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Adaptive generalized projective synchronization of two different chaotic systems with unknown parameters*

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This paper presents a general method of the generalized projective synchronization and the parameter identification between two different chaotic systems with unknown parameters. This approach is based on Lyapunov stability theory, and employs a combination of feedback control and adaptive control. With this method one can achieve the generalized projective synchronization and realize the parameter identifications between almost all chaotic (hyperchaotic) systems with unknown parameters. Numerical simulations results are presented to demonstrate the effectiveness of the method.

Keywords: different chaotic systems, generalized projective synchronization, parameter identification, unknown parameters

PACC: 0545

1. Introduction

Since the idea of synchronizing two identical chaotic systems with different initial conditions was first introduced by Pecora and Carroll in 1990,[1] chaotic synchronization has been intensively studied.[2−6] Different types of chaotic synchronization methods have been achieved, i.e. complete synchronization, generalized synchronization, phase synchronization, lag synchronization, generalized synchronization as well as generalized projective synchronization (GPS).[7−12] Among all kinds of synchronization schemes, projective synchronization, which was first reported by Mainieri and Rehacek[13] in partially linear systems and developed by many authors,[14−18] is the most noticeable one. Because this kind of synchronization belongs to a subclass of the generalized synchronization, it unifies different types of synchronization phenomena such as anti-phase synchronization and complete synchronization, as well as a specific case of generalized synchronization into one type by using a scaling factor.[19] The performance of projective synchronization has been investigated[14−18] due to its proportionality between synchronized dynamical states. In the application to secure communications, this feature can be used to extend binary digital to M-nary digital communication[20,21] for achieving fast communication, so the research on projective synchronization has important theoretical significance and application value.

A chaotic system is extremely sensitive to tiny variations of parameters. But parameters of some systems in practical circumstances cannot be exactly known in advance, The effect of these uncertainties will destroy the synchronization and even break it. Therefore, it is important and necessary to study projective synchronization in such systems with unknown parameters.

In this paper, we investigate the generalized projective synchronization between two different chaotic systems with unknown parameters and the parameter identification. Based on the adaptive technique, a novel nonlinear controller is designed such that the parameter identification is realized, and two different chaotic (hyperchaotic) systems with uncertain parameters achieve the globally GPS simultaneously. The conclusion is proved theoretically via the Lyapunov method. Finally, some numerical simulations are given to demonstrate the effectiveness and feasibility of the proposed technique.

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2. Design of GPS method

We consider a chaotic system in the form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{F}(\boldsymbol{x})\boldsymbol{\alpha} + \boldsymbol{f}(\boldsymbol{x}), \tag{1}$$

where $\boldsymbol{x} \in \boldsymbol{R}^n$ is the state vector, $\boldsymbol{A} \in \boldsymbol{R}^{n\times n}$is constant matrix, $\boldsymbol{f}(\boldsymbol{x}) \in \boldsymbol{R}^n, \boldsymbol{F}(\boldsymbol{x}) \in \boldsymbol{R}^{n\times m}$, $\boldsymbol{\alpha} \in \boldsymbol{R}^m$ is the unknown parameter vector. Equation (1) is regarded as a drive system. And a response system with a controller $\boldsymbol{u} \in \boldsymbol{R}^n$ is as follows:

$$\dot{\boldsymbol{y}} = \boldsymbol{B}\boldsymbol{y} + \boldsymbol{g}(\boldsymbol{y}) + \boldsymbol{u} = \boldsymbol{G}(\boldsymbol{y})\boldsymbol{\beta} + \boldsymbol{g}(\boldsymbol{y}) + \boldsymbol{u}, \tag{2}$$

where $\boldsymbol{y} \in \boldsymbol{R}^n$ is the state vector, $\boldsymbol{B} \in \boldsymbol{R}^{n\times n}$ is constant matrix, $\boldsymbol{g}(\boldsymbol{y}) \in \boldsymbol{R}^n, \boldsymbol{G}(\boldsymbol{y}) \in \boldsymbol{R}^{n\times p}, \boldsymbol{\beta} \in R^p$ is the unknown parameter vector.

Definition 1[22] For two different systems described by Eqs.(1) and (2), we say they are globally generalized projective synchronous with respect to the scaling matrix $\boldsymbol{\Lambda} = \lambda \boldsymbol{I}_{n\times n}$ (λ is a nonzero constant and $\boldsymbol{I}$ is the identity matrix) if there exists a vector controller $\boldsymbol{u}(\boldsymbol{x}, \boldsymbol{y}, t)$ such that all trajectories ($\boldsymbol{x}(t)$, $\boldsymbol{y}(t)$) in Eqs.(1) and (2) with any initial conditions ($\boldsymbol{x}(0)$, $\boldsymbol{y}(0)$) in $\boldsymbol{P} = \boldsymbol{R}^n_{\boldsymbol{x}} \times \boldsymbol{R}^n_{\boldsymbol{y}} \subset \boldsymbol{R}^n \times \boldsymbol{R}^n$ approach the manifold $\boldsymbol{M} = \{\boldsymbol{x}(t), \boldsymbol{y}(t) : \boldsymbol{x}(t) = \lambda \boldsymbol{y}(t)\}$ with $\boldsymbol{M} \subset \boldsymbol{P}$ as time t goes to infinity, that is to say, $\lim\limits_{t\to\infty} ||\boldsymbol{e}(t)|| = \lim\limits_{t\to\infty} ||\boldsymbol{x}(t) - \lambda \boldsymbol{y}(t)|| = 0$, which implies that the error dynamical system between the drive system and the response system is globally asymptotically stable.

