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Realization of fractional-order Liu chaotic system by a new circuit unit

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A new circuit unit for the analysis and the synthesis of the chaotic behaviours in a fractional-order Liu system is proposed in this paper. Based on the approximation theory of fractional-order operator, an electronic circuit is designed to describe the dynamic behaviours of the fractional-order Liu system with $\alpha = 0.9$. The results between simulation and experiment are in good agreement with each other, thereby proving that the chaos exists indeed in the fractional-order Liu system.

Keywords: fractional-order Liu system, circuit simulation, circuit experiment
PACC: 0545

1. Introduction

Study of fractional-order chaotic systems has been a focal point of renewed interest of many researchers in the past few decades.[1−14] As is well known, The fractional calculus has an over 300-year history as a mathematical topic, but its applications in physics and engineering have become a hot point of interest just recently.[2] More recently, the behaviours of fractional-order chaotic systems have been extensively studied. For example, in Ref.[3], it is shown that the fractional-order Chua circuit with an order as low as 2.7 can produce a chaotic attractor. In Ref.[4], the fractional-order Genesio–Tesi system is analysed as an example to investigate the conditions of system parameters under which this fractional-order system can give rise to a chaotic attractor. Ahmad and Sprott[5] pointed out that chaos can exist in the fractional-order 'jerk' model with an order as low as 2.1. In Ref.[6], chaos and hyperchaos in fractional-order Rössler equations are studied numerically. It is shown that the lowest orders for chaos and hyperchaos to exist in such systems are 2.4 and 3.8 respectively. However, the previous studies mainly focused on numerical simulations to describe the dynamical behaviours of the fractional-order systems from different points. As is well known, the results on the same system may be different from each other by using different simulating softwares or simulating methods. Therefore, it is very important and valuable to design an experimental circuit to prove the numerical simulations results.

In Refs.[7, 8], the fractional-order Liu systems are designed by a chain shape unit and a tree shape unit separately. The system is analysed by both numerical simulations and experiments. But we should try to use different methods to achieve the fractional-order Liu system. In this paper, an electronic circuit model of mixed shape unit is designed for the analysis and the synthesis of the chaotic behaviours of the fractional-order Liu system. In the end, both numerical simulations and experiments are used to verify the theoretical results.

2. Fractional derivative and its approximation

There are many definitions of fractional derivative.[2] Among them, the most reputed definition should be the Riemann–Liouville definition which is described as follows:

$$\frac{\mathrm{d}^{\alpha} f(t)}{\mathrm{d}t^{\alpha}} = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^{n}}{\mathrm{d}t^{n}} \int_{0}^{t} \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} \mathrm{d}\tau \qquad (1)$$

for $n-1 \leq \alpha < n$, where $\Gamma(\cdot)$ is the Gamma function. Considering the initial values to be zero, the Laplace

transform of the Riemann–Liouville fractional derivative can be reduced to

$$L\left\{\frac{\mathrm{d}^{\alpha}f(t)}{\mathrm{d}t^{\alpha}}\right\}=s^{\alpha}L\left\{f(t)\right\}. \tag{2}$$

Therefore, the fractional integral operator of order "α" can be represented by the transfer function $F(s)=\dfrac{1}{s^{\alpha}}$ in the frequency domain.

The standard definition of fractional differ-integral does not allow the direct implementation of the fractional operators in time-domain simulations. Thus, in order to effectively analyse such systems, it is necessary to develop approximations to the fractional operators by using the standard integer order operators. In Ref.[9], an effective algorithm is developed to approximate the fractional-order transfer functions. In Table 1 of Ref.[5], approximations of $\dfrac{1}{s^{\alpha}}$ with $\alpha=0.1-0.9$ in steps of 0.1 are given within an error of approximately 2 dB. We use these approximations to realize the fractional-order Liu system by using a circuit.

