Vol 17 No 11, November 2008
1674-1056/2008/17(11)/4022-05

Chinese Physics B

© 2008 Chin. Phys. Soc.
and IOP Publishing Ltd

New robust chaotic system with exponential quadratic term

Bao Bo-Cheng(包伯成)[a)b)†], Li Chun-Biao(李春彪)[a)c)],
Xu Jian-Ping(许建平)[d)], and Liu Zhong(刘 中)[a)]

[a)] *Department of Electronic Engineering, Nanjing University of Science and Technology, Nanjing* 210094, *China*
[b)] *School of Electrical and Information Engineering, Jiangsu Teachers University of Technology, Changzhou* 213001, *China*
[c)] *Jiangsu Institute of Economic & Trade Technology, Nanjing* 210007, *China*
[d)] *School of Electrical Engineering, Southwest Jiaotong University, Chengdu* 610031, *China*

(Received 29 February 2008; revised manuscript received 19 March 2008)

This paper proposes a new robust chaotic system of three-dimensional quadratic autonomous ordinary differential equations by introducing an exponential quadratic term. This system can display a double-scroll chaotic attractor with only two equilibria, and can be found to be robust chaotic in a very wide parameter domain with positive maximum Lyapunov exponent. Some basic dynamical properties and chaotic behaviour of novel attractor are studied. By numerical simulation, this paper verifies that the three-dimensional system can also evolve into periodic and chaotic behaviours by a constant controller.

Keywords: attractor, chaotic system, exponential quadratic term, constant controller
PACC: 0545

1. Introduction

Lorenz system and Rössler system are famous chaotic systems of three-dimensional autonomous ordinary differential equations.[1−5] There are two quadratic nonlinear terms in Lorenz system,[1−3,5] and one quadratic nonlinear term in Rössler system.[4,5] Since these systems were proposed, much attention has been paid to find out new chaotic systems, such as Chen system,[6] Lü system,[7,8] and other new chaotic systems.[9−13] Recently, Liu *et al* proposed a modified Lorenz system, which can show butterfly-shaped or reversed butterfly-shaped attractors;[11−13] Wang *et al* obtained a hyperchaotic attractor from three-dimensional modified Lorenz chaotic system.[14−16]

The chaotic system has been found to be very useful and can be applied in the fields of nonlinear circuit, communication security and fibre laser communication and so on.[17−19] There are increasing interests in exploiting chaotic dynamics in engineering applications.

It is known that Colpitts oscillator can exhibit chaotic dynamical behaviours,[20,21] which is captured by a third-order autonomous circuit model consisting of one nonlinear element via an exponential function. In this paper, we will introduce a new three-dimensional autonomous dissipative system of ordinary differential equations, with a total of six terms on the right-hand side, of which there are only one quadratic nonlinear term and one exponential quadratic nonlinear term. This system can generate a robust chaos with novel shaped chaotic attractor, and can be controlled from chaotic to periodic by a constant controller.

2. Robust chaotic system

2.1. Chaotic attractor

The new three-dimensional chaotic system is constructed by introducing an exponential quadratic term in three-dimensional Lorenz-like equations, which is described by

$$\begin{cases} \dot{x} = y - x, \\ \dot{y} = xz + c, \\ \dot{z} = \mu - \exp(xy), \end{cases} \tag{1}$$

where μ is a real constant, c is a constant control parameter, and x, y, z are the state variables.

This system is found to be chaotic in a very wide parameter range and has many interesting complex

†E-mail: mervinbao@126.com

dynamical behaviours. For example, when μ =5 and c=0, it is chaotic and displays a double-scroll chaotic attractor with novel Lorenz-unlike or Colpitts-unlike shaped, as shown in Fig.1. Its Lyapunov exponents are LE_1=0.4642, LE_2=0, LE_3=–1.4642, and the Lyapunov dimension is d_L=2.317 for initial value (1, 1, 0). Figure 2 shows the Poincaré mapping on y=–1 section, with a sheet of the attractor visualized. Figure 3 displays the waveform of state variable $x(t)$ of system (1), which is aperiodic.