Theorem 1 For drive system (1) and response system (2), let $\boldsymbol{e}(t) = \boldsymbol{x}(t) - \lambda \boldsymbol{y}(t)$, if the vector controller in Eq.(2) is given by

$$\begin{aligned}\boldsymbol{u} =& \boldsymbol{f}(\boldsymbol{x})/\lambda - \boldsymbol{g}(\boldsymbol{y}) - \boldsymbol{G}(\boldsymbol{x})\tilde{\boldsymbol{\beta}}/\lambda \\ &+ \boldsymbol{F}(\boldsymbol{y})\tilde{\boldsymbol{\alpha}} + \boldsymbol{k}\boldsymbol{e}/\lambda,\end{aligned} \tag{3}$$

where $\boldsymbol{k} = \mathrm{diag}(k_i) \in \boldsymbol{R}^{n\times n}$, $\tilde{\boldsymbol{\alpha}}$ and $\tilde{\boldsymbol{\beta}}$ represent the estimate vectors of unknown parameter vectors $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ respectively, which are updated according to the following laws:

$$\begin{aligned}&\dot{\tilde{\boldsymbol{\alpha}}} = \lambda[\boldsymbol{F}(\boldsymbol{y})]^{\mathrm{T}}\boldsymbol{e} - (\tilde{\boldsymbol{\alpha}} - \boldsymbol{\alpha}),\\ &\dot{\tilde{\boldsymbol{\beta}}} = -[\boldsymbol{G}(\boldsymbol{x})]^{\mathrm{T}}\boldsymbol{e} - (\tilde{\boldsymbol{\beta}} - \boldsymbol{\beta}),\\ &\dot{k}_i = \gamma_i e_i^2,\ \gamma_i > 0,\ i = 1, 2, \ldots, n.\end{aligned} \tag{4}$$

Then, the response system (2) associated with the proposed control law (3) and the drive system (1) can globally asymptotically achieve the generalized projective synchronization, and the parameter $\tilde{\boldsymbol{\alpha}}$ approaches to $\boldsymbol{\alpha}$, $\tilde{\boldsymbol{\beta}}$ approaches to $\boldsymbol{\beta}$, that is, $\lim\limits_{t\to\infty} \boldsymbol{e} = \lim\limits_{t\to\infty} (\boldsymbol{x} - \lambda \boldsymbol{y}) = 0$, $\lim\limits_{t\to\infty} \hat{\boldsymbol{\alpha}} = \lim\limits_{t\to\infty} (\tilde{\boldsymbol{\alpha}} - \boldsymbol{\alpha}) = 0$, and $\lim\limits_{t\to\infty} \hat{\boldsymbol{\beta}} = \lim\limits_{t\to\infty} (\tilde{\boldsymbol{\beta}} - \boldsymbol{\beta}) = 0$.

Proof The dynamic equation of synchronization error can be obtained easily from Eqs.(1) and (2), which is expressed as follows:

$$\dot{\boldsymbol{e}} = \dot{\boldsymbol{x}} - \lambda \dot{\boldsymbol{y}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{f}(\boldsymbol{x}) - \lambda \boldsymbol{B}\boldsymbol{y} - \lambda \boldsymbol{g}(\boldsymbol{y}) - \lambda \boldsymbol{u}.$$

Let $\hat{\boldsymbol{\alpha}} = \tilde{\boldsymbol{\alpha}} - \boldsymbol{\alpha}$ and $\hat{\boldsymbol{\beta}} = \tilde{\boldsymbol{\beta}} - \boldsymbol{\beta}$. Substituting the selected controller $\boldsymbol{u}(\boldsymbol{x}, \boldsymbol{y})$ (3) into the error system yields the following new error system:

$$\begin{aligned}\dot{\boldsymbol{e}} =& \boldsymbol{A}\boldsymbol{x} - \lambda \boldsymbol{B}\boldsymbol{y} + \boldsymbol{G}(\boldsymbol{x})(\hat{\boldsymbol{\beta}} + \boldsymbol{\beta}) - \lambda \boldsymbol{F}(\boldsymbol{y})(\hat{\boldsymbol{\alpha}} + \boldsymbol{\alpha}) - \boldsymbol{k}\boldsymbol{e}\\ =& \boldsymbol{A}\boldsymbol{x} - \lambda \boldsymbol{B}\boldsymbol{y} + \boldsymbol{G}(\boldsymbol{x})\hat{\boldsymbol{\beta}} + \boldsymbol{B}\boldsymbol{x} - \lambda \boldsymbol{F}(\boldsymbol{y})\hat{\boldsymbol{\alpha}} - \lambda \boldsymbol{A}\boldsymbol{y} - \boldsymbol{k}\boldsymbol{e}\\ =& (\boldsymbol{A}\boldsymbol{x} - \lambda \boldsymbol{A}\boldsymbol{y}) + (\boldsymbol{B}\boldsymbol{x} - \lambda \boldsymbol{B}\boldsymbol{y})\\ &+ \boldsymbol{G}(\boldsymbol{x})\hat{\boldsymbol{\beta}} - \lambda \boldsymbol{F}(\boldsymbol{y})\hat{\boldsymbol{\alpha}} - \boldsymbol{k}\boldsymbol{e}\\ =& \boldsymbol{A}\boldsymbol{e} + \boldsymbol{B}\boldsymbol{e} + \boldsymbol{G}(\boldsymbol{x})\hat{\boldsymbol{\beta}} - \lambda \boldsymbol{F}(\boldsymbol{y})\hat{\boldsymbol{\alpha}} - \boldsymbol{k}\boldsymbol{e}.\end{aligned} \tag{5}$$

