \bar{\xi})^2 - \beta(\xi + \bar{\xi})^4 + \gamma(\xi - \bar{\xi})^2\omega_{10}^2]\\ &\quad + \frac{\delta\varepsilon}{2}(\xi + \bar{\xi})(z + \bar{z})\Big\}.\end{aligned} \quad (5)$$

For the simplification of Eq.(5), a fifth-order nonlinear transformation is required

$$\begin{aligned}\xi &= \eta + h_1(\eta, \bar{\eta}, z, \bar{z}) + h_2(\eta, \bar{\eta}, z, \bar{z}) + h_3(\eta, \bar{\eta}, z, \bar{z})\\ &\quad + h_4(\eta, \bar{\eta}, z, \bar{z}) + h_5(\eta, \bar{\eta}, z, \bar{z}),\end{aligned} \quad (6)$$

where $h_n(\eta, \bar{\eta}, z, \bar{z}) = \sum_{i=0}^{n}\sum_{j=0}^{n-i}\sum_{k=0}^{n-i-j} \Gamma_{i,j,k,n-i-j-k}\eta^i\bar{\eta}^j z^k\bar{z}^{n-i-j-k}$, $n = 1, 2, \ldots, 5$.

These transformation coefficients $\Gamma_{j_1 j_2 j_3 j_4}$ are suitably chosen to eliminate the non-resonance terms[18] in the final expression at the principal parametric resonance $\Omega \approx 2\omega_{10}$. So the normal form of Eq.(2) is

$$\begin{aligned}\dot{\eta} &= \frac{\Omega^2 + 4\omega_0^2}{4\varepsilon\mu - 4\mathrm{i}\Omega}\eta + \frac{\mathrm{i}\varepsilon(24\mathrm{i}\kappa + 4\alpha\Omega + 3\gamma\Omega^3)}{8\varepsilon\mu - 8\mathrm{i}\Omega}\eta^2\bar{\eta}\\ &\quad - \frac{\delta\varepsilon}{2\varepsilon\mu - 2\mathrm{i}\Omega}\bar{\eta}z + \frac{\mathrm{i}\varepsilon(10\mathrm{i}\sigma + \beta\Omega)}{\varepsilon\mu - \mathrm{i}\Omega}\eta^3\bar{\eta}^2.\end{aligned} \quad (7)$$

Next, η and z require to be expressed in the following polar form $\eta = \frac{1}{2}a\mathrm{e}^{\mathrm{i}(\frac{\Omega}{2}t+\theta)}$, $z = \mathrm{e}^{\mathrm{i}\Omega t}$. Then separating the real and imaginary parts of the foregoing equation, we obtain the averaged equations

$$\begin{aligned}\dot{a} &= -\frac{a\varepsilon}{32(\varepsilon^2\mu^2 + \Omega^2)}[-8\mu\Omega^2 + 2a^4(10\varepsilon\mu\sigma + \beta\Omega^2) + a^2(24\varepsilon\kappa\mu + 4\alpha\Omega^2 + 3\gamma\Omega^4)\\ &\quad + 16\delta(\varepsilon\mu\cos(2\theta) + \Omega\sin(2\theta)) - 32\mu\omega_0^2],\\ a\dot{\theta} &= \frac{a}{32(\varepsilon^2\mu^2 + \Omega^2)}\{\Omega\{2a^4\varepsilon(\beta\varepsilon\mu - 10\sigma) - 8(2\varepsilon^2\mu^2 + \Omega^2) + a^2\varepsilon[-24\kappa + \varepsilon\mu(4\alpha + 3\gamma\Omega^2)] + 32\omega_0^2\}\\ &\quad - 16\delta\varepsilon\Omega\cos(2\theta) + 16\delta\varepsilon^2\mu\sin(2\theta)\}.\end{aligned} \quad (8)$$

