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A new Rösslor hyperchaotic system and its realization with systematic circuit parameter design*

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Based on two modified Rösslor hyperchaotic systems, which are derived from the chaotic Rösslor system by introducing a state feedback controller, this paper proposes a new switched Rösslor hyperchaotic system. The switched system contains two different hyperchaotic systems and can change its behaviour continuously from one to another via a switching function. On the other hand, it presents a systematic method for designing the circuit of realizing the proposed hyperchaotic system. In this design, circuit state equations are written in normalized dimensionless form by rescaling the time variable. Furthermore, an analogous circuit is designed by using the proposed method and built for verifying the new hyperchaos and the design method. Experimental results show a good agreement between numerical simulations and experimental results.

Keywords: hyperchaos, Rösslor system, circuit implementation, parameter design
PACC: 0545

1. Introduction

The discovery of chaos is one of the main achievements in the modern science.[1] Lorenz chaotic system[2] has led to a new era in the study of chaos and its applications. Based on the Lorenz equation, many researchers from different disciplines such as mathematics, physics, etc. extensively investigated the dynamical property of chaotic systems.[3−16]

In 1976, Rösslor found a chaotic model called Rösslor system,[3] which is a simple and prototypical equation with chaos similar to the Lorenz model. In 1979, hyperchaos was further introduced by Rösslor in Ref.[4], who named hyperchaos as a chaotic phenomenon with more than one positive Lyapunov exponents, implying that its dynamics is expanded in different directions simultaneously. It shows that hyperchaos has an increasing randomness and higher unpredictability that can be used to improve the security of chaotic communication system. On the other hand, from the engineering viewpoint of secure communication, a large number of chaotic/hyperchaotic functions are required for the security of the message to be transmitted, because only a few chaotic systems can be easily reconstructed by the interceptor and the message can be easily extracted by using well-known nonlinear signal processing techniques. Therefore, the theoretical design and circuit realization of various hyperchaotic signals are still a nontrivial task with interesting impact on both basic research and engineering applications and have recently become a focal research topic.[17−20]

One major task of applications of chaos-based information systems is the circuit realization of chaos/hyperchaos for the generation of various chaotic attractors. This stimulates the research of generating complex signals by using electronic circuits. In this endeavour, some hyperchaotic systems and their circuit realization have been proposed[17−20] and can be found in the literature. However, there does not seem to be any systematic method in the literatures for designing circuit parameters of a continuous hyperchaotic /chaotic system.

Considering these problems mentioned above, we present yet another new switched hyperchaotic system based on the chaotic Rösslor system, which contains two hyperchaotic systems and can transit continuously between one and another. Furthermore, a systematic method is developed here for realizing the new hyperchaotic system. The design method can be also applied to implement other continuous autonomous

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hyperchaotic/chaotic systems in analogue form.

This paper is organized as follows. In Section 2, a new hyperchaotic Rösslor system is presented, and its basic dynamical properties are also investigated in brief. Section 3 introduces a switched hyperchaotic system. In Section 4, a systematic parameter design approach is described for realizing the hyperchaotic Rösslor system via an electronic circuit. Then, the experimental results of the system are reported in Section 4. Finally, some concluding remarks are given in Section 5.

2. A new hyperchaotic Rösslor system

Rösslor system[3] is described as

$$\begin{cases} \dot{x} = -y - z, \\ \dot{y} = x + ay, \\ \dot{z} = xz - cz + k. \end{cases} \tag{1}$$

Based on the Rösslor chaotic system, one constructed a new fourth-order hyperchaotic system by introducing a state variable u and a corresponding system of differential equations as follows:

$$\begin{cases} \dot{x} = -y - z, \\ \dot{y} = x + ay - du, \\ \dot{z} = bxz - cz + k, \\ \dot{u} = exu - fz + g, \end{cases} \tag{2}$$

where a, b, c, d, e, f, g are constants, and k is a constant control parameter to be determined.

To create hyperchaos from system (2), it should possesses two positive Lyapunov exponents along with one zero and one negative Lyapunov exponents. Our numerical experiments show that system (2) has a hyperchaotic attractor with two positive Lyapunov exponents, L_1=0.042, L_2=0.019 (L_3=0, L_4=–1.262) for a=0.2, b=28/13, c=7, d=1.5, e=0.01, f=0.1, g=0.18 and k=0.2, as shown in Figs.1(a)–1(f).