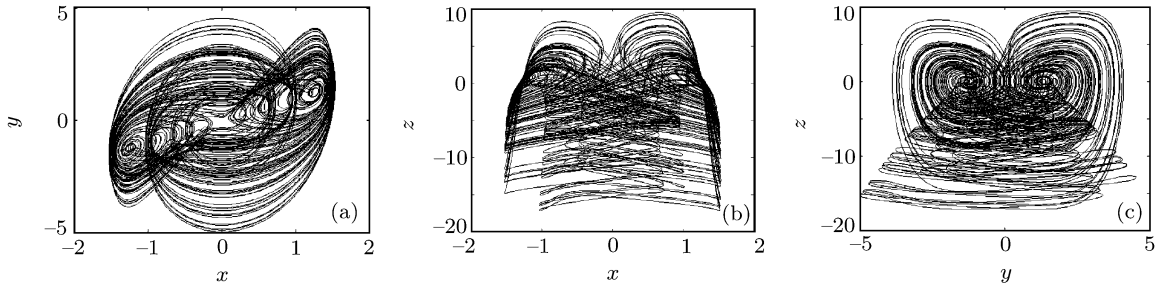


Fig.1. Phase portrait of new chaotic system in (a) $x-y$, (b) $x-z$, and (c) $y-z$ planes with μ=5 and c=0 for initial value (1, 1, 0).

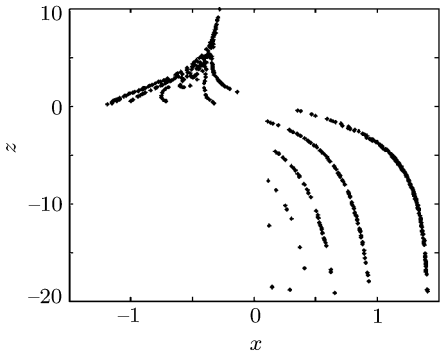


Fig.2. Poincaré mapping on y=–1 section.

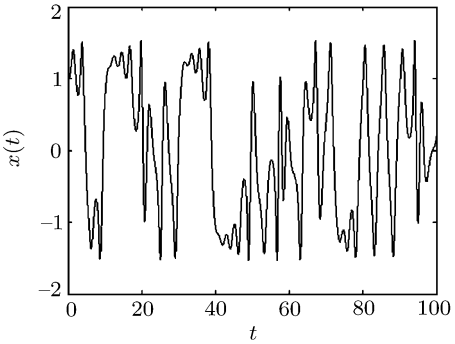


Fig.3. Waveform of the state variable $x(t)$.

2.2. Some basic properties

The proposed system has following basic properties.

(1) Symmetry and invariance

When c=0, it is easy to know that system (1) is invariant under the following transformation:

$$(x, y, z) \to (-x, -y, z), \tag{2}$$

i.e., system (1) is symmetrical about the z-axis. Furthermore, the symmetry exists for all values of the system parameters.

(2) Dissipativity and existence of attractor

For system (1), we have

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -1. \tag{3}$$

Thus, system (1) is dissipative, with an exponential contraction rate $\dot{V} = -V$, i.e., an initial volume element V_0 is contracted by the flow into a volume element $V_0 e^{-t}$ in time t. This means that each volume containing the system trajectory shrinks to zero at exponential rate of 1 as $t \to \infty$. Therefore, all system orbits will ultimately be confined to a specific subset of zero volume, and the asymptotic motion settles onto an attractor.

2.3. Equilibria and stability

The equilibrium of system (1) can be obtained by solving following equations:

$$y - x = 0, \quad xz + c = 0, \quad \text{and} \quad \mu - \exp(xy) = 0. \tag{4}$$

Obviously, if $\mu > 1$, system (1) has two equilibria:

$$S_1 = (\sqrt{\ln \mu},\ \sqrt{\ln \mu},\ -c/\sqrt{\ln \mu}),$$

$$S_2 = (-\sqrt{\ln \mu},\ -\sqrt{\ln \mu},\ c/\sqrt{\ln \mu}).$$

By linearization around the equilibria $S^* = (x^*,\ y^*,\ z^*)$, the Jacobian matrix of system (1) is

given by

$$\boldsymbol{J}_{S^*} = \begin{bmatrix} -1 & 1 & 0 \\ z^* & 0 & x^* \\ -\mu y^* & -\mu x^* & 0 \end{bmatrix}. \quad (5)$$

Equation (5) then yields following characteristic equation:

$$f(\lambda) = \lambda^3 + \lambda^2 + (\mu x^{*2} - z^*)\lambda + 2\mu x^{*2} = 0. \quad (6)$$

The coefficients of the above polynomial equation are all non-zero. Then according to Routh–Hurwitz condition, the real parts of the roots of Eq.(6) are negative if and only if

$$2\mu x^{*2} > 0,\ (\mu x^{*2} - z^*) - 2\mu x^{*2} > 0,$$

i.e.,

$$z* + \mu x^{*2} < 0. \quad (7)$$

To make the equilibrium $S^* = (x^*,\ y^*,\ z^*)$ unstable, thereby yielding the possibility for chaos to occur, the coefficients of system (1) need be chosen to satisfy the following conditions, which are obtained by the Routh–Hurwitz criterion:

$$\mu > 1 \quad \text{and} \quad z^* + \mu x^{*2} > 0. \quad (8)$$

Therefore, for the case when there is no constant controller c, i.e. c=0, if $\mu > 1$, two equilibria of this system are unstable.

