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Adaptive projective synchronization with different scaling factors in networks*

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We study projective synchronization with different scaling factors (PSDF) in N coupled chaotic systems networks. By using the adaptive linear control, some sufficient criteria for the PSDF in symmetrical and asymmetrical coupled networks are separately given based on the Lyapunov function method and the left eigenvalue theory. Numerical simulations for a generalized chaotic unified system are illustrated to verify the theoretical results.

Keywords: dynamical networks, adaptive control, synchronization with different scaling factors (PSDF), scaling factor

PACC: 0545

1. Introduction

Chaotic synchronization has aroused much interest in different research fields since some pioneer work was published.[1] Several types of chaos synchronization are well known, which include complete synchronization,[2−7] phase synchronization,[8] anti-synchronization,[9] lag synchronization,[10] generalized synchronization,[11−13] projective synchronization,[14−26] etc. One of the most interesting types of synchronous behaviours of coupled chaotic systems is projective synchronization which was illustrated for the Lorenz and disc dynamo systems by Mainieri and Rehacek.[14] They reported that this phenomenon could be induced by the partially linear property of chaotic system. Xu[15] has shown that it is hard to estimate the scaling factor because it is dependent on not only the initial condition but also the underlying chaotic dynamics. From the application viewpoint, it is necessary to control the system in order to offer the occasion to select and direct the solution of synchronization in a defined way. In Refs.[15–17], the authors introduced three types of feedback controllers into the drive system to manipulate the scaling factor onto a desired value, separately. In Ref.[18], the authors used a simple and physically available controller proposed firstly in Ref.[19] to manipulate the scaling factor onto a desired value. In Ref.[20], the authors further extended the projective synchronization feature to general nonlinear systems rather than partially linear systems by applying a controller to the response system in consideration of potential application of secure communications.[21−23] Some recent results regarding the issue of projective synchronization (PS) have been reported in Refs.[24–26].

Complex dynamical networks have become a focal research topic in recent years,[27−35] which are ubiquitous in nature and society, arising from fields such as sociology, physics and biology. The criteria of synchronization in symmetric coupling networks have been reported in the literature, in other words, the coupling configuration matrix is symmetric. In fact, the coupling between nodes of many real world networks may be non-symmetric, such as the food web, metabolic networks, and epidemic networks. Therefore, the study on synchronization in networks with non-symmetric coupling has been carried out. And some criteria based on the left eigenvalue theory for synchronization without any symmetric assumption are presented in Refs.[27–29].

Recently, Liu *et al*[30] and Hu *et al*[31] studied the projective synchronization in drive–response dynamical networks (DRDNs) by means of a common scaling factor, by which the identical synchronization can be achieved in the response networks, while the drive and the response networks vectors synchronize with each other up to a common scaling factor. How-

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ever, it is common that the dynamical system on each node in networks presents different behaviours in the real world. In this paper, we present the adaptive projective synchronization with different scaling factors (PSDF) in complex networks based on the left-eigenvalue theory. An additional drive system is not needed and the coupling topology matrix is not necessarily symmetrical in our model networks, either. Furthermore, the difficulty in estimating a value of coupling coefficient for the coupled systems is avoided by using such a kind of adaptive scheme.

The rest of this paper is organized as follows. First, in Section 2, we present the coupled dynamical network model and introduce some necessary definitions and lemmas. Then, in Section 3, some sufficient criteria for PSDF in coupled dynamical networks with N nodes are derived. Illustrative examples on the chaotic unified system are shown to support the theoretical results in Section 4. Finally, some conclusions drawn from the present study are given in Section 5.