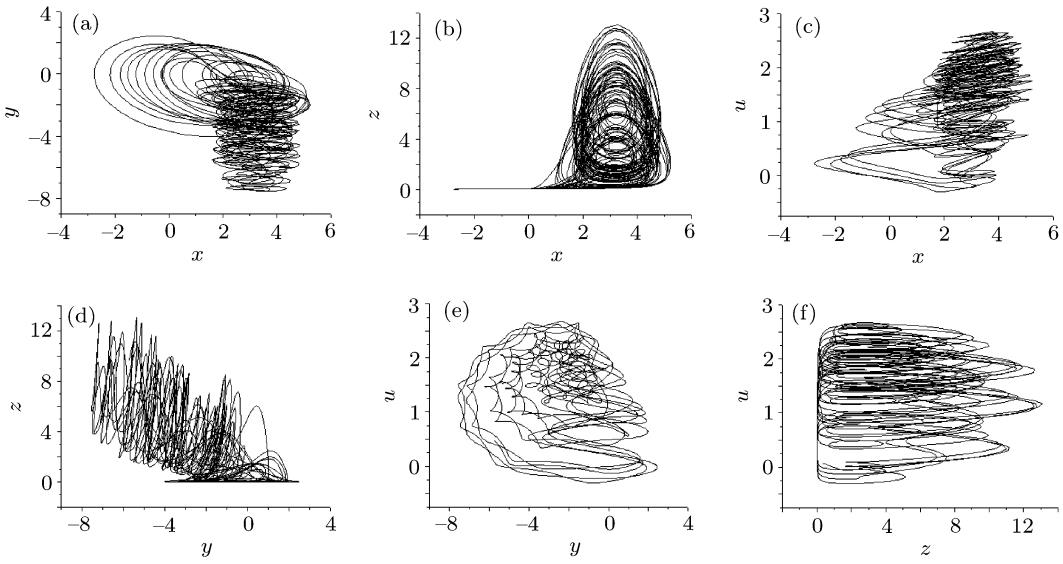


Fig.1. Hyperchaotic Rösslor attractors obtained from numerical simulations.

Basic dynamics of the new hyperchaotic system can be described via the following Lyapunov exponent spectrum (to clearly see positive Lyapunov exponents, vertical axes are limited in a certain range, so we cannot see the fourth curve, as shown in Fig.2(a)) and bifurcation diagram, which are shown in Figs.2(a) and 2(b), where a=0.2, b=28/13, c=7, d=1.5, e=0.01, f=0.1 and g=0.18 are fixed but k is varied. According to Fig.2, the dynamical behaviours of system (2) are summarized as follows.

Obviously, when $k \in (0.01; 0.34]$, the system has two positive Lyapunov exponents, implying a hyperchaotic orbit, as shown in Fig.3(a); while for $k \in [0.36; 0.42] \cup [0.55, 0.58]$, the system becomes chaotic with a single positive Lyapunov exponent, and its attractor is shown in Fig.3(b). In the region of $k \in [0.6; 0.61)$, $L_1 = L_2 = 0$, $L_3 < 0$, $L_4 < 0$, implying a quasi-periodic orbit shown in Fig.3(c). In the region of $k \in$

(0.42; 0.55)∪(0.58; 0.6)∪[0.61;1], the maximum Lyapunov exponent equals zero, implying a periodic orbit shown in Fig.3(d). The whole evolution process can be clearly seen from the bifurcation diagram of x, as shown in Fig.2(b). From the bifurcation diagram, one can see that system (2) eventually evolves into a periodic orbit.

