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Nonlinear transport of Bose–Einstein condensates in a double barrier potential*

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The stable nonlinear transport of the Bose–Einstein condensates through a double barrier potential in a waveguide is studied. By using the direct perturbation method we have obtained a perturbed solution of Gross–Pitaevskii equation. Theoretical analysis reveals that this perturbed solution is a stable periodic solution, which shows that the transport of Bose–Einstein condensed atoms in this system is a stable nonlinear transport. The corresponding numerical results are in good agreement with the theoretical analytical results.

Keywords: the nonlinear transport, Bose–Einstein condensate, perturbed solution
PACC: 0365, 0547

1. Introduction

In recent decades, the field of Bose–Einstein condensate (BEC) of atomic vapours has undergone a rapid theoretical and experimental development, providing a rich, new and important phenomenology and also allowing the testing of concepts originating from other fields, such as the theory of superfluidity, nonlinear optics and atomic physics.[1] The rapid progress in the fabrication and manipulation of ultra-cold Bose–Einstein condensates (BECs) has led to investigating the phenomena of complex condensed matters in perfectly controllable environments (for example, the creation of vortex lattices[2] and the quantum phase transition from a superfluid to a Mott insulator state in optical lattices[3]) through some interesting experiments.[4] Recently, the technology of guiding cold atoms by using hollow optical fibres,[5] magnetic guides[6] and microchips[7] has been applied to the study of guided Bose–Einstein condensed beams, i.e. guided continuous atom lasers, and many new and important results have been reported. For example, wave-guides have been designed for a BEC using a blue-detuned hollow laser beam;[8] Bose condensation has been obtained over a microchip;[9,10] a Bose–Einstein wave-packet has been propagated in a microfabricated magnetic waveguide;[11] a continuous beam of cold atoms has been loaded into a magnetic guide, as a first step in order to perform evaporative cooling and condensation in the guide.[12,13] Very recently, Paul *et al*[4] discussed the transport of BECs in a magnetic waveguide and the transmission modes of BECs have been constructed by Leboeuf and Pavloff[1] and Pavloff.[12] However, their studies are based on the numerical method. In this paper, we give a quantitative analytical method to discuss the nonlinear transport of BECs in a double barrier potential. Our plan in this paper is as follows. In Section 2, we set up a theoretical framework. The motion of condensate is described by using an effective one-dimensional Gross–Pitaevskii equation (GPE). In Section 3, we study the propagation of a condensate through a double barrier potential (Gaussian potential) in a magnetic waveguide. We obtain the perturbation solution of the system by using the direct perturbation method, and we prove that this solution is analytically bounded as $x \to \pm\infty$. This leads the atomic number density and the velocity field of the system to be all stable, and it reveals that the transport of BECs is a stable nonlinear transport. We numerically calculate the GPE by using 'Mathematica' in Section 4. We plot the phase orbit and spatial evolutions of the system. These numerical results are in agreement with the theoretical analytical results. Finally, we present our conclusions in Sec-

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tion 5, where we discuss our results and emphasize the important functions of the system parameters in controlling the transports of BECs.

2. An effective one-dimensional equation

We consider Bose–Einstein condensed atoms at zero temperature propagating through a double barrier potential in a magnetic waveguide. Let x be the coordinate along the axis of the guide (which is possibly bent), then the mean-field dynamics of the system is described by the effective one-dimensional Gross–Pitaevkii equation[4,14−16]

$$\begin{aligned} &\mathrm{i}\hbar\frac{\partial\psi(x,t)}{\partial t} \\ &= \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) + g_0\left|\psi(x,t)^2\right|\right]\psi(x,t), \quad (1) \end{aligned}$$