2. Model description and preliminaries

A class of chaotic systems called partially linear system is as follows:

$$\begin{aligned} \dot{\boldsymbol{u}} &= M(z)\cdot\boldsymbol{u}, \\ \dot{z} &= f(\boldsymbol{u}, z), \end{aligned} \tag{1}$$

where the state vector can be divided into two parts $(\boldsymbol{u}, z)$, the equation for z is nonlinearly related to the other variables, and the equation for the rate of change of the vector $\boldsymbol{u}$ is linearly related to $\boldsymbol{u}$ through a matrix $\boldsymbol{M}$ that is dependent on the variable z. The PS may occur when system (1) is linked through the variable z in the form

$$\begin{aligned} \dot{\boldsymbol{u}}^{\mathrm{d}} &= \boldsymbol{M}(z)\cdot\boldsymbol{u}^{\mathrm{d}}, \\ \dot{z} &= f(\boldsymbol{u}^{\mathrm{d}}, z), \\ \dot{\boldsymbol{u}}^{\mathrm{r}} &= \boldsymbol{M}(z)\cdot\boldsymbol{u}^{\mathrm{r}}, \end{aligned} \tag{2}$$

where the superscripts d and r denote the drive and the response systems, respectively.

Hu etc extended model (2) to the drive–response dynamical network model described by the following equations:

$$\begin{aligned} \dot{\boldsymbol{u}}^{\mathrm{d}} &= \boldsymbol{M}(z)\cdot\boldsymbol{u}^{\mathrm{d}}, \\ \dot{z} &= f(\boldsymbol{u}^{\mathrm{d}}, z), \\ \dot{\boldsymbol{u}}_i^{\mathrm{r}} &= \boldsymbol{M}(z)\cdot\boldsymbol{u}_i^{\mathrm{r}} + c\sum_{j=1}^{N} a_{ij}\boldsymbol{u}_j^{\mathrm{r}}, \\ & i = 1, 2, \ldots, N, \end{aligned} \tag{3}$$

where $\boldsymbol{u}_i^{\mathrm{r}} (i = 1, 2, \ldots, N)$ denote the multi- dimensional state variables of the response networks systems, N is the number of coupled nodes, the constant $c(c > 0)$ is the coupling strength, and $\boldsymbol{A} = (a_{ij})_{N\times N}$ is the real symmetric matrix reflecting the topology of network. The system is said to realize the PS, if there exists a common scaling factor constant $\alpha(\neq 0)$ such that $\lim_{t\to\infty} \left\|\boldsymbol{u}_i^{\mathrm{r}} - \boldsymbol{u}^{\mathrm{d}}\right\| = 0$ for any $i(1 \le i \le N)$.

In this paper, we do not need the drive system, and we further consider another reduced complex dynamical network consisting of N linear projective coupled identical nodes, with full diagonal coupling, and each node is an $(n+1)$-dimensional dynamical oscillator that may be chaotic. The N dynamical nodes share one common variable z. Without loss of generality, we suppose that the signal z comes from the first system. The equation of state for the network is

$$\begin{aligned} \dot{\boldsymbol{u}}_i &= \boldsymbol{M}(z)\cdot\boldsymbol{u}_i + c(t)\sum_{j=1}^{N} a_{ij}\left(\frac{\alpha_i}{\alpha_j}\boldsymbol{u}_j - \boldsymbol{u}_i\right), \\ & i = 1, 2, \ldots, N, \\ \dot{z} &= f(\boldsymbol{u}_1, z), \end{aligned} \tag{4}$$

where $\alpha_i(\neq 0, i = 1, 2, \ldots, N)$ denote the scaling factor of node i, $\boldsymbol{u}_i$ $(\neq 0, i = 1, 2, \ldots, N)$ denote the multi-dimensional state variables of network systems, N is the number of coupled nodes, and $c(t)$ is the time-varying coupling strength. In the present paper, $\boldsymbol{A} = (a_{ij})_{N\times N}$ is the same as that mentioned above, while $\boldsymbol{A}$ is irreducible and not necessarily assumed to be symmetric, where $a_{ij} \ge 0, i \ne j$; $a_{ii} = -\sum_{j=1(j\ne i)}^{N} a_{ij} (i = 1, 2, \ldots, N)$.