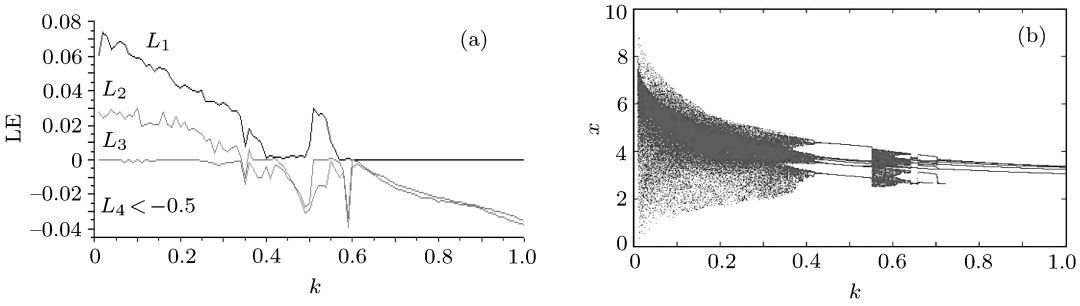
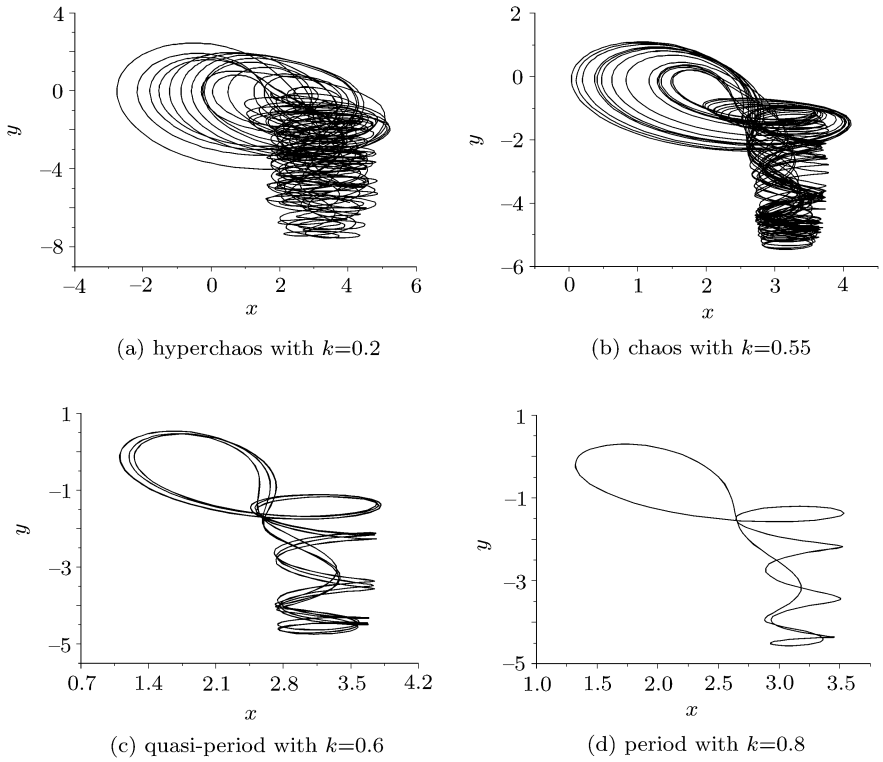


Fig.2. Lyapunov exponent spectrum and Bifurcation diagram obtained from numerical simulations: (a) Lyapunov exponent spectrum; (b) bifurcation diagram.



(a) hyperchaos with k=0.2 (b) chaos with k=0.55

(c) quasi-period with k=0.6 (d) period with k=0.8

Fig.3. Phase portraits with variable parameter k.

3. Switched hyperchaotic Rösslor system

Of particular interest is the fact that this chaotic system still display a hyperchaotic attractor when its parameter e=0. That is, system (2) with e=0 is described by

$$\begin{cases} \dot{x} = -y - z, \\ \dot{y} = x + ay - du, \\ \dot{z} = bxz - cz + k, \\ \dot{u} = -fz + g. \end{cases} \tag{3}$$

When a= 0.2, b=28/13, c=7, d=1.5, f=0.1, g=0.18 and k=0.2, this system has a hyperchaotic attrac-

tor with two positive Lyapunov exponents (L_1=0.046, L_2=0.035).