For the case when there is a constant controller c, i.e. $c \neq 0$, for the equilibrium S_1,

$$c < \mu(\ln \mu)^{3/2}, \quad (9)$$

and for the equilibrium S_2,

$$c > -\mu(\ln \mu)^{3/2}. \quad (10)$$

Thus, when the control parameter c drops into following region:

$$-\mu(\ln \mu)^{3/2} < c < \mu(\ln \mu)^{3/2}, \quad (11)$$

two equilibria are unstable, i.e., the system is unstable.

When μ=5 and c=0, the characteristic roots are λ_1=–1.7291, $\lambda_{2,3} = 0.3646 \pm 3.092$i, thus two equilibria are all saddle-foci, Hopf bifurcation can appear at two equilibria of system (1).

When μ=5 and $c \neq 0$, system (1) is unstable in the control parameter region $-10.209 < c \leq 10.209$.

3. Dynamics analysis

The basic dynamics of the new chaotic system can be summarized in the following Lyapunov exponent spectra and bifurcation diagrams.

3.1. μ increasing when c=0

Figure 4 shows the Lyapunov exponent spectra and the corresponding bifurcation diagram of the state variable x versus parameter μ when system (1) has no constant controller, i.e. c=0.

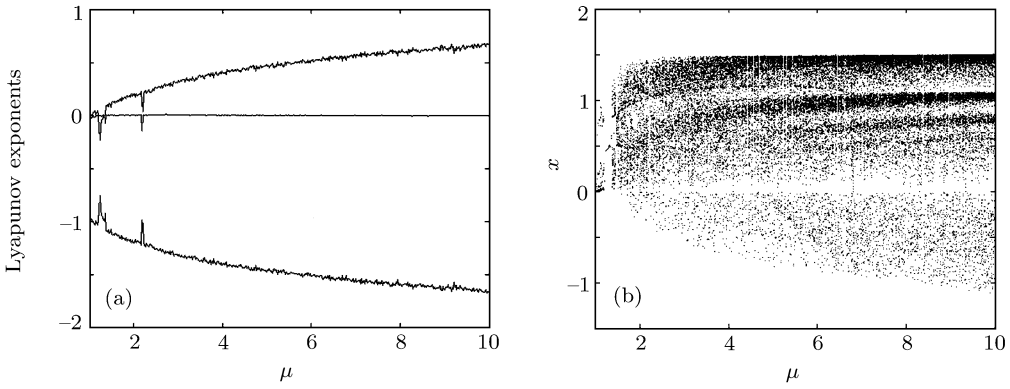


Fig.4. Lyapunov exponent spectra and bifurcation diagram of system (1) with c=0 and $1 \leq \mu \leq 10$. (a) Lyapunov exponents, (b) bifurcation diagram of x.

When $1 < \mu < 1.2$, system (1) is chaotic with a positive Lyapunov exponent LE_1. When $1.2 \leq \mu < 1.4$, the system has a zero valued exponent along with two negative Lyapunov exponents, this implies that the system becomes periodic. As μ increases further in the region $\mu \geq 1.4$, the system locates almost in the chaotic range, while only a periodic window appears near μ=2.2. Therefore, the chaos of the new

system is robust stable in a wide parameter domain, which is useful and can be applied to communication security due to its robust stable chaos without any parameter perturbation effect. Several typical periodic and chaotic orbits are obtained from simulations, as shown in Fig.5.

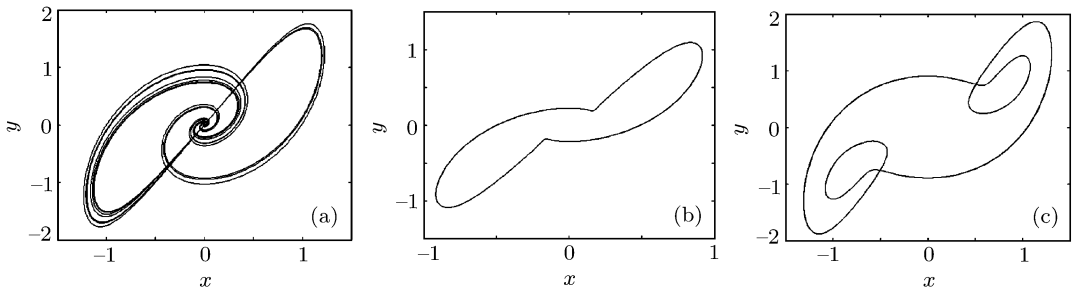


Fig.5. Periodic and chaotic orbits: (a) chaos (μ=1.1), (b) period (μ=1.3), (c) period of inner periodic window (μ=2.2).