Letting $\tilde{\boldsymbol{u}} = \left[\frac{\boldsymbol{u}_1}{\alpha_1}, \frac{\boldsymbol{u}_2}{\alpha_2}, \ldots, \frac{\boldsymbol{u}_N}{\alpha_N}\right]^{\mathrm{T}} \in \boldsymbol{R}^{N\times n}$ and using Kronecker product, system (4) can be rewritten in the vector form as follows:

$$\begin{aligned} \dot{\tilde{\boldsymbol{u}}} &= \boldsymbol{M}(z)\cdot\tilde{\boldsymbol{u}} + c(t)\boldsymbol{A}\tilde{\boldsymbol{u}}, \\ \dot{z} &= f(\boldsymbol{u}_1, z), \end{aligned} \tag{5}$$

where $\boldsymbol{M}(z) = \boldsymbol{I}_N \otimes \boldsymbol{M}(z)$, $\boldsymbol{A} = A \otimes \boldsymbol{I}_N$, and $\boldsymbol{I}_n$ and $\boldsymbol{I}_N$ denote n-order and N-order identity matrices, respectively. Note that the scaling factors $\alpha_i(\neq 0, i = 1, 2, \ldots, N)$ can be different from each other. We give the definition of PS in networks:

Definition 1 The networks of system (4) is said to be projective synchronized with different scaling factors (PSDF) $\alpha_i(\neq 0, i = 1, 2, \ldots, N)$, if

$\lim_{t\to\infty}\left\|\frac{\boldsymbol{u}_j}{\alpha_j}-\frac{\boldsymbol{u}_i}{\alpha_i}\right\|=0$ for any $i,j=1,2,\ldots,N$. The set $S=\left\{(\boldsymbol{u}_1,\boldsymbol{u}_2,\ldots,\boldsymbol{u}_N)^{\mathrm{T}}\left|\frac{\boldsymbol{u}_1}{\alpha_1}=\frac{\boldsymbol{u}_2}{\alpha_2}=\ldots=\frac{\boldsymbol{u}_N}{\alpha_N}\right.\right\}$ is called projective synchronization manifold with the scaling factors $\alpha_i\neq 0,(i=1,2,\ldots,N)$.

Remark 1 It is easy to see that this definition of PSDF accords with complete synchronization in networks when all the scaling factors are equal.

Before stating main results, we need the following Lemma which can be induced and found in Refs.[27,30].

Lemma 1 Suppose $\boldsymbol{A}=(a_{ij})_{i,j=1}^{N}$ is an irreducible matrix, $a_{ij}\geq 0, i\neq j$; $a_{ii}=-\sum_{j=1(j\neq i)}^{N}a_{ij}$, $i=1,2,\ldots,N$.

(1) the eigenvalues of $\boldsymbol{A}$ have the order $0=\lambda_1>\lambda_2\geq\lambda_3\geq\ldots\geq\lambda_N$;

(2) $(1,1,\ldots,1)^{\mathrm{T}}$ is the right eigenvector of A and corresponding to eigenvalue 0 with multiplicity 1;

(3) Let $\boldsymbol{\xi}=(\boldsymbol{\xi}_1,\ldots,\boldsymbol{\xi}_N)\in\boldsymbol{R}^N$ is the left eigenvector corresponding to eigenvalue 0. If A is irreducible, then all $\xi_i>0, i=1,2,\ldots,N$;

(4) Define $\boldsymbol{\Xi}=\mathrm{diag}(\xi_1,\ldots,\xi_N)$, then $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{\Xi}+\boldsymbol{\Xi}\boldsymbol{A}$ will be a symmetrical and zero row sum matrix, and the eigenvalues μ_i of $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{\Xi}+\boldsymbol{\Xi}\boldsymbol{A}$ have the order $0=\mu_1>\mu_2\geq\mu_3\geq\ldots\geq\mu_N$.

In the following, we always assume that A is irreducible and $\sum_{i=1}^{N}\xi_i=1,\ \xi_i>0$, which is called the normalized left eigenvector.

3. Adaptive PS with different scaling factors in networks

For any irreducible matrix $\boldsymbol{A}_{N\times N}$ satisfying the condition of Lemma 1, let $\boldsymbol{\Xi}=\mathrm{diag}(\xi_1,\ldots,\xi_N)$, then we will have $\boldsymbol{B}=\boldsymbol{\Xi}-\boldsymbol{\xi}^{\mathrm{T}}\boldsymbol{\xi}$, where $\boldsymbol{\xi}=(\xi_1,\ldots,\xi_N)\in\boldsymbol{R}^N$ is the normalized left eigenvector corresponding to eigenvalue 0. It is easy to check that $-\boldsymbol{B}$ is symmetrical and zero row sum matrix and the eigenvalues of $-\boldsymbol{B}$ are all negative expect an eigenvalue 0 with multiplicity 1.