According to Eq.(3), the system has only one equilibrium point:

$$\tilde{x} = \frac{ch-k}{bh}, \quad \tilde{y} = -h,$$
$$\tilde{z} = h, \quad \tilde{u} = \frac{ch-k}{bhd} - \frac{ah}{d}, \tag{4}$$

here $h = g/f$. But system (2) has three equilibrium points with coordinates

$$\tilde{x} = \frac{c\tilde{z}-k}{b\tilde{z}}, \quad \tilde{y} = -\tilde{z}, \quad \tilde{u} = \frac{c\tilde{z}-k}{bd\tilde{z}} - \frac{a\tilde{z}}{d},$$
$$\tilde{z} = A\tilde{z}^3 + B\tilde{z}^2 - 2kc\tilde{z} + k^2 = 0,$$

where $A = b^2df - abce$; $B = c^2e + abk + b^2dg$.

It is easy to verify that systems (1), (2) and (3) are not topologically equivalent since the former has two equilibria but the middle has three equilibria and the latter has only one. Moreover, it is found that the structure of system (3) is simpler than hyperchaotic Rösslor system (4) by comparing Eq.(3) with Ref.[4].

Based on Eqs.(2) and (3), we can design a switched hyperchaotic system, which can transit from one attractor to another via a switching function. The switched system is given by

$$\begin{cases} \dot{x} = -y - z, \\ \dot{y} = x + ay - du, \\ \dot{z} = bxz - cz + k, \\ \dot{u} = S(u)exu - fz + g, \end{cases} \tag{5}$$

where

$$S(u) = \begin{cases} 1, & u \geq m, \\ 0, & u < m, \end{cases} \tag{6}$$

and m is a constant. Obviously, when the state variable of the switched system, $u \geq m$, it runs on system (2), while when $u < m$, it runs on system (3). Hence, as $t \to \infty$, the switched system changes its behaviour continuously between the hyperchaotic systems (2) and (3), leading a more stochastic system. So the switched system can provide more complex signals for chaos-based secure communications. Time signal and phase portrait of the switched system are shown in Fig.4, where m=1.5.

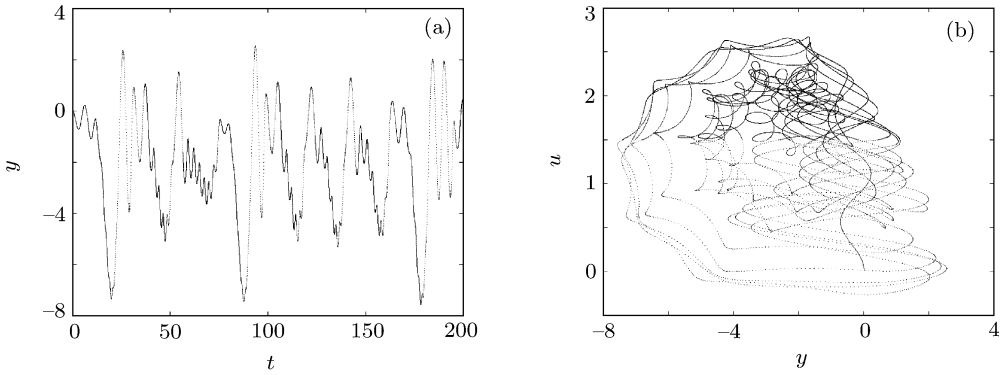


Fig.4. Wave form and phase portrait of switched system (5): (a) waveform of y; (b) phase portrait on $y-u$ plane. Solid lines denote the wave (phase portrait) of system (2) and dashed lines denote that of system (3).

4. Circuit design and realization

In this section, an electronic circuit is designed to physically realize the new switched hyperchaotic system (5), as shown in Fig.5. The circuit performs the original hyperchaotic equation (5) directly, and it consists of four channels to realize the integration, addition and subtraction of the four state variables x, y, z and u, respectively. Analogue multipliers AD633 are employed to perform the required multiplication operations, which correspond to the cross-product nonlinear terms in Eq.(5). The switching function S(u) can be realized by the analogue multiplexer CD4052, which is controlled by the voltage U_4. When the switch is turn on, the circuit corresponds to system (2), when it is turned off, then the circuit corresponds to system (3).