Constructing the Lyapunov function in the form

$$\begin{aligned}\boldsymbol{V}(\boldsymbol{e}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}) =& \frac{1}{2}(\boldsymbol{e}^{\mathrm{T}}\boldsymbol{e} + \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} + \hat{\boldsymbol{\beta}}^{\mathrm{T}}\hat{\boldsymbol{\beta}})\\ &+ \frac{1}{2}\sum_{i=1}^{n}\frac{1}{\gamma_i}(k_i - k^*)^2, \quad (k^* > 0),\end{aligned} \tag{6}$$

And differentiating V with respect to time t along the trajectories of (5), then we will obtain

$$\begin{aligned}\dot{\boldsymbol{V}}(\boldsymbol{e}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}) =& \boldsymbol{e}^{\mathrm{T}}\dot{\boldsymbol{e}} + \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\dot{\hat{\boldsymbol{\alpha}}} + \hat{\boldsymbol{\beta}}^{\mathrm{T}}\dot{\hat{\boldsymbol{\beta}}} + \sum_{i=1}^{n}\frac{1}{\gamma_i}(k_i - \boldsymbol{k}^*)\dot{k}_i\\ =& \boldsymbol{e}^{\mathrm{T}}(\boldsymbol{A} + \boldsymbol{B} - \boldsymbol{k})\boldsymbol{e} + \boldsymbol{e}^{\mathrm{T}}\boldsymbol{G}(\boldsymbol{x})\hat{\boldsymbol{\beta}}\\ &- \boldsymbol{e}^{\mathrm{T}}\lambda \boldsymbol{F}(\boldsymbol{y})\hat{\boldsymbol{\alpha}} + \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\lambda[\boldsymbol{F}(\boldsymbol{y})]^{\mathrm{T}}\boldsymbol{e} - \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}}\\ &- \hat{\boldsymbol{\beta}}^{\mathrm{T}}[\boldsymbol{G}(\boldsymbol{x})]^{\mathrm{T}}\boldsymbol{e} - \hat{\boldsymbol{\beta}}^{\mathrm{T}}\hat{\boldsymbol{\beta}} + \boldsymbol{e}^{\mathrm{T}}(\boldsymbol{k} - k^*\boldsymbol{I})\boldsymbol{e}\\ =& \boldsymbol{e}^{\mathrm{T}}(\boldsymbol{A} + \boldsymbol{B} - k^*\boldsymbol{I})\boldsymbol{e} - \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} - \hat{\boldsymbol{\beta}}^{\mathrm{T}}\hat{\boldsymbol{\beta}}\\ =& -\boldsymbol{e}^{\mathrm{T}}\frac{1}{2}[(k^*\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{B})\\ &+ (k^*\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{B})^{\mathrm{T}}]\boldsymbol{e} - \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} - \hat{\boldsymbol{\beta}}^{\mathrm{T}}\hat{\boldsymbol{\beta}}\\ =& -\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e} - \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} - \hat{\boldsymbol{\beta}}^{\mathrm{T}}\hat{\boldsymbol{\beta}}.\end{aligned}$$

Obviously, there exists a positive constant k^*, which makes matrix $\boldsymbol{P}$ positive. Since $\boldsymbol{P}$ is positively definite, its minimum eigenvalue $\lambda_{\min}(\boldsymbol{P}) > 0$, then

$$\begin{aligned}\dot{\boldsymbol{V}}(\boldsymbol{e}, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}) =& -\boldsymbol{e}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e} - \hat{\boldsymbol{\alpha}}^{\mathrm{T}}\hat{\boldsymbol{\alpha}} - \hat{\boldsymbol{\beta}}^{\mathrm{T}}\hat{\boldsymbol{\beta}}\\ \le& -\lambda_{\min}(\boldsymbol{P})||\boldsymbol{e}||^2 - ||\hat{\boldsymbol{\alpha}}||^2 - ||\hat{\boldsymbol{\beta}}||^2 < 0.\end{aligned}$$

Based on the Lyapunov stability theory, we have

$$\lim_{t\to\infty} \boldsymbol{e} = \lim_{t\to\infty} (\boldsymbol{x} - \lambda \boldsymbol{y}) = 0,$$

$$\lim_{t\to\infty} \hat{\boldsymbol{\alpha}} = \lim_{t\to\infty} (\tilde{\boldsymbol{\alpha}} - \boldsymbol{\alpha}) = 0,$$

and

$$\lim_{t\to\infty} \hat{\boldsymbol{\beta}} = \lim_{t\to\infty} (\tilde{\boldsymbol{\beta}} - \boldsymbol{\beta}) = 0.$$

3. Application of the above-mentioned scheme

To illustrate the effectiveness of the proposed adaptive generalized projective synchronization scheme, two examples are considered and their numerical simulations are performed.