Let $\boldsymbol{B}=\boldsymbol{B}\otimes\boldsymbol{I}_n$, $\boldsymbol{\Theta}=\boldsymbol{\Xi}\otimes\boldsymbol{I}_N$, $\boldsymbol{I}_n$ and I_N represent n-order and N-order identity matrices, respectively.

Lemma 2 $\tilde{\boldsymbol{u}}\in S=\left\{(\boldsymbol{u}_1,\boldsymbol{u}_2,\ldots,\boldsymbol{u}_N)^{\mathrm{T}}\left|\frac{\boldsymbol{u}_1}{\alpha_1}=\frac{\boldsymbol{u}_2}{\alpha_2}=\ldots=\frac{\boldsymbol{u}_N}{\alpha_N}\right.\right\}$ if and only if $\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{B}\tilde{\boldsymbol{u}}=0$.

In fact, for any value of $\tilde{\boldsymbol{u}}$, $\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{B}\tilde{\boldsymbol{u}}=\frac{1}{2}\sum_{i,j=1}^{N}\xi_i\xi_j\left(\frac{\boldsymbol{u}_i}{\alpha_i}-\frac{\boldsymbol{u}_j}{\alpha_j}\right)^{\mathrm{T}}\left(\frac{\boldsymbol{u}_i}{\alpha_i}-\frac{\boldsymbol{u}_j}{\alpha_j}\right)$ holds if and only if $\frac{\boldsymbol{u}_i}{\alpha_i}=\frac{\boldsymbol{u}_j}{\alpha_j}$ for any $i,j=1,2,\ldots,N$.

Then, we give the scheme to PSDF with multi-scaling factors $\alpha_i\neq 0,(i=1,2,\ldots,N)$ in complex networks and prove the following theorem.

Theorem 1 Suppose that $\boldsymbol{A}=(a_{ij})_{i,j=1}^{N}$ is the irreducible matrix satisfying that $a_{ij}\geq 0, i\neq j$, and $a_{ii}=-\sum_{j=1(j\neq i)}^{N}a_{ij}, i=1,2,\ldots,N$, the maximum eigenvalue of $M(z(t))$ is bounded for any time t, and k is any positive constant larger than 0, i.e. $k>0$, then the following coupled systems with an adaptive coupling strength $c(t)$:

$$\begin{aligned}&\dot{\tilde{\boldsymbol{u}}}=\boldsymbol{M}(z)\cdot\tilde{\boldsymbol{u}}+c(t)\boldsymbol{A}\tilde{\boldsymbol{u}},\\&\dot{z}=f(\boldsymbol{u}_1,z),\\&\dot{c}(t)=-\frac{k}{2}\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{\Theta}\boldsymbol{A}\tilde{\boldsymbol{u}}.\end{aligned}\tag{6}$$

can achieve the PSDF with scaling factors $\alpha_i\neq 0,(i=1,2,\ldots,N)$.

Proof Take a sufficiently large constant l, i.e. $l\gg 0$, define a Lyapunov function as

$$V=\frac{1}{2}\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{B}\tilde{\boldsymbol{u}}+\frac{1}{k}(l-c(t))^2,\tag{7}$$

and suppose that $\bar{\lambda}(t)$ is the maximum eigenvalue of $M(z(t))$ and $\bar{M}$ is the up bound of $\bar{\lambda}(t)$, note that $BA=\Xi A$ and B is semi-positive definitive, then we will have