Considering the property of multiplier AD633, whose gain is tenth of the two-input product, for this type of realization (Fig.5) the corresponding state equations are given by

$$\begin{cases} \dfrac{\mathrm{d}U_1}{\mathrm{d}t} = -\dfrac{R_1}{C_1R_4}\left(\dfrac{U_2}{R_2}+\dfrac{U_3}{R_3}\right), \\ \dfrac{\mathrm{d}U_2}{\mathrm{d}t} = \dfrac{R_7}{C_2R_{11}}\left(\dfrac{U_1}{R_8}+\dfrac{U_2}{R_9}-\dfrac{U_4}{R_{10}}\right), \\ \dfrac{\mathrm{d}U_3}{\mathrm{d}t} = \dfrac{R_{14}}{C_3R_{18}}\left(\dfrac{U_1U_3}{10R_{15}}-\dfrac{U_3}{R_{16}}+\dfrac{U_{\mathrm{b1}}}{R_{17}}\right), \\ \dfrac{\mathrm{d}U_4}{\mathrm{d}t} = \dfrac{R_{21}}{C_4R_{25}}\left(\dfrac{U_1U_4}{10R_{22}}-\dfrac{U_3}{R_{23}}+\dfrac{U_{\mathrm{b2}}}{R_{24}}\right). \end{cases} \tag{7}$$

Obviously, the state voltages in Eq.(7), U_1, U_2, U_3, U_4, correspond to the state variables in Eq.(5), x, y, z, u, respectively. Let $R_4 = R_{11} = R_{18} = R_{25} = R$, $C_1 = C_2 = C_3 = C_4 = C$, we may write Eq.(7) in normalized dimensionless form by rescaling $\tau = t/(RC)$ and then redefining τ as t. Thus,

$$\begin{cases} \dfrac{\mathrm{d}U_1}{\mathrm{d}t} = -\dfrac{R_1}{R_2}U_2 - \dfrac{R_1}{R_3}U_3, \\ \dfrac{\mathrm{d}U_2}{\mathrm{d}t} = \dfrac{R_7}{R_8}U_1 + \dfrac{R_7}{R_9}U_2 - \dfrac{R_7}{R_{10}}U_4, \\ \dfrac{\mathrm{d}U_3}{\mathrm{d}t} = \dfrac{R_{14}}{10R_{15}}U_1U_3 - \dfrac{R_{14}}{R_{16}}U_3 + \dfrac{R_{14}}{R_{17}}U_{\mathrm{b1}}, \\ \dfrac{\mathrm{d}U_4}{\mathrm{d}t} = \dfrac{R_{21}}{10R_{22}}U_1U_4 - \dfrac{R_{21}}{R_{23}}U_3 + \dfrac{R_{21}}{R_{24}}U_{\mathrm{b2}}. \end{cases} \tag{8}$$

Comparing Eq.(5) with the normalized Eq.(8), we obtain

$$\begin{cases} \dfrac{R_1}{R_2} = \dfrac{R_1}{R_3} = 1,\ \dfrac{R_7}{R_8} = 1,\ \dfrac{R_7}{R_9} = a, \dfrac{R_7}{R_{10}} = d, \\ \dfrac{R_{14}}{10R_{15}} = b,\ \dfrac{R_{14}}{R_{16}} = c,\ \dfrac{R_{14}}{R_{17}}U_{\mathrm{b1}} = k, \\ \dfrac{R_{21}}{10R_{22}} = e,\ \dfrac{R_{21}}{R_{23}} = f,\ \dfrac{R_{21}}{R_{24}}U_{\mathrm{b2}} = g, \end{cases} \tag{9}$$