In this paper, an electronic circuit model of mixed shape unit is designed as shown in Fig.1 to realize the approximations of $\dfrac{1}{s^{\alpha}}$ with $\alpha=0.1-0.9$. It is one of three basic circuit units to realize the fractional-order chaotic system. The mixed shape unit has the same important status as the chain shape unit and the tree shape unit.

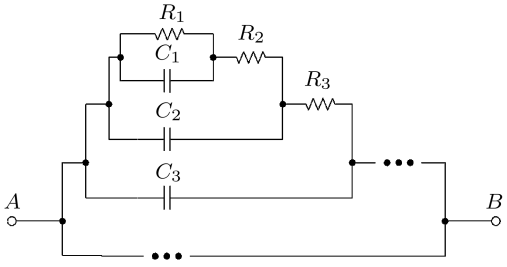


Fig.1. The mixed shape unit of $\dfrac{1}{s^{\alpha}}$.

We take $\alpha=0.9$ for example to show the design process of the mixed shape unit. From Table 1 in Ref.[5], we can obtain an approximation of $\dfrac{1}{s^{0.9}}$ with an error of 2 dB as follows:

$$\frac{1}{s^{0.9}}\approx\frac{2.2675(s+1.292)(s+215.4)}{(s+0.01292)(s+2.154)(s+359.4)}. \tag{3}$$

The mixed shape unit is designed as shown in Fig.2 to implement Eq.(3) in the Laplace domain.

Thus, the transfer function $H(s)$ between A and B can be obtained as follows:

$$H(s)=\frac{\dfrac{1}{C_3}s^2+\left(\dfrac{R_1+R_2}{R_1R_2C_1C_3}+\dfrac{R_2+R_3}{R_2R_3C_2C_3}\right)s+\dfrac{R_1+R_2+R_3}{R_1R_2R_3C_1C_2C_3}}{s^3+\left(\dfrac{R_1+R_2}{R_1R_2C_1}+\dfrac{R_2+R_3}{R_2R_3C_2}+\dfrac{1}{R_3C_3}\right)s^2+\dfrac{(C_1+C_2+C_3)R_1+(C_2+C_3)R_2+C_3R_3}{R_1R_2R_3C_1C_2C_3}s+\dfrac{1}{R_1R_2R_3C_1C_2C_3}}. \tag{4}$$

Comparing $10^{-6}H(s)$ with Eq.(3), we can obtain $R_1=61.5233\,\mathrm{M\Omega}$, $R_2=1.55226\,\mathrm{M\Omega}$, $R_3=0.0156514\,\mathrm{M\Omega}$, $C_1=0.506198\,\mu\mathrm{F}$, $C_2=0.299629\,\mu\mathrm{F}$ and $C_3=0.441014\,\mu\mathrm{F}$. In the same process, the relevant values of resistances and capacitances in Fig.1 can be obtained to realize $\dfrac{1}{s^{\alpha}}$ with $\alpha=0.1-0.9$ in steps of 0.1 in Table 1 of Ref.[5] in Laplace domain, which are shown in Tables 1 and 2 respectively.

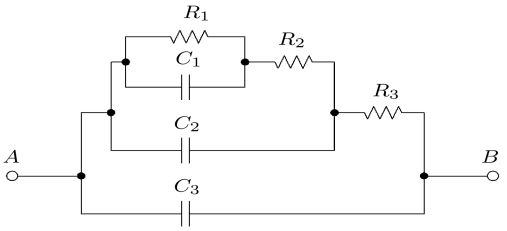


Fig.2. The mixed shape unit of $\dfrac{1}{s^{0.9}}$.

Table 1. The values of resistances in a mixed shape unit of $\frac{1}{s^\alpha}$ (2 dB).