where m is the atomic mass, $g_0 = 2a_\mathrm{s}\hbar\omega_\perp$ is the interatomic scattering pseudopotential with a_s and $\omega_\perp$ being the s-wave scattering length and trapping frequency, respectively. In Eq.(1), $V(x)$ represents the effect of the obstacle, which restricts the study to the case of a localized perturbation with limit $\lim\limits_{x\to\pm\infty} V(x) = 0$.[12,13] Such an obstacle can be realized by a double barrier potential, such as the Gaussian potential[4,17]

$$V(x) = V_0\left[\mathrm{e}^{-(x+L/2)^2/\sigma^2} + \mathrm{e}^{-(x-L/2)^2/\sigma^2}\right], \quad (2)$$

where V_0 is the amplitude of the external longitudinal potential, L is the width of this one-dimensional square potential well, and σ is the beam waist (see the Fig.1).

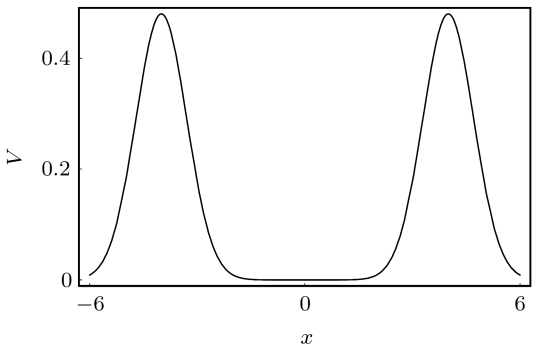


Fig.1. The barrier potential with parameters: $V_0 = 0.48$, $L = 4$, and $\sigma = 1$.

Equation (1) is equivalent to two hydrodynamic equations corresponding to the conservation of mass and the conservation of momentum, respectively,[18]

$$\frac{\partial\rho}{\partial t} + \frac{\partial J(x,t)}{\partial x} = 0,$$

and

$$\frac{\partial J(x,t)}{\partial t} + \frac{\partial T(x,t)}{\partial x} = 0, \quad (3)$$

where

$$\rho = m\psi^*\psi \quad (4)$$

and

$$J(x.t) = \frac{\mathrm{i}\hbar}{2}\left(\psi\frac{\partial\psi^*}{\partial x} - \psi^*\frac{\partial\psi}{\partial\psi}\right) \quad (5)$$

are the mass density and momentum flux, respectively, and the stress tensor is given by[12]

$$\begin{aligned} T(x,t) = &-\mathrm{Im}\left(\psi^*\frac{\partial\psi}{\partial t}\right) + \frac{\hbar^2}{2m}\left|\frac{\partial\psi}{\partial x}\right|^2 \\ &-V(x)n(x,t) - \varepsilon[n(x,t)], \end{aligned}$$

and

$$\varepsilon(n) = \int_0^n g_0\rho\mathrm{d}\rho = \frac{1}{2}g_0n^2 = \frac{1}{2}g_0\left|\psi(x,t)\right|^4, \quad (6)$$

where $n(x,t) = |\psi(x,t)|^2$ is the longitudinal density. The instantaneous drag force on an obstacle is given by[4,12]

$$F_\mathrm{d} = \int_{-\infty}^{\infty}\mathrm{d}x\left|\psi(x,t)\right|^2\frac{\mathrm{d}V(x)}{\mathrm{d}x}. \quad (7)$$

Note that the drag force F_d can characterize the superfluidity of the flow passing through the obstacle. For example, we compute the drag force that is exerted by the atom laser on the obstacle. A finite drag implies dissipation, whereas $F_\mathrm{d} = 0$ corresponds to a superfluid flow.