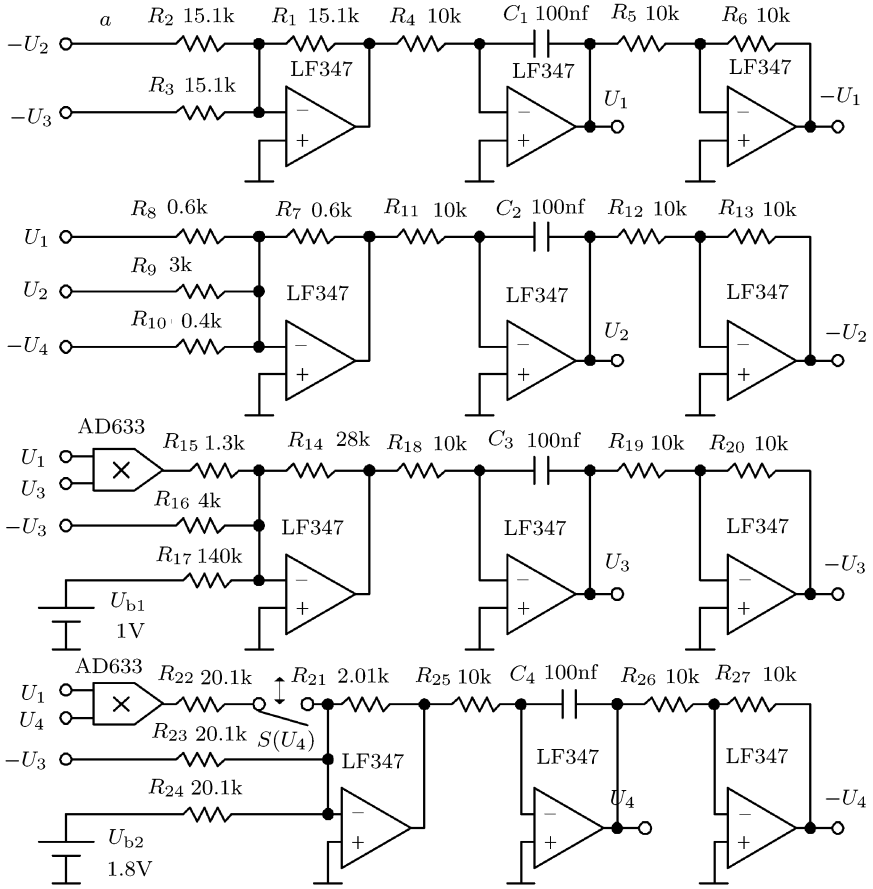
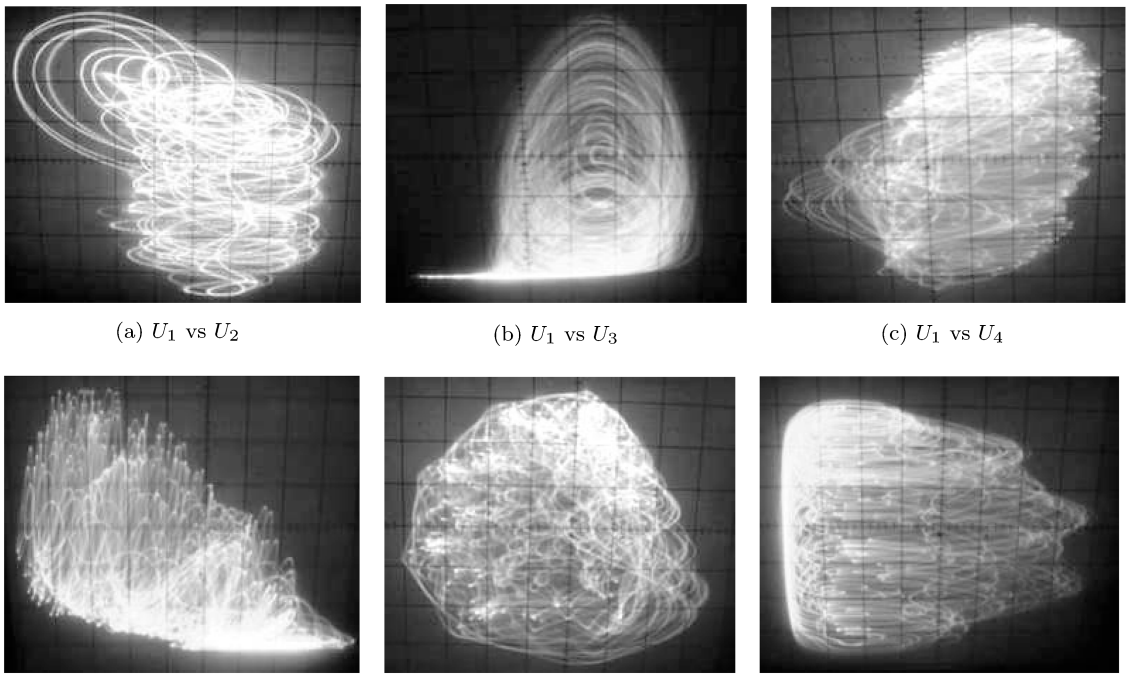