α	$R_1/\mathrm{M\Omega}$	$R_2/\mathrm{M\Omega}$	$R_3/\mathrm{M\Omega}$	$R_4/\mathrm{M\Omega}$	$R_5/\mathrm{M\Omega}$	$R_6/\mathrm{M\Omega}$
0.1	0.6286	0.3845	0.5718			
0.2	0.9931	0.6624	0.3881	0.4685		
0.3	1.5210	1.1580	0.6317	0.3475	0.3246	
0.4	2.4000	1.9940	1.0210	0.4855	0.2400	0.1696
0.5	4.0450	3.3690	1.5450	0.6346	0.2669	0.1398
0.6	7.4190	5.4540	2.0210	0.6582	0.2163	0.07983
0.7	14.810	7.8440	1.9390	0.4253	0.09333	0.02149
0.8	31.100	7.7630	0.8377	0.08447	0.008578	
0.9	61.520	1.5520	0.01565			

Table 2. The values of capacitances in a mixed shape unit of $\frac{1}{s^\alpha}$ (2 dB).

α	$C_1/\mu\mathrm{F}$	$C_2/\mu\mathrm{F}$	$C_3/\mu\mathrm{F}$	$C_4/\mu\mathrm{F}$	$C_5/\mu\mathrm{F}$	$C_6/\mu\mathrm{F}$
0.1	15.750	0.1569	0.000631			
0.2	28.680	2.6770	0.2736	0.01259		
0.3	24.100	4.3490	0.9258	0.2109	0.02512	
0.4	16.370	3.8810	1.1800	0.3760	0.1304	0.02818
0.5	9.7780	2.6940	1.0150	0.4088	0.1836	0.06310
0.6	5.2080	1.6310	0.7165	0.3366	0.1793	0.09261
0.7	2.4880	0.9160	0.4571	0.2392	0.1408	0.1068
0.8	1.0970	0.5073	0.2833	0.1722	0.1884	
0.9	0.5062	0.2996	0.4410			

3. The fractional-order Liu chaotic system and its circuit realization

In Ref.[10], a new chaotic attractor is introduced and we call it the Liu chaotic system, which is described by

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= a(y-x),\\ \frac{\mathrm{d}y}{\mathrm{d}t} &= bx - kxz,\\ \frac{\mathrm{d}z}{\mathrm{d}t} &= -cz + hx^2.\end{aligned} \tag{5}$$

When $a = 10$, $b = 40$, $c = 2.5$, $k = 1$ and $h = 4$, system (5) has a chaotic attractor as shown in Fig.3.

Now, consider the fractional-order Liu system

$$\begin{aligned}\frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha} &= a(y-x),\\ \frac{\mathrm{d}^\alpha y}{\mathrm{d}t^\alpha} &= bx - kxz,\\ \frac{\mathrm{d}^\alpha z}{\mathrm{d}t^\alpha} &= -cz + hx^2.\end{aligned} \tag{6}$$

where α is the fractional-order satisfying $0 < \alpha \leq 1$. In this paper, we take $\alpha = 0.9$ for example to show the characteristic of fractional-order Liu chaotic system and the realization of fractional-order Liu system by a new circuit unit. When $\alpha = 0.9$, the numerical simulation phase portraits of 2.7-order Liu chaotic system are shown in Fig.4.

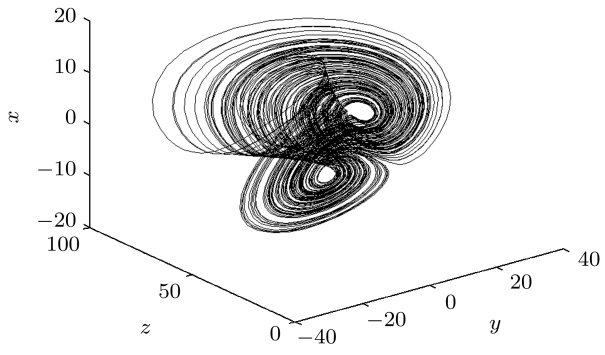


Fig.3. Chaotic attractor of the Liu system.

The designed circuitry to realize Eq.(6) is presented in Fig.5, where AD633 is a multiplier with an output coefficient of 0.1 and LM741 is an operational amplifier.