3.2. c increasing when μ=5

Figure 6 shows Lyapunov exponent spectra and the corresponding bifurcation diagram of the state variable x versus the control parameter c varying in the region [–3, 3]. From Fig.6(a) and 6(b), symmetric dynamical behaviours can be clearly observed. The forward bifurcation appears in the region [–3, 0], and the reverse bifurcation appears in the region [0, 3]. Here the main dynamical behaviours in the region $[-3, 0]$ are considered.

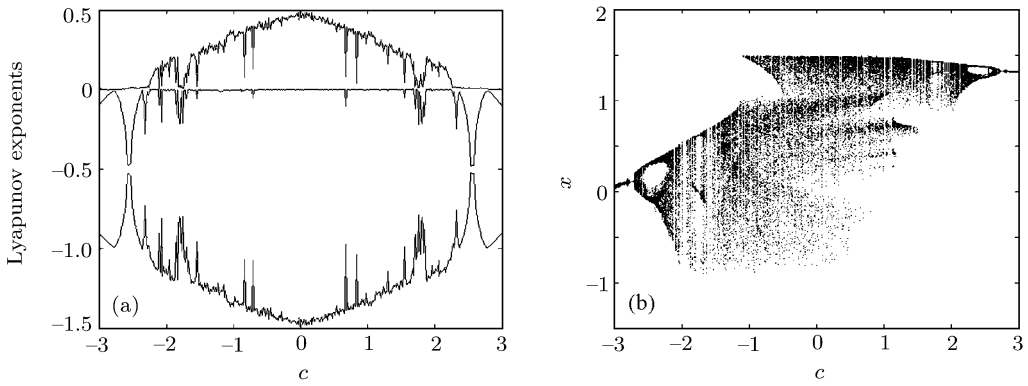


Fig.6. Lyapunov exponent spectra and bifurcation diagram of system (1) with μ=5 and $-3 \leq c \leq 3$. (a) Lyapunov exponents, (b) bifurcation diagram of x.

From Fig.6 and Eq.(11), it is clear that $-10.209 < c \leq -2.78$ is a very wide periodic region, in which system (1) oscillates with period-1 orbit; $-2.78 < c \leq -2.32$ is period-2 orbit region. As c increases in the range of $-2.32 < c \leq 0$, system (1) is chaotic and has five periodic windows. The maximum window is period-3 window in $-1.86 < c \leq -1.72$. The periodic window plays an important role in the evolution of dynamical behaviours of system (1).

Some typical period-1 orbit, period-2 orbit, chaotic orbit and period-3 orbit of inner period-window are obtained from simulations, as shown in Fig.7. Comparing Fig.7(c) with Fig.7(e), we find that these attractors of system (1) are similar and symmetrical. The corresponding Lyapunov exponents of Fig.7 are list in Table 1.

Table 1. Lyapunov exponents for several typical orbits of system (1).

c	LE_1	LE_2	LE_3
–5	0	–0.4835	–0.5219
–2.5	0	–0.2503	–0.7584
–1.75	0	–0.0229	–0.9808
–1	0.3669	0	–1.3642
1.75	0	–0.0346	–0.9710
2	0.1601	0	–1.1598