At the singular point $\dot{a}=0,\ \dot{\theta}=0$, eliminating θ from Eq.(8) gives the possible stationary solutions as the result: one is $a=0$, the other is the polynomial

$$a^8+c_6a^6+c_4a^4+c_2a^2+c_0=0, \tag{9}$$

where

$$\begin{aligned}
c_0 &= -16[4\varepsilon^2(\delta^2-\mu^2\Omega^2)-(\Omega^2-4\omega_0^2)^2]/[\varepsilon^2(100\sigma^2+\beta^2\Omega^2)],\\
c_2 &= -8\{\Omega^2[-12\kappa+\varepsilon\mu(4\alpha+3\gamma\Omega^2)]+48\kappa\omega_0^2\}/[\varepsilon(100\sigma^2+\beta^2\Omega^2)],\\
c_4 &= \{320\sigma\Omega^2+\varepsilon[576\kappa^2-64\beta\mu\Omega^2+\Omega^2(4\alpha+3\gamma\Omega^2)^2]-1280\sigma\omega_0^2\}/[4\varepsilon(100\sigma^2+\beta^2\Omega^2)],\\
c_6 &= (240\kappa\sigma+4\alpha\beta\Omega^2+3\beta\gamma\Omega^4)/(100\sigma^2+\beta^2\Omega^2).
\end{aligned}$$

3. Transition set and bifurcation diagram

It is hard to unfold Eq.(9) directly because of the hyper codimensional singularity occurring there. Thus, multiplying a on both sides of Eq.(9), and constructing a bifurcation problem with Z_2 symmetry to determine the phase portraits in various regions of the parameter plane, we obtain

$$G(a,c_0,c_2,c_4,c_6)=a^9+c_6a^7+c_4a^5+c_2a^3+c_0a. \tag{10}$$

Now Eq.(10) is an odd function of a; i.e. it is $G(-a,c_0,c_2,c_4,c_6)=-G(a,c_0,c_2,c_4,c_6)$ in symbol. We will show in the present work that as Z_2 symmetric bifurcation problems the above singularity has codimension three.

Let $g(a,c_0)=r(v,c_0)a$ be in $\varepsilon_{a,c_0}(\mathbb{Z}_2)$. Then g is strongly Z_2-equivalent to $(\Delta_1v^4+\Delta_2c_0)a$ at $v=a^2=0,c_0=0$, if and only if

$$r=\frac{\partial r}{\partial v}=\left(\frac{\partial}{\partial v}\right)^2 r=\left(\frac{\partial}{\partial v}\right)^3 r=0, \tag{11}$$

and $\mathrm{sgn}\left(\dfrac{\partial}{\partial v}\right)^4 r(0,0)=\Delta_1$, $\mathrm{sgn}r_{c_0}(0,0)=\Delta_2$, $\Delta_1,\Delta_2=\pm1$ as usual.[16]

In terms of the singularity theory,[16] the Golubitsky–Schaeffer (GS) normal form $(\Delta_1v^4+\Delta_2c_0)a$ has Z_2-codimension 3; a universal unfolding is provided by

$$(\Delta_1v^4+\Delta_2c_0+c_2v+c_4v^2+c_6v^3)a. \tag{12}$$

So Eq.(10) is strongly Z_2-equivalent to $(\Delta_1v^4+\Delta_2c_0)a$, and the universal unfolding is consistent with Eq.(12) at the point of Z_2-codimension 3, where (c_2,c_4,c_6) each take a function of unfolding parameters. On the other hand, (c_2,c_4,c_6) are derived from the physical parameters of the system as well. Therefore, the double functions can convenience researcher's analysing the bifurcation behaviours of the SNOS.

The three sources of nonpersistence (bifurcation, hysteresis, and double limit points) form the transition set $\sum(\mathbb{Z}_2)$, as follows (Different bifurcation phenomena occur for $a=0$ and $a\neq0$, that means the sources of nonpersistence have to be considered separately.)