Example 1 Consider the generalized projective synchronization between the Chen system[23]

$$\begin{aligned}\dot{x}_1 &= \alpha_1(x_2 - x_1),\\ \dot{x}_2 &= (\alpha_3 - \alpha_1)x_1 - x_1x_3 + \alpha_3x_2,\\ \dot{x}_3 &= x_1x_2 - \alpha_2x_3\end{aligned} \tag{7}$$

and the Lorenz system

$$\begin{aligned}\dot{y}_1 &= \beta_1(y_2 - y_1) + u_1,\\ \dot{y}_2 &= \beta_2y_1 - y_2 - y_1y_3 + u_2,\\ \dot{y}_3 &= y_1y_2 - \beta_3y_3 + u_3.\end{aligned} \tag{8}$$

Compare systems (7) and (8) with Eqs.(1) and (2), one can obtain that

$$\boldsymbol{F}(\boldsymbol{x}) = \begin{pmatrix} x_2 - x_1 & 0 & 0\\ -x_1 & 0 & x_1 + x_2\\ 0 & -x_3 & 0\end{pmatrix},$$

$$\boldsymbol{\alpha} = \begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix},$$

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix}0\\ -x_1x_3\\ x_1x_2\end{pmatrix},$$

$$\boldsymbol{G}(\boldsymbol{y}) = \begin{pmatrix} y_2 - y_1 & 0 & 0\\ 0 & y_1 & 0\\ 0 & 0 & -y_3\end{pmatrix},$$

$$\boldsymbol{\beta} = \begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\end{pmatrix},$$

and

$$\boldsymbol{g}(\boldsymbol{y}) = \begin{pmatrix}0\\ -y_2 - y_1y_3\\ y_1y_2\end{pmatrix}.$$

According to Theorem 1, the controller $\boldsymbol{u}$ is taken as

$$\begin{aligned}\boldsymbol{u} = \begin{pmatrix}u_1\\ u_2\\ u_3\end{pmatrix} &= \boldsymbol{f}(\boldsymbol{x})/\lambda - \boldsymbol{g}(\boldsymbol{y}) - \boldsymbol{G}(\boldsymbol{x})\tilde{\boldsymbol{\beta}}/\lambda + \boldsymbol{F}(\boldsymbol{y})\tilde{\boldsymbol{\alpha}} + \boldsymbol{k}\boldsymbol{e}/\lambda\\ &= \frac{1}{\lambda}\begin{pmatrix}0\\ -x_1x_3\\ x_1x_2\end{pmatrix} - \begin{pmatrix}0\\ -y_2 - y_1y_3\\ y_1y_2\end{pmatrix} + \frac{1}{\lambda}\begin{pmatrix} x_2 - x_1 & 0 & 0\\ 0 & x_1 & 0\\ 0 & 0 & -x_3\end{pmatrix}\begin{pmatrix}\tilde{\beta}_1\\ \tilde{\beta}_2\\ \tilde{\beta}_3\end{pmatrix}\\ &\quad + \begin{pmatrix} y_2 - y_1 & 0 & 0\\ -y_1 & 0 & y_1 + y_2\\ 0 & -y_3 & 0\end{pmatrix}\begin{pmatrix}\tilde{\alpha}_1\\ \tilde{\alpha}_2\\ \tilde{\alpha}_3\end{pmatrix} + \frac{1}{\lambda}\begin{pmatrix}k_1e_1\\ k_2e_2\\ k_3e_3\end{pmatrix}\end{aligned}$$

with

$$\dot{\hat{\boldsymbol{\alpha}}} = \dot{\tilde{\boldsymbol{\alpha}}} = \lambda[\boldsymbol{F}(\boldsymbol{y})]^{\mathrm{T}}\boldsymbol{e} - \hat{\boldsymbol{\alpha}} = \lambda[(y_2 - y_1)e_1 - y_1e_2,\ -y_3e_3,\ (y_1 + y_2)e_2]^{\mathrm{T}} - \hat{\boldsymbol{\alpha}}$$

and

$$\dot{\hat{\boldsymbol{\beta}}} = \dot{\tilde{\boldsymbol{\beta}}} = -[\boldsymbol{G}(\boldsymbol{x})]^{\mathrm{T}}\boldsymbol{e} - \hat{\boldsymbol{\beta}} = -[(x_2 - x_1)e_1,\ x_1e_2,\ -x_3e_3]^{\mathrm{T}} - \hat{\boldsymbol{\beta}}.$$