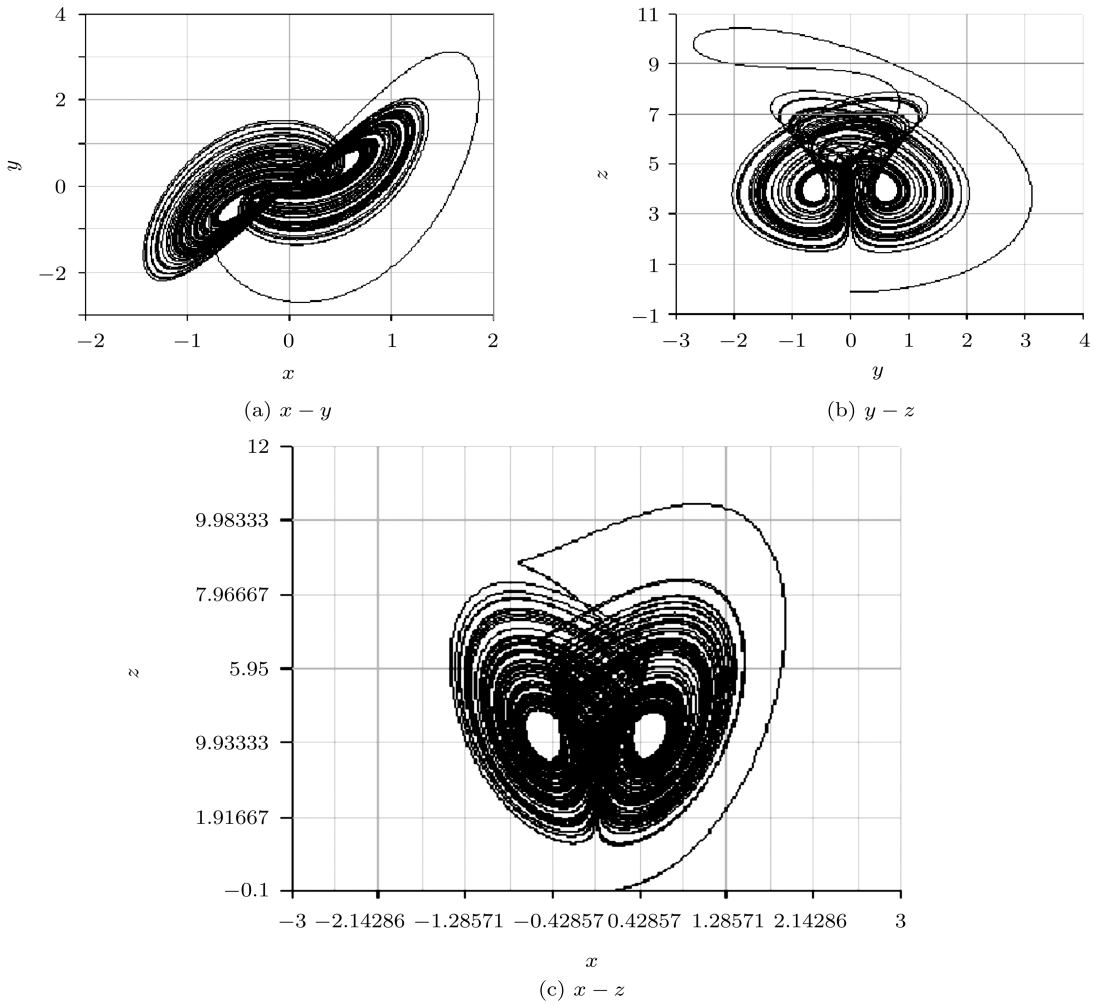
From the circuit as shown in Fig.5, the circuitry equations in Laplace domain are described by

$$\frac{X(s)}{F(s)} = \frac{R_4}{C_0 R_5}\left(\frac{Y(s)}{R_6} - \frac{R_2 X(s)}{R_1 R_3}\right),$$

$$\frac{Y(s)}{F(s)} = \frac{R_9}{C_0 R_{10}}\left(\frac{X(s)}{R_7} - \frac{kR_{16}X(s)Z(s)}{R_8 R_{15}}\right),$$