(I) Bifurcation set, $B_0(\mathbb{Z}_2)=B_1(\mathbb{Z}_2)=\emptyset$;

(II) Hysteresis set, $H_0(\mathbb{Z}_2):c_2=0$; $H_1(\mathbb{Z}_2):c_2=3c_6v^2+8v^3,\ c_4=-3c_6v-6v^2$ Hysteresis set in the physical parametric conditions, in the case of $\varepsilon\neq0$

$$\begin{aligned}
&\Omega^2[12\kappa-\varepsilon\mu(4\alpha+3\gamma\Omega^2)]=48\kappa\omega_0^2,\\
&80v^2\varepsilon\sigma(9\kappa+10v\sigma)+4[v^2\beta(3\alpha+2v\beta)\varepsilon\\
&-24\kappa+8\alpha\varepsilon\mu]\Omega^2\\
&+3\gamma\varepsilon(3v^2\beta+8\mu)\Omega^4+384\kappa\omega_0^2=0,\\
&96\varepsilon(6\kappa^2+30v\kappa\sigma+25v^2\sigma^2)\\
&+8[2\alpha^2\varepsilon+6v\alpha\beta\varepsilon+\beta\varepsilon(3v^2\beta-8\mu)+40\sigma]\Omega^2\\
&+12(2\alpha+3v\beta)\gamma\varepsilon\Omega^4+9\gamma^2\varepsilon\Omega^6=1280\sigma\omega_0^2;
\end{aligned}$$

(III) Double limit points set, $\mathscr{DL}(\mathbb{Z}_2)=\mathscr{DL}_{\mathrm{I}}+\mathscr{DL}_{\mathrm{II}}+\mathscr{DL}_{\mathrm{III}}$

$$\mathscr{DL}_{\mathrm{I}}:\begin{cases}G(0,c_0,c_2,c_4,c_6)=0,\\ G(v,c_0,c_2,c_4,c_6)=G_a(v,c_0,c_2,c_4,c_6)=0,\\ v=a^2\neq0\end{cases}\Rightarrow\begin{cases}c_2=2a^3+c_6a^2,\\ c_4=-3a^2-2c_6a.\end{cases}$$

$$\mathscr{DL}_{\mathrm{II}}:\begin{cases} G(v_i,c_0,c_2,c_4,c_6)=0,\\ G_{v_i}(a,c_0,c_2,c_4,c_6)=0,\ (i=1,2)\\ (v_1\neq v_2)\end{cases}\Rightarrow\begin{cases} c_2=\dfrac{1}{8}(4c_4c_6-c_6^3),\\ c_0<0,\ c_2<0,\ c_4\geq 0,\ c_6<0.\end{cases}$$

and

$$\mathscr{DL}_{\mathrm{III}}:\begin{cases} G(a_i,c_0,c_2,c_4,c_6)=0,\\ G_{a_i}(a_i,c_0,c_2,c_4,c_6)=0,\ (i=1,2)\\ (a_1=-a_2)\end{cases}\Rightarrow \mathscr{DL}_{\mathrm{III}}=0.$$

(Double limit points set in the physical parametric conditions can be figured out similarly.)

(IV) Transition set, $\sum(\mathbb{Z}_2)=B_0(\mathbb{Z}_2)\cup B_1(\mathbb{Z}_2)\cup H_0(\mathbb{Z}_2)\cup H_1(\mathbb{Z}_2)\cup\mathscr{DL}(\mathbb{Z}_2)$, where the subscript 1 indicates a phenomenon occurring for $a\neq 0$; this is to be distinguished from subscript 0 showing the bifurcation phenomenon occurring for $a=0$. For this Z_2-codimension 3 bifurcation, (c_2,c_4,c_6)divide the whole parameter space $\mathbb{R}^3$ into different parts, and the bifurcation behaviours of the problem in each of the unfolding regions are persistent as shown in Fig.1.