Fig.5. Circuit of implementing the switched Rösslor hyperchaotic system.

and

$$\begin{cases} R_2 = R_3 = R_1,\ R_8 = R_7,\ R_9 = \dfrac{R_7}{a}, R_{10} = \dfrac{R_7}{d}, \\ R_{15} = \dfrac{R_{14}}{10b},\ R_{16} = \dfrac{R_{14}}{c},\ R_{17} = \dfrac{R_{14}}{k}U_{\mathrm{b1}}, \\ R_{22} = \dfrac{R_{21}}{10e},\ R_{23} = \dfrac{R_{21}}{f},\ R_{24} = \dfrac{R_{21}}{g}U_{\mathrm{b2}}. \end{cases} \tag{10}$$

Thus, if we take a=0.2, b=28/13, c=7, d=1.5, e=0.01, f=0.1, g=0.18, k=0.2 as well as R_1=15.1 kΩ, R_7=0.6 kΩ, R_{14}=28 kΩ, R_{21}=2.01 kΩ, U_{b1} = 1.0V and U_{b2}=1.8V, then the parameters calculating from Eq.(10) are: $R_2 = R_3$=15.1 kΩ, R_8=0.6 kΩ, R_9=3 kΩ, R_{10}=0.4 kΩ, R_{15}=1.3 kΩ, R_{16}=4 kΩ, R_{17}=140 kΩ, R_{22}=20.1 kΩ, R_{23}=20.1 kΩ, R_{24}=20.1 kΩ, which are labelled in Fig.5, and others are also shown in Fig.5. The adjustable parameters a, c, d, e, f and k in system (5) can be implemented by adjusting the linear resistors R_9, R_{10}, R_{15}, R_{16}, R_{22} and R_{23}, and parameters b and g in system (5) can be implemented by adjusting the voltage U_{b1} and U_{b2}. When R_9=0.6 kΩ, R_{10}=3 kΩ, R_{15}=1.3 kΩ, R_{16}=4 kΩ, $R_{22} = R_{23}$ = 20.1 kΩ, U_{b1}=1.0V and U_{b2}=1.8V, various hyperchaotic attractors obtained from our laboratory experiments are shown in Figs.6(a)–6(f).



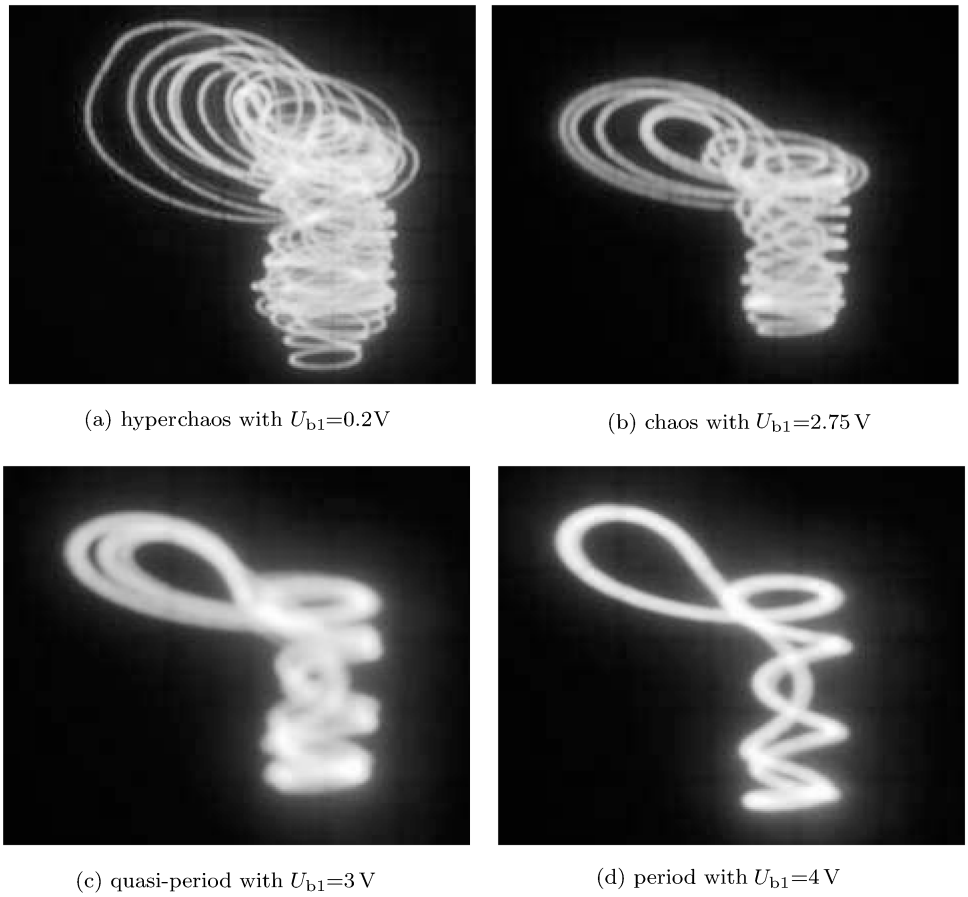
(a) U_1 vs U_2 (b) U_1 vs U_3 (c) U_1 vs U_4