In the simulations, we choose the unknown parameter vectors $\boldsymbol{\alpha}$=(35, 3, 28)$^{\mathrm{T}}$, and $\boldsymbol{\beta}$=(10, 28, 8/3)$^{\mathrm{T}}$, the initial conditions $\boldsymbol{x}(0) = (3, -3.5, -1)^{\mathrm{T}}$, $\boldsymbol{y}(0) = (2, 3, 7)^{\mathrm{T}}$, $\tilde{\boldsymbol{\alpha}}(0) = (25, 2, 20)^{\mathrm{T}}$and $\tilde{\boldsymbol{\beta}}(0) = (5, 35, 1)^{\mathrm{T}}$. Let $k_1 = k_3 = 0$, $\dot{k}_2 = \gamma_2e_2^2$, $\gamma_2 = 1$, and $k_2(0) = 0$. When $\lambda = -0.5$, the numerical results

are illustrated in Fig.1. Figure 1(a) shows that the errors converge to zero as time increases, which implies that the generalized projective synchronization can be realized. Figures 1(b) and 1(c) display the evolutions of $\hat{\boldsymbol{\alpha}} = \tilde{\boldsymbol{\alpha}} - \boldsymbol{\alpha}$ and $\hat{\boldsymbol{\beta}} = \tilde{\boldsymbol{\beta}} - \boldsymbol{\beta}$. Obviously, with time passing, these parameters tend to zero, that is to say, the parameter identification is realized simultaneously. Figure 1(d) depicts the projection of the synchronized attractors, which illustrates an anti-phase synchronization with $\lambda = -0.5$.

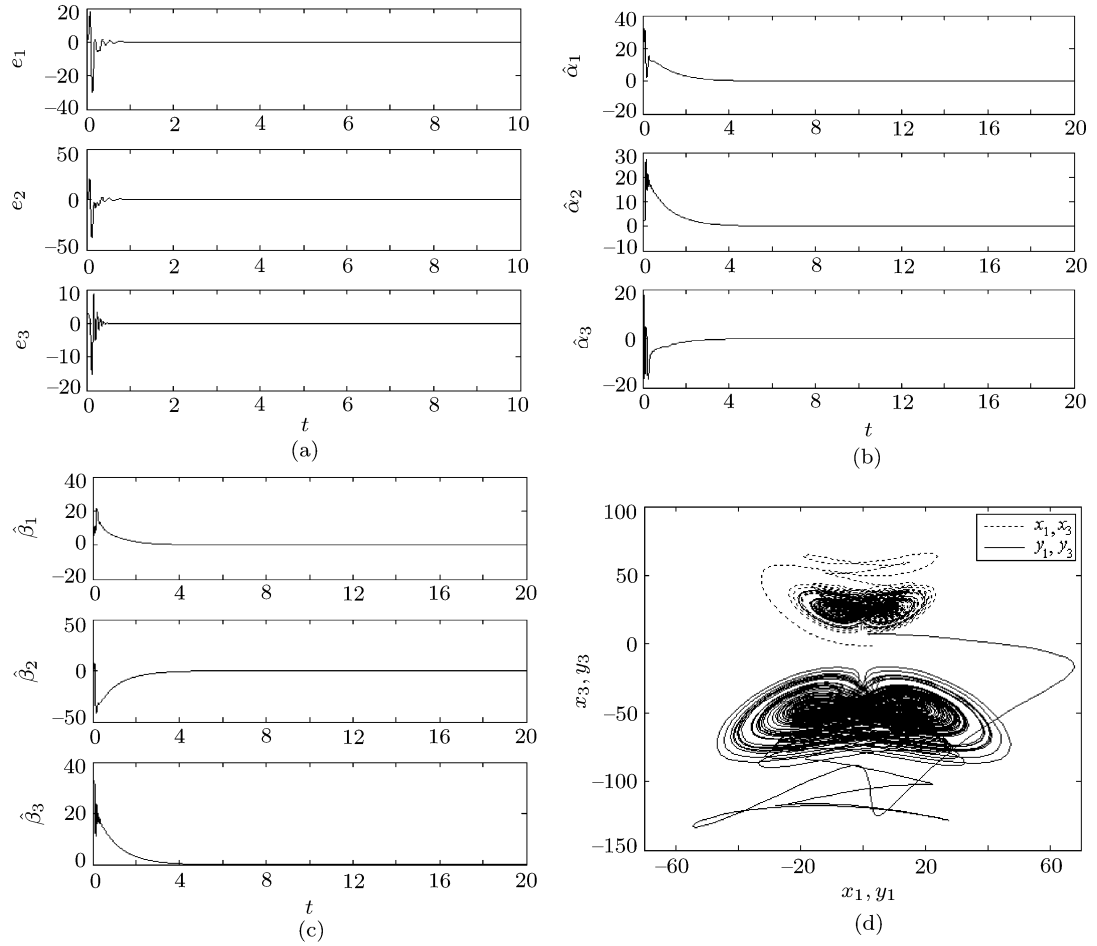
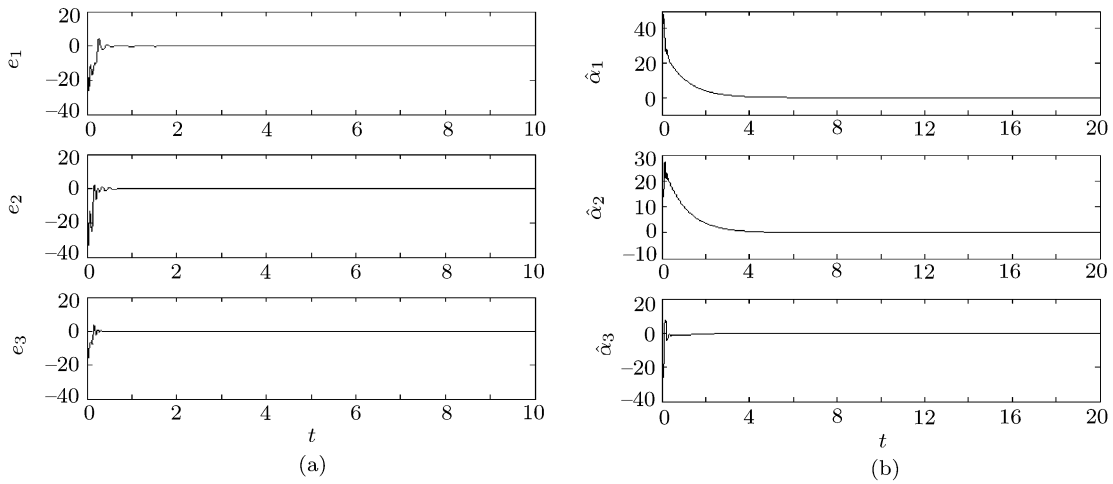