$$\begin{aligned}&\dot{V}\\&=\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{B}[\boldsymbol{M}(z)\tilde{\boldsymbol{u}}+c(t)\boldsymbol{A}\tilde{\boldsymbol{u}}]+(l-c(t))(\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{\Theta}\boldsymbol{A}\tilde{\boldsymbol{u}})\\&=\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{M}(z)\tilde{\boldsymbol{u}}+l\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{\Theta}\boldsymbol{A}\tilde{\boldsymbol{u}}\\&\leq\bar{M}\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{B}\tilde{\boldsymbol{u}}+l\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{\Theta}\boldsymbol{A}\tilde{\boldsymbol{u}}\\&=\bar{M}\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{B}\tilde{\boldsymbol{u}}-\bar{M}\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{\Delta}\tilde{\boldsymbol{u}}+\bar{M}\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{\Delta}\tilde{\boldsymbol{u}}+l\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{\Theta}\boldsymbol{A}\tilde{\boldsymbol{u}},\end{aligned}\tag{8}$$

where $\boldsymbol{\Delta}=\boldsymbol{\Delta}\otimes I_n$, with $\boldsymbol{\Delta}=(|\gamma_i|+1)\ \mathrm{diag}\ (1,1,\ldots,1)_N$ and $|\gamma_i|$ being the largest absolute value of $\boldsymbol{B}$'s eigenvalue. Then $\boldsymbol{B}-\boldsymbol{\Delta}$ is a negative definitive matrix.

Let $\tilde{\boldsymbol{u}}_j=\left(\frac{\boldsymbol{u}_1^j}{\alpha_1},\frac{\boldsymbol{u}_2^j}{\alpha_2},\ldots,\frac{\boldsymbol{u}_N^j}{\alpha_N}\right)$ for $j=1,2,\ldots,n$, then we will have

$$\bar{M}\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{\Delta}\tilde{\boldsymbol{u}}=\bar{M}\sum_{j=1}^{n}\tilde{\boldsymbol{u}}_j^{\mathrm{T}}(|\gamma_i|+1)\tilde{\boldsymbol{u}}_j,$$

and

$$l\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{\Theta}\boldsymbol{A}\tilde{\boldsymbol{u}}=l\sum_{j=1}^{n}\tilde{\boldsymbol{u}}_j^{\mathrm{T}}\Xi\boldsymbol{A}\tilde{\boldsymbol{u}}_j.\tag{9}$$

According to Lemma 1 (4), it can be seen that matrix $\boldsymbol{A}^{\mathrm{T}}\boldsymbol{\Xi}+\boldsymbol{\Xi}\boldsymbol{A}$ is a diagonal and zero row sum ma-

trix. Now, let $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$ be the normalized eigenvectors of the matrix $\frac{1}{2}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{\Xi}+\boldsymbol{\Xi}\boldsymbol{A})$ with corresponding eigenvalues $0 = \mu_1 > \mu_2 \geq \mu_3 \geq \ldots \geq \mu_N$. Write $\tilde{\boldsymbol{u}}_j^*$ as the projection of $\tilde{\boldsymbol{u}}_j$ on the subspace L spanned by $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n$. Then

$$\begin{aligned} & l\sum_{j=1}^{n}\tilde{\boldsymbol{u}}_j^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{A}\tilde{\boldsymbol{u}}_j \\ = & \, l\sum_{j=1}^{n}\tilde{\boldsymbol{u}}_j^{\mathrm{T}}\left[\frac{1}{2}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{\Xi}+\boldsymbol{\Xi}\boldsymbol{A})\right]\tilde{\boldsymbol{u}}_j \\ \leq & \, l\mu_2\sum_{j=1}^{n}\tilde{\boldsymbol{u}}_j^{*\mathrm{T}}\tilde{\boldsymbol{u}}_j^*. \end{aligned} \tag{10}$$

and

$$\begin{aligned} & \bar{M}\sum_{j=1}^{n}\tilde{\boldsymbol{u}}_j^{\mathrm{T}}(|\gamma_i|+1)\tilde{\boldsymbol{u}}_j \\ = & \, \bar{M}\sum_{j=1}^{n}\tilde{\boldsymbol{u}}_j^{*\mathrm{T}}(|\gamma_i|+1)\tilde{\boldsymbol{u}}_j^* \\ = & \, \bar{M}(|\gamma_i|+1)\sum_{j=1}^{n}\tilde{\boldsymbol{u}}_j^{*\mathrm{T}}\tilde{\boldsymbol{u}}_j^*. \end{aligned} \tag{11}$$