In order to determine the transmission of stationary-state solutions of Eq.(1) and better understand the BEC dynamics, we employ the time-dependent variational macroscopic condensate wave function[13,19,20]

$$\psi(x,t) = \phi(x)\mathrm{e}^{\mathrm{i}\theta(x)}\mathrm{e}^{-\mathrm{i}\mu t/\hbar}, \quad (8)$$

where μ is the chemical potential, the amplitude $\phi(x)$ and the phase $\theta(x)$ are the real functions. The atomic number density is $n(x) = \phi^2(x)$, which is normalized by the total number of atoms $\int\phi^2(x)\mathrm{d}x = \int n(x)\mathrm{d}x = N$. Substitute expression (8) into Eq.(1), then the stationary scattering state

Gross–Pitaevskii equation (1) will split into two coupled equations corresponding to its imaginary and real parts as

$$-\frac{\hbar^2}{2m}\left[\frac{\mathrm{d}^2\phi(x)}{\mathrm{d}x^2}+\phi(x)\left(\frac{\mathrm{d}\theta}{\mathrm{d}x}\right)^2\right] + \left[V(x)+g_0\phi^2(x)\right]\phi(x)=\mu\phi(x), \tag{9}$$

and

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2}+\frac{2}{\phi(x)}\frac{\mathrm{d}\phi(x)}{\mathrm{d}x}\frac{\mathrm{d}\theta}{\mathrm{d}x}=0. \tag{10}$$

Integrating Eq.(10) yields easily the relationship between the superfluid velocity $v(x)$ with the density $n(x)$ as

$$v(x)=\frac{\hbar}{m}\frac{\mathrm{d}\theta}{\mathrm{d}x}=\frac{\hbar}{m}\frac{a}{\phi^2(x)}=\frac{\hbar}{m}\frac{a}{n(x)}, \tag{11}$$

where $a=\theta_x(x_0)\phi^2(x_0)$ is an integration constant determined by the velocity and atomic number density at the boundary point x_0. The atomic current density

$$J(x)=n(x)v(x)=\frac{a\hbar}{m}\equiv J_\mathrm{t} \tag{12}$$

is a constant, indicating the flux conservation. For stationary flows, the stress tensor (6) becomes[12]

$$T(x)=\frac{1}{2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}x}\right)^2+W[n(x)]-V(x)n(x),$$

and

$$W[n(x)]=-\varepsilon[n(x)]+\mu n(x)+\frac{J_\mathrm{t}^2}{2n(x)}, \tag{13}$$

and the drag force (7) becomes

$$F_\mathrm{d}=\int_{-\infty}^{\infty}\mathrm{d}x\,|\phi(x)|^2\,\frac{\mathrm{d}V(x)}{\mathrm{d}x}, \tag{14}$$

which provides a measure of the proximity of the scattering state to a resonant state. Far from any resonance, the amount of reflection from the potential is rather large, leading to a finite drag due to the associated momentum transfer, while a vanishing drag is expected near a resonance where the Bose–Einstein condensate is perfectly transmitted through a quantum dot.[4] Combining Eqs.(9), (11) and (12), the real part gives a Schrödinger-like equation for $\phi(x)$

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\phi(x)}{\mathrm{d}x^2}+\left[V(x)+g_0\phi^2(x)+\frac{m}{2}\frac{J_\mathrm{t}^2}{\phi^4(x)}\right]\phi(x) = \mu\phi(x). \tag{15}$$

The numerical solution of the nonlinear equation (15) is relatively easy to obtain,[4,12,13] but the analytical solution is very complex and it is difficult to find the exact solution. In the next section, we solve Eq.(15) by using a theoretical analytical method (the direct perturbation method). Furthermore, in order to describe a realistic propagation process of Bose–Einstein condensed atoms in the double barrier potential, we introduce the transmission coefficient τ that is associated with a given scattering state: τ can be determined by the ratio of the atomic current J_t in the presence of the double barrier potential (i.e., the transmitted current) to the incident current J_i obtained in the absence of the potential. The incident current of the scattering state is approximately determined by[4]

$$J_\mathrm{i}\approx\frac{\sqrt{\mu-g_0n_\mathrm{av}}}{2m}(n_\mathrm{av}+\sqrt{n_\mathrm{max}n_\mathrm{min}}), \tag{16}$$

where n_max and n_min denote the maxima and minima, respectively, of the longitudinal upstream density, and $n_\mathrm{av}=(n_\mathrm{man}+n_\mathrm{min})/2$. Combine expressions (12) and (16), then the transmission coefficient τ will be given by

$$\tau=\frac{J_\mathrm{t}}{J_\mathrm{i}}=\frac{2a\hbar}{\sqrt{\mu-g_0n_\mathrm{av}}(n_\mathrm{av}+\sqrt{n_\mathrm{max}n_\mathrm{min}})}. \tag{17}$$