Fig.1. The GPSs of systems (7) and (8) with $\lambda = -0.5$, where panel (a) is for GPS errors between the Chen system and the Lorenz system with unknown parameters, panels (b) and (c) for time evolutions of parameters of $\hat{\boldsymbol{\alpha}} = \tilde{\boldsymbol{\alpha}} - \boldsymbol{\alpha}$, and $\hat{\boldsymbol{\beta}} = \tilde{\boldsymbol{\beta}} - \boldsymbol{\beta}$ respectively, and panel (d) for the projection of the synchronized attractors on plane $(x_1 - x_3)$.

When $\lambda = 3$, the numerical results are illustrated in Fig.2.



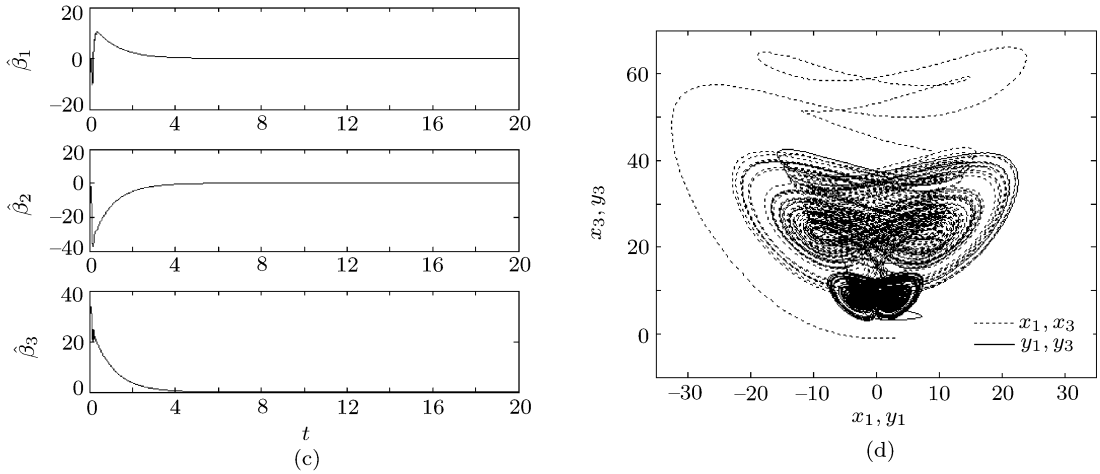


Fig.2. The GPSs of systems (7) and (8) with λ=3, where panel (a) is for GPS errors between the Chen system and the Lorenz system with unknown parameters, panels (b) and (c) for the time evolutions of parameters of $\hat{\boldsymbol{\alpha}} = \tilde{\boldsymbol{\alpha}} - \boldsymbol{\alpha}$ and $\hat{\boldsymbol{\beta}} = \tilde{\boldsymbol{\beta}} - \boldsymbol{\beta}$ respectively, and panel (d) for the projection of the synchronized attractors on plane $(x_1 - x_3)$.

Example 2 Consider the generalized projective synchronization between the hyperchaotic Lü system[24]

$$\begin{aligned}
\dot{x}_1 &= \alpha_1(x_2 - x_1) + x_4,\\
\dot{x}_2 &= -x_1x_3 + \alpha_3x_2,\\
\dot{x}_3 &= x_1x_2 - \alpha_2x_3,\\
\dot{x}_4 &= x_1x_3 + \alpha_4x_4
\end{aligned} \tag{9}$$

and the new hyperchaotic system[25]

$$\begin{aligned}
\dot{y}_1 &= -\beta_1y_1 + y_1y_3 + y_3 + u_1,\\
\dot{y}_2 &= \beta_2y_2 - y_1y_3 + u_2,\\
\dot{y}_3 &= y_1y_2 - \beta_3y_3 - \beta_4y_4 + u_3,\\
\dot{y}_4 &= -\beta_5y_1 + u_4.
\end{aligned} \tag{10}$$