It is clear that if l is sufficiently large, then

$$\bar{M}(|\gamma_i|+1)\sum_{j=1}^{n}\tilde{\boldsymbol{u}}_j^{*\mathrm{T}}\tilde{\boldsymbol{u}}_j^* + l\mu_2\sum_{j=1}^{n}\tilde{\boldsymbol{u}}_j^{*\mathrm{T}}\tilde{\boldsymbol{u}}_j^* \leq 0. \tag{12}$$

Substituting it into inequality (8), we have

$$\begin{aligned} \dot{V} \leq & \, \bar{M}\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{B}\tilde{\boldsymbol{u}} - \bar{M}\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{\Delta}\tilde{\boldsymbol{u}} \\ \leq & \, \bar{M}\tilde{\boldsymbol{u}}^{\mathrm{T}}(\boldsymbol{B}-\boldsymbol{\Delta})\tilde{\boldsymbol{u}} \leq 0. \end{aligned} \tag{13}$$

Similarly, it can be seen that $\dot{c}(t) \geq 0$ and $c(t) = 0$ if and only if $\frac{\boldsymbol{u}_1}{\alpha_1} = \frac{\boldsymbol{u}_2}{\alpha_2} = \ldots = \frac{\boldsymbol{u}_N}{\alpha_N}$, so $c(t) > 0$ for $t > 0$.

It is obvious that $\dot{V} = 0$ if and only if $\frac{\boldsymbol{u}_1}{\alpha_1} = \frac{\boldsymbol{u}_2}{\alpha_2} = \ldots = \frac{\boldsymbol{u}_N}{\alpha_N}$. According to the well-known Lyapunov–LaSalle type theorem for functional equations, the trajectory of coupled systems, starting with an initial value, converges asymptotically to the largest invariant set H_1 contained in $H_2 = \dot{V}(t) = 0$ as $t \to \infty$, where $H_1 = \{[\tilde{\boldsymbol{u}}, c] : \tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{B}\tilde{\boldsymbol{u}} = 0, \quad c = c_0 \in R^+\}$. Therefore, $\tilde{\boldsymbol{u}}^{\mathrm{T}}\boldsymbol{B}\tilde{\boldsymbol{u}} \to 0$ and $c(t) \to c_0$.

Remark 2 Compared with the method proposed in Ref.[32] where there is a common scaling factor, the present model and scheme with different scaling factors is more general. We need not have the drive system in the model and the coupling matrix $\boldsymbol{A}$ is not necessarily symmetrical in networks. Furthermore, the difficulty in estimating the value of the coupling coefficient c_0 can be avoided with the adaptive scheme.

4. Numerical examples

In this section, we place a general unified chaotic system[23,32] on nodes of networks

$$\begin{aligned} \dot{x} &= (25\theta+10)(y-x), \\ \dot{y} &= (28-35\theta)x - xz + (29\theta-1)y, \\ \dot{z} &= xy - \frac{\theta+8}{3}z, \end{aligned} \tag{14}$$

where $\theta \in [-0.016, 1.15]$. System (14) belongs to the generalized Lorenz system when $-0.016 \leq \theta < 0.8$ and the generalized Chen system when $0.8 \leq \theta < 1.15$. And system (14) is the original Lorenz system for $\theta = 0$, the original Chen system for $\theta = 1$, and the Lü system for $\theta = 0.8$. The networks of unified chaotic systems are

$$\begin{aligned} \dot{x}_i &= (25\theta+10)(y_i - x_i) \\ &\quad + c_x\sum_{j=1(j=i)}^{5}\left(\frac{\alpha_i}{\alpha_j}x_j - x_i\right), \\ \dot{y}_i &= (28-35\theta)x_i - x_i z_i + (29\theta-1)y_i \\ &\quad + c_y\sum_{j=1(j=i)}^{5}\left(\frac{\alpha_i}{\alpha_j}y_j - y_i\right), \\ \dot{z} &= x_1 y_1 - \frac{\theta+8}{3}z, \quad (i = 1, 2, \ldots, 6) \\ \dot{c}_x &= -\frac{k}{2}x^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{A}\boldsymbol{x}, \\ \dot{c}_y &= -\frac{k}{2}y^{\mathrm{T}}\boldsymbol{\Xi}\boldsymbol{A}\boldsymbol{y}, \end{aligned} \tag{15}$$