$$\frac{Z(s)}{F(s)} = \frac{R_{13}}{C_0 R_{14}}\left(-\frac{R_{16}Z(s)}{R_{11}R_{15}} + \frac{kX^2(s)}{R_{12}}\right), \quad (7)$$

where k is the AD633's output coefficient, and $k = 0.1$. Let R_1, R_2, R_3, R_4, R_6, R_7, R_{13}, R_{15}, and R_{16} each be $10\,\mathrm{k\Omega}$, R_5, R_{10}, and R_{14} each be $100\,\mathrm{k\Omega}$, R_9 and R_{11} each be $40\,\mathrm{k\Omega}$, $R_8 = 4\,\mathrm{k\Omega}$, $R_{12} = 2.5\,\mathrm{k\Omega}$, and $C_0 = 1\,\mu\mathrm{F}$.



(a) $x - y$ (b) $y - z$

(c) $x - z$

Fig.4. Chaotic attractors of the 2.7-order Liu system.

Because of $F(s) = \dfrac{1}{s^{0.9}}$, the frequency form of system (7) is obtained as follows:

$$\begin{aligned}
\frac{\mathrm{d}^{0.9}x}{\mathrm{d}t^{0.9}} &= 10(y - x),\\
\frac{\mathrm{d}^{0.9}y}{\mathrm{d}t^{0.9}} &= 40x - 10xz,\\
\frac{\mathrm{d}^{0.9}z}{\mathrm{d}t^{0.9}} &= -2.5z + 4x^2. \qquad (8)
\end{aligned}$$

Considering the accuracies of resistances and capacitances in the circuitry experiment, we choose $R_\mathrm{a} = 61.5\,\mathrm{M\Omega}$, $R_\mathrm{b} = 1.55\,\mathrm{M\Omega}$, $R_\mathrm{c} = 15.7\,\mathrm{k\Omega}$, $C_\mathrm{a} = 0.5\,\mu\mathrm{F}$, $C_\mathrm{b} = 0.3\,\mu\mathrm{F}$, and $C_\mathrm{c} = 0.44\,\mu\mathrm{F}$ to realize the mixed shape model in Fig.5, and obtain its phase portraits as shown in Fig.6. By comparing Fig.4 with Fig.6, a good qualitative agreement between the numerical simulation and the experimental measurement is observed.