(d) U_2 vs U_3 (e) U_2 vs U_4 (f) U_3 vs U_4

Fig.6. Attractors measured in the experimental setup. The supplies of all active devices are ±15 volts. The scales of both vertical and horizontal axes are 3V/div.

Let the parameter k, i.e., the voltage of U_{b1} be varying, the circuit will work in different states, that is, bifurcations from hyperchaos to chaos, quasi-period until period occur in the circuit. When U_{b1}=1 V, the circuit exhibits a hyperchaotic oscillation shown in Fig.7(a); when U_{b1}=2.75 V, the circuit exhibits a chaotic oscillation shown in Fig.7(b); while when U_{b1}=3 V, the circuit exhibits a quasi-periodic oscillation shown in Fig.7(c) and when U_{b1}=4 V, a periodic oscillation appears in the circuit, as shown in Fig.7(d).

Comparing Figs.6 and 7 with Figs.1 and 3 respectively, the proposed systematic design method for implementing continuous hyperchaotic system is verified by a good agreement between computer simulations and experimental results.



(a) hyperchaos with U_{b1}=0.2V (b) chaos with U_{b1}=2.75 V

(c) quasi-period with U_{b1}=3 V (d) period with U_{b1}=4 V

Fig.7. Experimental phase portraits in bifurcations from hyperchaos to chaos, quasi-period until period. The supplies of all active devices are ±15 volts. The scales of both vertical and horizontal axes are 3V/div.

5. Conclusions and discussions

A new switched hyperchaotic Rösslor system has been introduced and discussed in this paper. The switched system is produced as a complex hyperchaotic generator which contains two different hyperchaotic systems and changes its behaviour continuously from one system to another. Some basic dynamical behaviours of the system are further explored by calculating its Lyapunov exponent spectrum and bifurcation diagrams. Furthermore, a new systematic parameter design method for realizing the hyperchaos via an analogue circuit has been proposed and developed. With this method, we can easily determine the designed circuit parameters, which accurately correspond to the original system parameters. Finally, an analogue circuit is built for realizing the new hyperchaotic system and verifying the design, showing a good agreement between system simulations and experimental results. The systematic method for designing hyperchaotic circuit parameters can be used to other continuous hyperchaotic/chaotic systems.

The attractive property of the proposed system is the transition characteristic in different hyperchaotic attractors, which might be useful for applications in chaotic communications and various chaos-based information systems. First, because of its random transition characteristic, it can generate more complex pseudo-noise signals for information security and secure communication systems. Second, in chaos-based communications, for example, the switched system provides two different hyperchaotic attractors for the chaos-shift-keying system that maps different symbols to different chaotic attractors, which can be produced from the switched system by controlling its switching function.

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