By using expression (17), we can calculate the transmission coefficient of stationary scattering states, if given the transmitted current J_t and the incident current J_i. For example, if we fix incident current $J_\mathrm{i}=1.6\omega_\perp$, where $\omega_\perp=2\pi\times10^3\mathrm{s}^{-1}$ for ^{87}Rb atoms ($a_\mathrm{s}=5.77\,\mathrm{nm}$) according to Ref.[4], then we have the transmitted current $J_\mathrm{t}=a\hbar/m\approx0.12a\omega_\perp$. Choosing the boundary velocity $v(x_0)=0$ for the ^{87}Rb atom and inserting it into Eq.(11) to yield the constant $a=0$, lead to the transmitted current $J_\mathrm{t}=0$ and the transmission coefficient $\tau=0$, which shows that there exists no transmission of atoms in this case. When the boundary velocity $v(x_0)=0.0535\,\mathrm{m/s}$ of the ^{87}Rb atom, from expression (11) we can obtain constants $\theta_x(x_0)=1$ and $a=\theta_x(x_0)\phi^2(x_0)=0.01$, and so the transmitted current $J_\mathrm{t}=0.0012\omega_\perp$. From expression (17) we can calculate the transmission coefficient τ to be 1.2×10^{-3}, which shows that the transport of BEC atoms in the double barrier potential is a nonlinear resonant transport with a very small transmission ratio in this case. With the velocity of the BEC atom increasing in the boundary, the transmission ratio of the atoms becomes more and more large. For example, when constant α is 5, the variation in boundary velocity $v(x_0)$ leads to a transmission coefficient (τ) of 0.6, i.e. $\tau=0.6$. Choose the constant $a=8.33$, then the transmission coefficient τ will be about 1,

i.e. $\tau \approx 1$, which means that all atoms in the double barrier potential are almost transmitted. However, the transmission ratio of BEC atoms is in the interval $0 < \tau < 1$ in general.

3. Analytical perturbation solution

From Eq.(15) we know that for the simple case, i.e. the current $J_t = 0$, which is just the parametrically driven Duffing equation with the time variable replaced by the spatial coordinate, the homoclinic solution of this system has been widely studied.[21−23] For the general case, $J_t \neq 0$, the system becomes more complex. Because it is difficult to find the exact solution of Eq.(15), we consider a weak external potential $V(x)$ as a perturbation to seek an approximate solution. By using the direct perturbation method, we expand the amplitude $\phi(x)$ to the first order approximation

$$\phi(x) = \phi_0(x) + \phi_1(x),$$

and

$$|\phi_0(x)| \gg |\phi_1(x)|. \tag{18}$$

Combining Eqs.(2), (15), and (18), and setting units such that $m = \hbar = 1$ in Eq.(15), we can obtain a zero-order equation

$$\frac{\mathrm{d}^2\phi_0(x)}{\mathrm{d}x^2} + 2\mu\phi_0(x) - 2g_0\phi_0^3(x) - \frac{J_t^2}{\phi_0^3(x)} = 0, \tag{19}$$

and a first-order perturbation equation

$$\frac{\mathrm{d}^2\phi_1(x)}{\mathrm{d}x^2} + 2\mu\phi_1(x) - 6g_0\phi_0^3(x) - \frac{3J_t^2}{\phi_0^4(x)}\phi_1(x) = h(x)$$