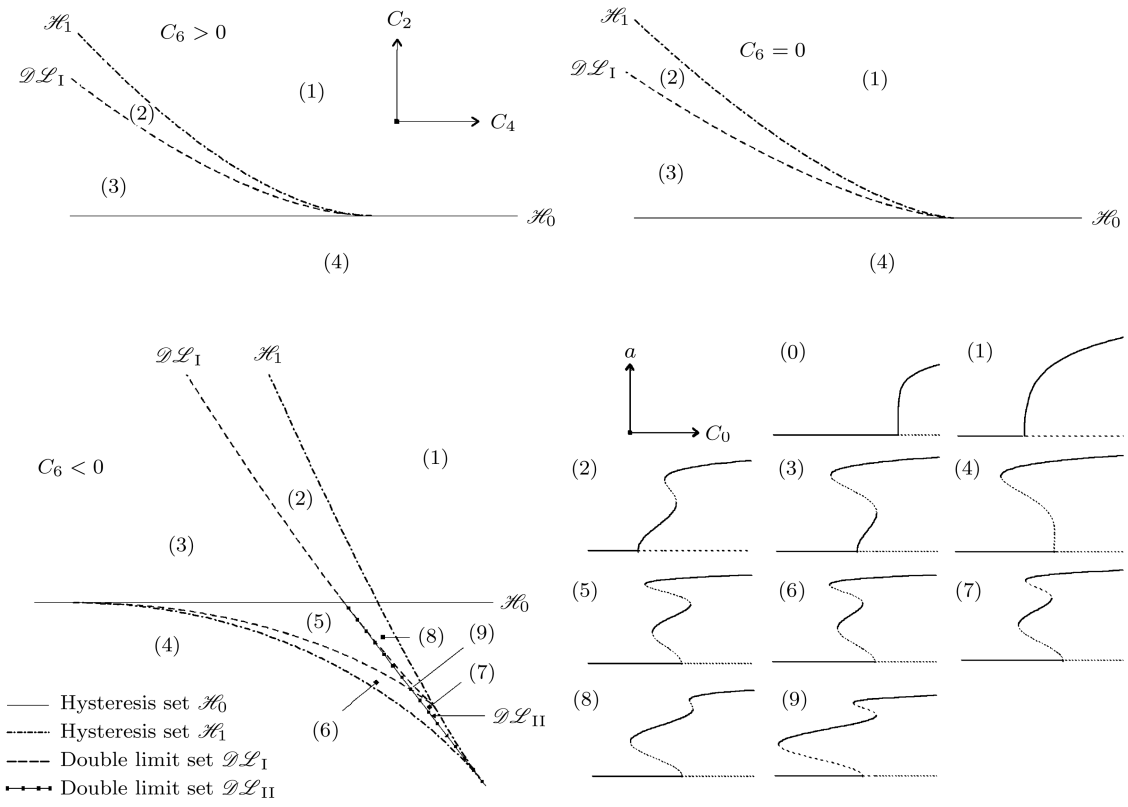


Fig.1. Bifurcation diagrams and transition set of Z_2-codimension 3 bifurcation problem.

4. Stability of trivial solution

For the convenience of determining the stability of the constant, we convert the averaged equation (8) form (the polar form) into a Cartesian form

$$\frac{1}{2}a\mathrm{e}^{\mathrm{i}\theta}=p+q\mathrm{i}. \tag{13}$$

Substituting Eq.(13) into Eq.(8) and separating the real part and the imaginary part yield the following real form equations:

$$\begin{aligned}
\dot{p} = & \frac{1}{32(\varepsilon^4\mu^2+\Omega^2)}\{p\varepsilon^2[-4\varepsilon^2\mu(4\delta+6a^2\kappa+5a^4\sigma)-2(2a^2\alpha+a^4\beta-4\mu)\Omega^2-3a^2\gamma\Omega^4] \\
& +q\Omega\{8\Omega^2+\varepsilon^2\{-16\delta+16\varepsilon^2\mu^2+a^4(-2\beta\varepsilon^2\mu+20\sigma)+a^2[24\kappa-\varepsilon^2\mu(4\alpha+3\gamma\Omega^2)]\}\} \\
& +32(p\varepsilon^2\mu-q\Omega)\omega_0^2\}, \\
\dot{q} = & \frac{1}{32(\varepsilon^4\mu^2+\Omega^2)}\{q\varepsilon^2[4\varepsilon^2\mu(4\delta-6a^2\kappa-5a^4\sigma)-2(2a^2\alpha+a^4\beta-4\mu)\Omega^2-3a^2\gamma\Omega^4] \\
& +p\Omega\{-8\Omega^2+\varepsilon^2\{-16\delta-16\varepsilon^2\mu^2+2a^4(\beta\varepsilon^2\mu-10\sigma)+a^2[-24\kappa+\varepsilon^2\mu(4\alpha+3\gamma\Omega^2)]\}\} \\
& +32(q\varepsilon^2\mu+p\Omega)\omega_0^2\},
\end{aligned} \tag{14}$$

where $a^2=4(p^2+q^2)$. The Jacobi matrix of Eq.(14) is

$$\boldsymbol{J}=\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \tag{15}$$

with

$$\begin{aligned}
A_{11} &= -\frac{\varepsilon}{32(\varepsilon^2\mu^2+\Omega^2)}[4\varepsilon\mu(4\delta+6a^2\kappa+5a^4\sigma)+2(2a^2\alpha+a^4\beta-4\mu)\Omega^2+3a^2\gamma\Omega^4-32\mu\omega_0^2], \\
A_{12} &= -\frac{\Omega}{32(\varepsilon^2\mu^2+\Omega^2)}\{8\Omega^2+\varepsilon\{-16\delta+16\varepsilon\mu^2-2a^4(\beta\varepsilon\mu-10\sigma)+a^2[24\kappa-\varepsilon\mu(4\alpha+3\gamma\Omega^2)]\}-32\omega_0^2\}, \\
A_{21} &= \frac{\Omega}{32(\varepsilon^2\mu^2+\Omega^2)}\{-8\Omega^2+\varepsilon\{-16\delta-16\varepsilon\mu^2+2a^4(\beta\varepsilon\mu-10\sigma)+a^2[-24\kappa+\varepsilon^2\mu(4\alpha+3\gamma\Omega^2)]\}+32\omega_0^2\}, \\
A_{22} &= \frac{\varepsilon}{32(\varepsilon^2\mu^2+\Omega^2)}[4\varepsilon\mu(4\delta-6a^2\kappa-5a^4\sigma)-2(2a^2\alpha+a^4\beta-4\mu)\Omega^2-3a^2\gamma\Omega^4+32\mu\omega_0^2].
\end{aligned}$$

For the trivial solution, the Jacobi matrix is

$$\boldsymbol{J}|_{a=0}=\frac{1}{4(\varepsilon^2\mu^2+\Omega^2)}\begin{pmatrix} \varepsilon\mu(-2\delta\varepsilon+\Omega^2+4\omega_0^2) & \Omega(-2\delta\varepsilon+2\varepsilon^2\mu^2+\Omega^2-4\omega_0^2) \\ -\Omega(2\delta\varepsilon+2\varepsilon^2\mu^2+\Omega^2-4\omega_0^2) & \varepsilon\mu(2\delta\varepsilon+\Omega^2+4\omega_0^2) \end{pmatrix}. \tag{16}$$

The characteristic equation for the trivial solution is

$$\lambda^2-\frac{\varepsilon\mu(\Omega^2+4\omega_0^2)}{2(\varepsilon^2\mu^2+\Omega^2)}\lambda+\frac{-4\delta^2\varepsilon^2+4\varepsilon^2\mu^2\Omega^2+(\Omega^2-4\omega_0^2)^2}{16(\varepsilon^2\mu^2+\Omega^2)}=0. \tag{17}$$