Compare systems (9) and (10) with Eqs.(1) and (2), one can obtain that

$$\boldsymbol{F}(\boldsymbol{x}) = \begin{pmatrix} x_2 - x_1 & 0 & 0 & 0\\ 0 & 0 & x_2 & 0\\ 0 & -x_3 & 0 & 0\\ 0 & 0 & 0 & x_4 \end{pmatrix},$$

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_4 \end{pmatrix}, \quad \boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} x_4\\ -x_1x_3\\ x_1x_2\\ x_1x_3 \end{pmatrix},$$

$$\boldsymbol{G}(\boldsymbol{y}) = \begin{pmatrix} -y_1 & 0 & 0 & 0 & 0\\ 0 & y_2 & 0 & 0 & 0\\ 0 & 0 & -y_3 & -y_4 & 0\\ 0 & 0 & 0 & 0 & -y_1 \end{pmatrix},$$

$$\boldsymbol{\beta} = \begin{pmatrix} \boldsymbol{\beta}_1\\ \boldsymbol{\beta}_2\\ \boldsymbol{\beta}_3\\ \boldsymbol{\beta}_4\\ \boldsymbol{\beta}_5 \end{pmatrix}, \quad \text{and} \quad \boldsymbol{g}(\boldsymbol{y}) = \begin{pmatrix} y_2y_3 + y_3\\ -y_1y_3\\ y_1y_2\\ 0 \end{pmatrix}.$$

According to Theorem 1, the controller $\boldsymbol{u}$ is designed as follows:

$$\boldsymbol{u} = \frac{1}{\lambda}\begin{pmatrix} x_4\\ -x_1x_3\\ x_1x_2\\ x_1x_3 \end{pmatrix} - \begin{pmatrix} y_2y_3 + y_3\\ -y_1y_3\\ y_1y_2\\ 0 \end{pmatrix} - \frac{1}{\lambda}\begin{pmatrix} -x_1 & 0 & 0 & 0 & 0\\ 0 & x_2 & 0 & 0 & 0\\ 0 & 0 & -x_3 & -x_4 & 0\\ 0 & 0 & 0 & 0 & -x_1 \end{pmatrix}\begin{pmatrix} \tilde{\beta}_1\\ \tilde{\beta}_2\\ \tilde{\beta}_3\\ \tilde{\beta}_4\\ \tilde{\beta}_5 \end{pmatrix}$$

$$+\begin{pmatrix} y_2-y_1 & 0 & 0 & 0 \\ 0 & 0 & y_2 & 0 \\ 0 & -y_3 & 0 & 0 \\ 0 & 0 & 0 & y_4 \end{pmatrix}\begin{pmatrix} \tilde{\alpha}_1 \\ \tilde{\alpha}_2 \\ \tilde{\alpha}_3 \\ \tilde{\alpha}_4 \end{pmatrix}+\frac{1}{\lambda}\begin{pmatrix} k_1e_1 \\ k_2e_2 \\ k_3e_3 \\ k_4e_4 \end{pmatrix}$$

with

$$\dot{\hat{\boldsymbol{\alpha}}}=\dot{\tilde{\boldsymbol{\alpha}}}=\lambda[(y_2-y_1)e_1,\ -y_3e_3, y_2e_2,\ y_4e_4]^{\mathrm{T}}-\hat{\boldsymbol{\alpha}}$$

and

$$\dot{\hat{\boldsymbol{\beta}}}=\dot{\tilde{\boldsymbol{\beta}}}=(x_1e_1,\ -x_2e_2,\ x_3e_3,\ x_4e_3,\ x_1e_4)^{\mathrm{T}}-\hat{\boldsymbol{\beta}}.$$

In the simulations, we choose the unknown parameter vectors, $\boldsymbol{\alpha}$=(36, 3, 20, 1)$^{\mathrm{T}}$, and $\boldsymbol{\beta}$=(10, 2.5, 4, 2, 0.25)$^{\mathrm{T}}$, the initial conditions $\boldsymbol{x}(0)=(3,-3.5,-1,\ 1)^{\mathrm{T}}$, $\boldsymbol{y}(0)$=(2, 3, 7, 1)$^{\mathrm{T}}$, $\tilde{\boldsymbol{\alpha}}(0)$=(25, 2, 20, 0.5)$^{\mathrm{T}}$, and $\tilde{\boldsymbol{\beta}}(0)$=(5, 1, 2, 1, 0)$^{\mathrm{T}}$.

Let $k_1=k_3=0$, $\dot{k}_2=\gamma_2e_2^2$, $\dot{k}_4=\gamma_4e_4^2$, $\gamma_2=\gamma_4=1$, $k_2(0)=k_4(0)=0$, when $\lambda=-2$, the numerical results are illustrated in Fig.3. Figure 3(a) shows that the errors converge to zero as time increases, which implies that the generalized projective synchronization can be realized. Figures 3(b) and 3(c) display the evolutions of $\hat{\boldsymbol{\alpha}}=\tilde{\boldsymbol{\alpha}}-\boldsymbol{\alpha}$ and $\hat{\boldsymbol{\beta}}=\tilde{\boldsymbol{\beta}}-\boldsymbol{\beta}$ respectively. Obviously, with time passing, these parameters tend to zero, that is to say, the parameter identification is realized simultaneously. Figures 3(d)