where $\boldsymbol{x} = \left(\frac{x_1}{\alpha_1}, \frac{x_2}{\alpha_2}, \ldots, \frac{x_6}{\alpha_6}\right)^{\mathrm{T}}$, and $\boldsymbol{y} = \left(\frac{y_1}{\alpha_1}, \frac{y_2}{\alpha_2}, \ldots, \frac{y_6}{\alpha_6}\right)^{\mathrm{T}}$, with $\boldsymbol{u}_i = (x_i, y_i)^{\mathrm{T}}$, $(i = 1, 2, \ldots, 6)$ being the state vectors of node i. Without loss of generality, we choose the scaling factors $(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) = (1, -3, -0.5, 5, -2.5, 2)$, $k = 0.005$. Its topology matrix is

$$\boldsymbol{A} = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -3 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 1 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -2 \end{pmatrix}$$

with all negative real part eigenvalues except 0 with multiplicity 1. Correspondingly, $\boldsymbol{\Xi}$ = diag (0.1765, 0.0588, 0.2941, 0.2353, 0.1176, 0.1176).

And $\boldsymbol{M}(z(t)) = \begin{pmatrix} -(25\theta+10) & 25\theta+10 \\ -z(t)+28-35\theta & 29\theta-1 \end{pmatrix}$, $\theta \in [-0.016, 1.15]$. We fix $\theta = 1$ and set the initial values of system (15) $(x_1, y_1, x_2, y_2, x_3, y_3, x_4, y_4, x_5, y_5, x_6, y_6, z, c_x, c_y)(t_0) = (1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 0, 0)$. According to Theorem 1 and Remark 1 in Ref.[28], $|z(t)|$ is eventually bounded at any time $t \in [0, +\infty)$. Then the maximum eigenvalue $|\lambda(t)|$ of $M(z(t))$ is bounded when $|z(t)|$ is bounded. Figure 1 shows that $|\lambda(t)|$ is bounded at any time t, which satisfies the condition of Theorem 1.

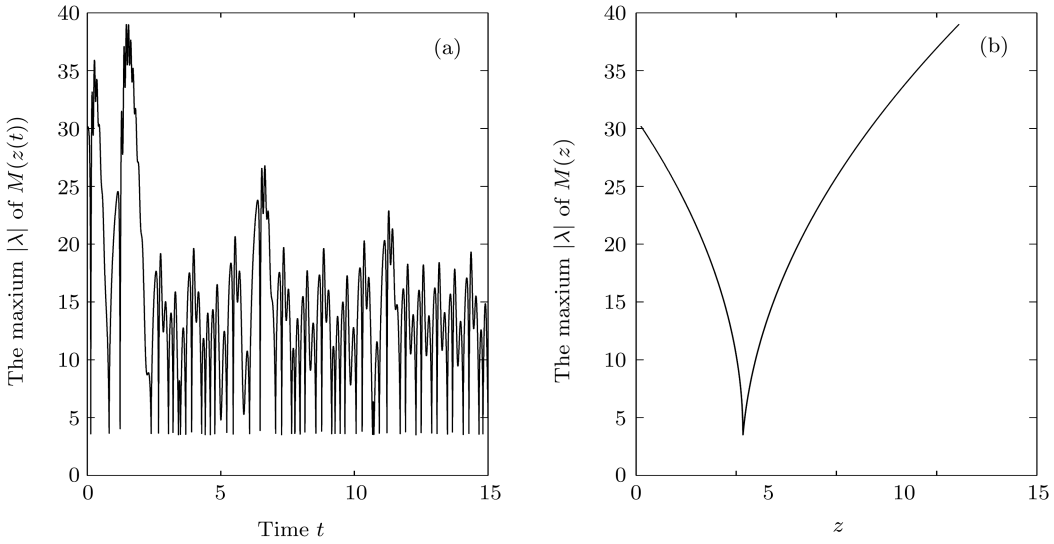


Fig.1. t vs maximum eigenvalue $|\lambda|$ of $M(z(t))$ (a) and z vs maximum eigenvalue $|\lambda|$ of $M(z)$ (b).