And in terms of the newly introduced transformation (18)

$$\varpi=\Omega^2-4\omega_0^2, \tag{18}$$

Equation (17) can be rewritten as

$$\lambda^2-\frac{\varepsilon\mu\left(2\Omega^2-\varpi\right)}{2\left(\varepsilon^2\mu^2+\Omega^2\right)}\lambda+\frac{\varpi^2-4\varepsilon^2\left(\delta^2-\mu^2\Omega^2\right)}{16\left(\varepsilon^2\mu^2+\Omega^2\right)}=0. \tag{19}$$

Results from the straightforward analysis of the possible singular points and their linearized stability are presented in Fig.2. The (ϖ,μ) parameter plane is divided into 8 disjoint regions, and their boundary conditions, Eq.(20), give the local bifurcation set at which the fixed points of the averaged equations undergo a change in stability

$$\begin{aligned}
& L_1:\varpi^2+4\varepsilon^2\mu^2\Omega^2=4\delta^2\varepsilon^2, \\
& L_2:\left(\varpi+2\varepsilon^2\mu^2\right)^2\Omega^2-4\delta^2\varepsilon^2\left(\varepsilon^2\mu^2+\Omega^2\right)=0, \\
& \qquad \mu=0.
\end{aligned} \tag{20}$$

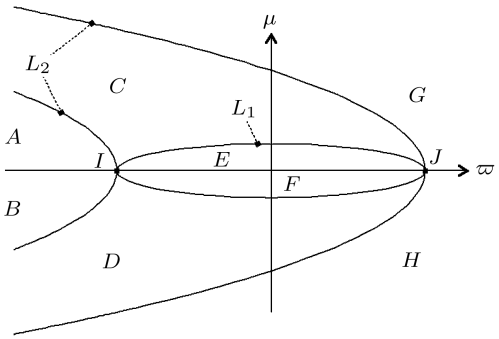


Fig.2. The local bifurcation set in parameter plane (ϖ, μ).

(a) In regions A, B, G, and H the singular point is the focus point. Moving from A (G) across the axis ϖ into region B (H), the zero solution gains the stability.

(b) In regions C and D, the origin is the node point. The trivial solution is unstable in region C, while stable in region D.

(c) Across the circle L_1, the origin undergoes a pitchfork bifurcation into the non-trivial solutions. The origin is the saddle point in regions E and F.

(d) Moving across the line $\varpi(-\infty, -2\delta\varepsilon)$ and $(2\delta\varepsilon, 2\Omega^2)$, into the stability of the zero solution changes with a complex-conjugate pair of eigenvalues crossing the imaginary axis. Along these lines we, therefore, expect Hopf bifurcation of the origin into a limit cycle.

(e) At points I and J the zero solution has both eigenvalues vanishing and the geometric and the algebraic multiplicities of the zero solution eigenvalues are different. It is a degenerate singular point of codimension 2 or 3 at these values of parameters.

5. Conclusion

The static bifurcation of a fifth order SNOS subject to parametrical excitation is analysed in terms of the undetermined fundamental frequency method. The averaged equations of the system are obtained by introducing the undetermined fundamental frequency into the normal form operation. The results of static bifurcation indicated by the universal unfolding with the character of Z_2-codimension 3 may explain the local dynamic behaviour of the oscillator. That also helps people construct the relationship between the unfolding parameters and physical parameters of the SNOS directly, and convenience their analysing the bifurcation behaviours. The stability of trivial solution is discussed based on the characteristic equation, and we divide the whole parameter plane into 8 regions in terms of the boundary conditions. In different regions, the singular point shares different types of origin, while the stability changes from one region to another.

The strategy presented in this work is sufficiently general, so it would be possible to apply the present method to considering other SNOS under combined parametrical and external excitations simultaneously, which will be the topics for the further research.

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