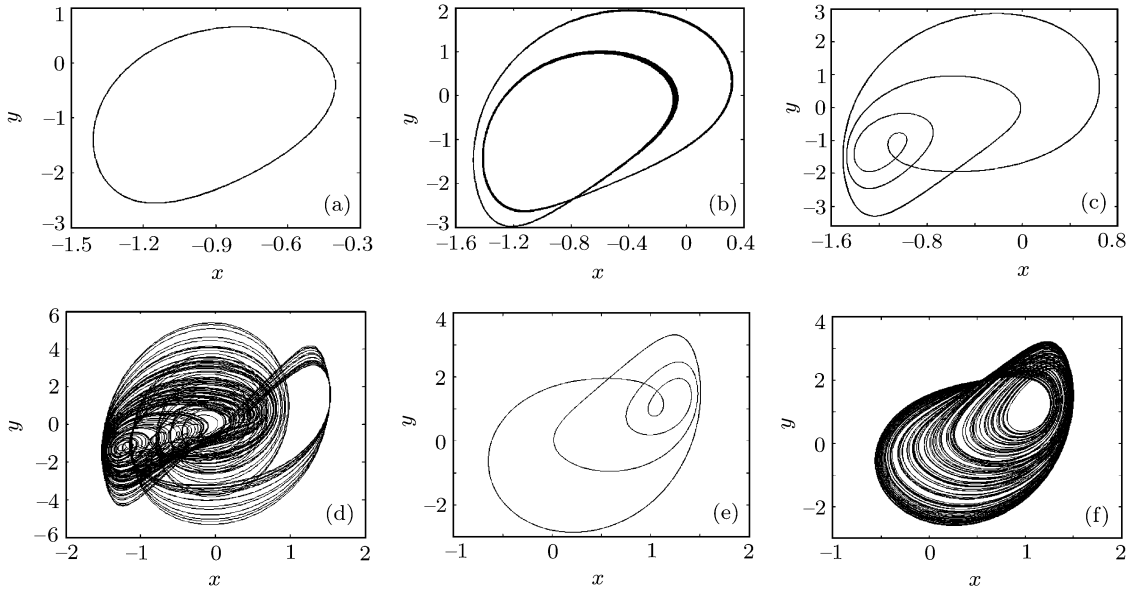


Fig.7. Periodic and chaotic orbits: (a) period-1 ($c = -5$), (b) period-2 ($c = -2.5$), (c) period-3 of inner period-window ($c = -1.75$), (d) chaos ($c = -1$), (e) period-3 of inner period-window ($c = 1.75$), (f) chaos ($c = 2$).

4. Conclusions

A new three-dimensional chaotic system with an exponential quadratic term is introduced in this paper, which displays a novel 2-scroll chaotic attractor. This system is an autonomous dissipative system with two equilibria. Some basic dynamical behaviours are further explored by calculating its Lyapunov exponent spectra and bifurcation diagrams. The attractive values of the proposed system is the existence of robust chaotic attractor over a wide variation range of parameter value, which can be applied to the chaotic communications and various chaos-based information systems. Moreover, the system can be effectually controlled by a constant parameter into a needful periodic area. Some complex characteristics of the system and its analogue electronic circuit implementation are left for further researches.

References

[1] Lorenz E N 1963 *J. Atmosph. Sci.* **20** 130
[2] Lorenz E N 1993 *The Essence of Chaos* (Seattle, WA: University of Washington Press) p148
[3] Stewart I 2002 *Nature* **406** 948
[4] Robinson R C 2004 *An Introduction to Dynamical Systems: Continuous and Discrete* (New York: Prentice-Hall) p256
[5] Rössler O E 1976 *Phys. Lett.* A **57** 397
[6] Chen G R and Ueta T 1999 *Int. J. Bifur. Chaos* **9** 1465
[7] Lü J H and Chen G R 2002 *Int. J. Bifur. Chaos* **12** 659
[8] Lü J H, Chen G R and Zhang S 2002 *Int. J. Bifur. Chaos* **12** 1001
[9] Wang G Y, Qiu S S, Li H W, Li C F and Z Y 2006 *Chin. Phys.* **15** 2872
[10] Liu L, Su Y C and Liu C X 2007 *Acta Phys. Sin.* **56** 1966 (in Chinese)
[11] Liu L, Su Y C and Liu C X 2006 *Int. J. Nonlin. Sci. Numerical Simulation* **7** 187
[12] Liu L, Liu C X and Zhang Y B 2007 *Chin. Phys. Lett.* **24** 2756
[13] Liu L, Su Y C and Liu C X 2007 *Chin. Phys.* **16** 1897
[14] Wang F Z, Chen Z Q, Wu W J and Yuan Z Z 2007 *Chin. Phys.* **16** 3238
[15] Wang F Q and Liu C X 2006 *Chin. Phys.* **15** 963
[16] Wang G Y, Liu J B and Zheng X 2007 *Chin. Phys.* **16** 2278
[17] Barbara C and Silvano C 2002 *Int. J. Circuit Theory and Applications* **30** 625
[18] Udaltsov V S, Goedgebuer J P, Larger L, Cuenot J B, Levy P and Rhodes W T 2003 *Optics and Spectroscopy* **95** 11
[19] Zhang S H and Shen K 2003 *Chin. Phys.* **12** 149
[20] Kennedy M P 1994 *IEEE Trans. Circuits and Syst.* I **41** 771
[21] Mykolaitis G, Tamaševičius A and Bumelienė S 2004 *Electron. Lett.* **40** 91