Figure 2 depicts the phase portrait of dynamical nodes in networks with different scaling factors respectively.

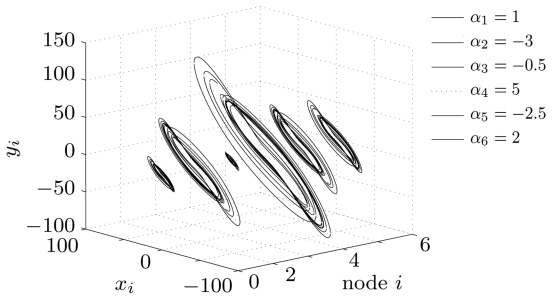


Fig.2. The portrait phase x_i vs y_i $(i = 1,2,3,4,5,6)$ of 6 nodes in networks of system (15) with the scaling factors $(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) = (1, -3, -0.5, 5, -2.5, 2)$.

Figure 3 displays that the network projective synchronization error $e = \sqrt{\sum_{i=1}^{5} \left(\frac{\boldsymbol{u}_i}{\alpha_i} - \frac{\boldsymbol{u}_{i+1}}{\alpha_{i+1}} \right)^2} \to 0$ quickly as $t \to \infty$, which implies that projective synchronization of networks with different scaling factors is achieved.

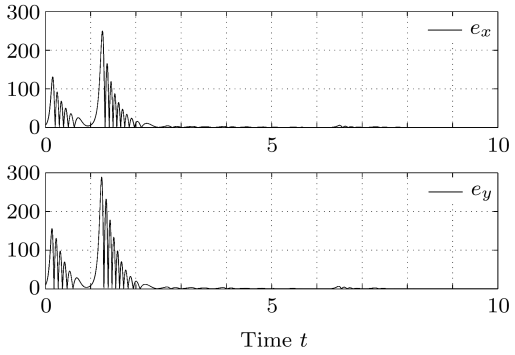


Fig.3. Global projective synchronization error $e_x = \sqrt{\sum_{i=1}^{5} \left(\frac{x_i}{\alpha_i} - \frac{x_{i+1}}{\alpha_{i+1}} \right)^2}$, $e_y = \sqrt{\sum_{i=1}^{5} \left(\frac{y_i}{\alpha_i} - \frac{y_{i+1}}{\alpha_{i+1}} \right)^2}$ in networks.

The two adaptive update laws are shown in Fig.4, and we can see $c_x \to 0.8142$, $c_y \to 1.2853$ quite quickly, as $t \to \infty$.

Remark 3 From the numerical experiments, we observe that if the value of k is tiny, the time to reach the PSDF in networks will be rather long.

Remark 4 In this illustrative example, the numerical results are perfect, any value of $\theta \in [-0.016, 1.15]$ can be chosen, and the PSDF always happens as we expected and observed even when some of the scaling factors $\alpha_i \neq 0, (i = 1, 2, \ldots, N)$ are pos-

itive and some are negative.

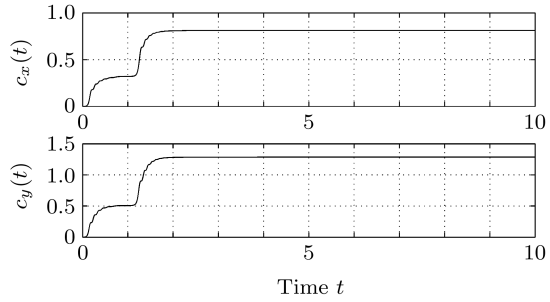


Fig.4. The update laws of $c_x(t)$ and $c_y(t)$.

5. Conclusions

Synchronization of coupled chaotic systems in networks is important for both theory and application. The schemes of adaptive PSDF in networks of symmetrical and asymmetrical coupling have been presented separately in this paper, which can avoid the difficulty in estimating the value of a coupling coefficient for the coupled systems. Strict theoretical proofs have been given based on the Lyapunov function method and the left eigenvalue theory. The interesting numerical simulations for a unified chaotic system sufficiently show the effectiveness of the proposed method.

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