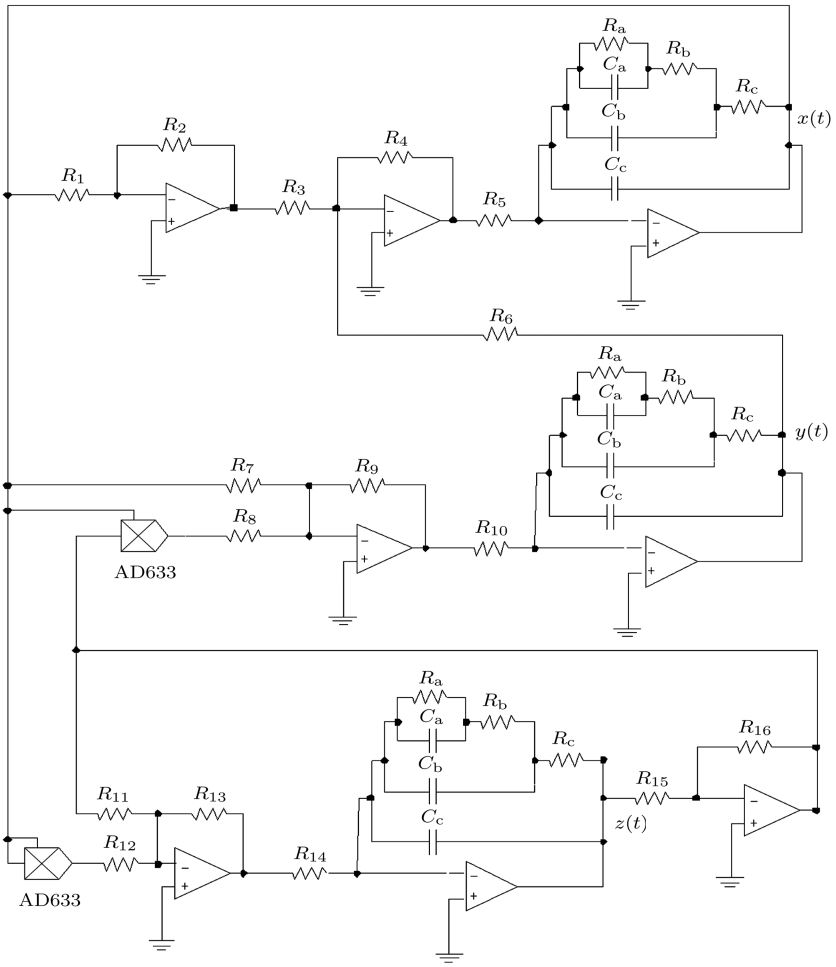
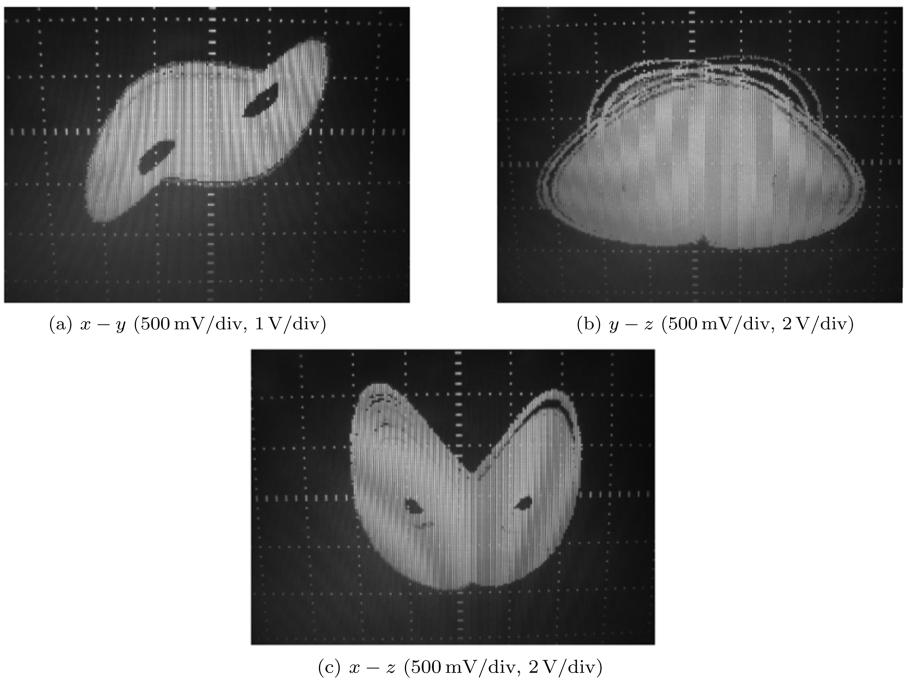


Fig.5. Circuit to realize the 2.7-order Liu system.



(a) $x-y$ (500 mV/div, 1 V/div)

(b) $y-z$ (500 mV/div, 2 V/div)

(c) $x-z$ (500 mV/div, 2 V/div)

Fig.6. Chaotic attractors of the 2.7-order Liu system realized experimentally.

4. Conclusions

In this paper, by utilizing the frequency domain technique based on the Bode diagrams, an electronic circuit model of the mixed shape is designed to realize the fractional-order operator. The chaotic behaviour of 2.7-order fractional-order Liu chaotic system is studied by numerical simulations and circuit experiments. In addition, the electronic circuit model of the mixed shape unit to realize the approximations of $\frac{1}{s^{0.9}}$ can be applied universally, even to other fractional-order systems.

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