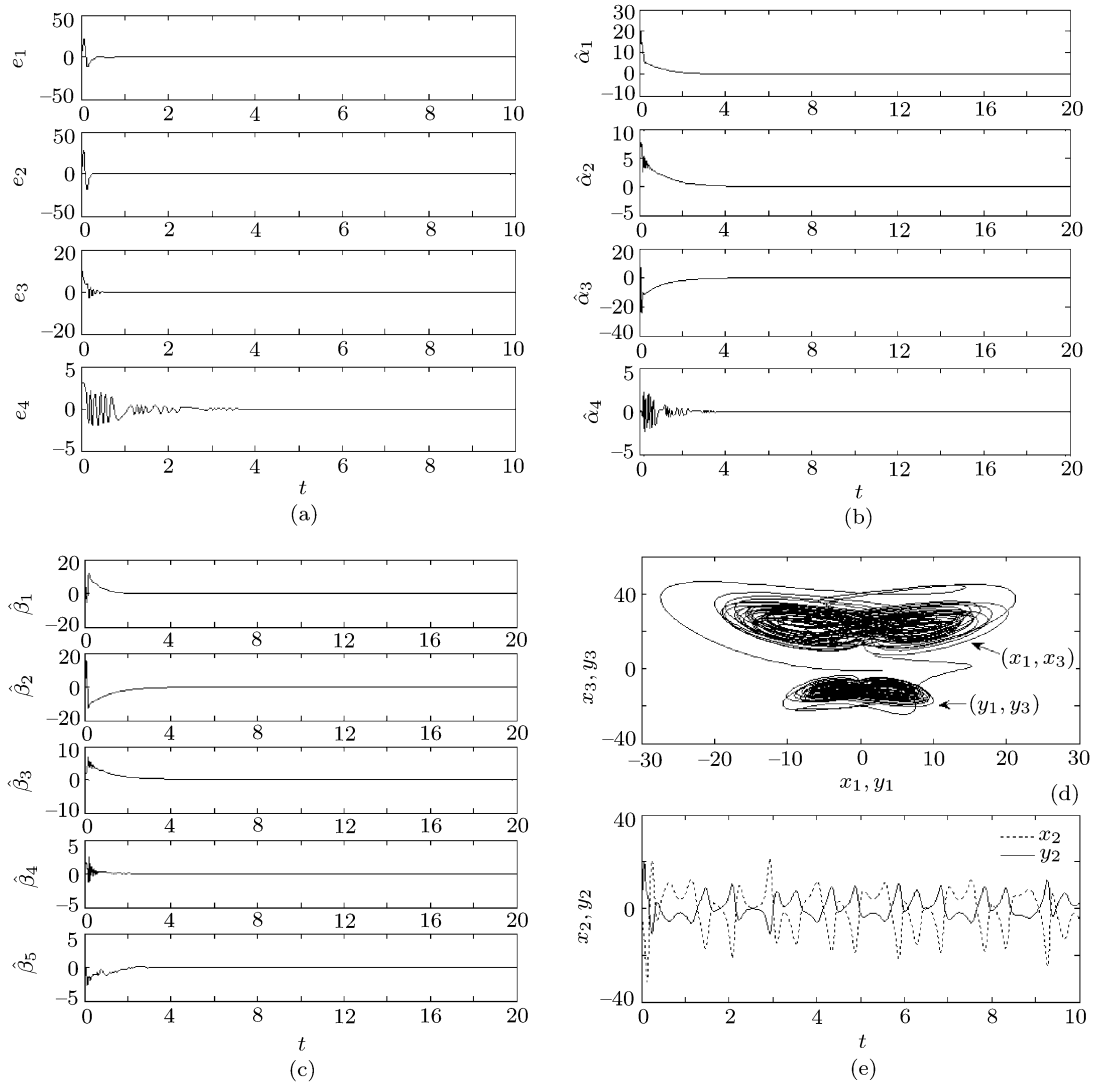


Fig.3. The GPSs of systems (9) and (10) with $\lambda=-2$, where panel (a) is for GPS errors between the hyperchaotic Lü system and the new hyperchaotic system with unknown parameters, panels (b) and (c) for time evolutions of parameters of $\hat{\boldsymbol{\alpha}}=\tilde{\boldsymbol{\alpha}}-\boldsymbol{\alpha}$ and $\hat{\boldsymbol{\beta}}=\tilde{\boldsymbol{\beta}}-\boldsymbol{\beta}$ respectively, panel(d) for the projection of the synchronized attractors on plane (x_1-x_3), and panel (e) for time evolutions of state vectors $\boldsymbol{x}_2$ and $\boldsymbol{y}_2$.

and 3(e) depict the projections of the synchronized attractors and time evolutions of state vectors respectively, which illustrates an anti-phase synchronization with $\lambda = -2$.

4. Conclusions

In this paper, we have discussed the problems about the GPS and the parameter identification between two different chaotic systems with unknown parameters. A novel nonlinear controller is designed based on the adaptive technique to realize the parameter identification, and two different chaotic (hyperchaotic) systems have achieved the globally GPS simultaneously. A sufficient condition for the globally GPS is given and proved theoretically via the Lyapunov method. Finally, the numerical simulations of the two examples have demonstrated the effectiveness and feasibility of the proposed technique.

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