$$h(x) \approx V_0[\mathrm{e}^{-(x+L/2)^2/\sigma^2} + \mathrm{e}^{-(x-L/2)^2/\sigma^2}]\phi_0(x), \tag{20}$$

where $h(x)$ can be regarded as a perturbation, because the amplitude V_0 is very small. The unperturbed equation (19) has the well-known analytic solution[23,24]

$$\phi_0(x) = \sqrt{b - (b-c)\mathrm{sech}^2\xi},$$

and

$$\xi = \sqrt{g_0(b-c)}x - C,$$

with

$$C = \sqrt{g_0(b-c)}x_0 - \mathrm{arsech}\left[\frac{b-\phi_0^2(x_0)}{b-c}\right]^{\frac{1}{2}}, \tag{21}$$

where C is an integration constant, the constants b and c obey

$$2b + c = \frac{\mu}{g_0}, \quad b^2 + 2bc = \frac{D}{g_0}, \quad b^2c = \frac{J_t^2}{g_0},$$

with

$$D = \frac{1}{2}\left(\frac{\mathrm{d}\phi_0}{\mathrm{d}x}|_{x=x_0}\right)^2 + \frac{C^2}{2\phi_0^2(x_0)} - \frac{1}{2}g_0\phi_0^4(x_0) + \mu\phi_0^2(x_0). \tag{22}$$

Applying expression (21) to expression (20), we construct a general solution[25]

$$\phi_1(x) = \phi_1'' \int_B^x \phi_1' h(x)\mathrm{d}x - \phi_1' \int_B^{\prime x} \phi_1''(x)\mathrm{d}x, \tag{23}$$

where B and B' are two integration constants, ϕ_1' and ϕ_1'' are two linear independent solutions of Eq.(20) for $h(x) = 0$

$$\phi_1' = \frac{\mathrm{d}\phi_0(x)}{\mathrm{d}x} = \sqrt{g_0(b-c)}(b-c)\frac{\mathrm{sech}^2\xi\tanh\xi}{\sqrt{b-(b-c)\mathrm{sech}^\xi}}, \tag{24}$$

and

$$\phi_1'' = \phi_1'\int(\phi_1')^{-2}\mathrm{d}x = \mathrm{sech}^2\xi\tanh\xi[(12b+28c)\xi - 32c\coth\xi + 8(b+c)\sinh(2\xi) + b\sinh(4\xi)]/[32g_0(b-c)^2 \times\sqrt{b-(b-c)\mathrm{sech}^2\xi}. \tag{25}$$

Generally, the first correction (23) is unbounded, because of the unboundedness of ϕ_1'' as coordinate tends to infinity. However, due to the Gaussian potential $V(x) = 0$ as $x \to \pm\infty$, perturbation solution (23) is a bounded one in any case and there exists no chaotic behaviour in this system. This shows that the transport of the BECs in a Gaussian potential is a stable nonlinear transport.

4. Numerical simulations of the system

In order to prove the exactitude of the analytical results in Section 3, we use the numerical method to solve the nonlinear equation (15). For simplicity, we

take a constant current, i.e. $J_t = 0.01$, and a set of experimental parameters i.e. $g_0 = 0.5, \mu = 2, V_0 = 0.35$, $L = 4$ and $\sigma = 1$. From Eq.(15) we numerically plot the phase orbit in an equivalent phase space $(\phi, \mathrm{d}\phi/\mathrm{d}x)$ as shown in Fig.2(a). From this phase diagram, we see that the phase orbit evolution with the coordinate in a finite region is periodic, and there appears no chaotic feature in this system, showing that the numerical results are in good agreement with the theoretical analytical results. Adopting the same parameters as those in Fig.2(a), from Eq.(15) we plot the spatial evolution of the solution as indicated in Fig.2(b). This spatial evolutional picture also shows the periodic behaviour of the BEC system. We are interested in the transport of BECs. By applying the numerical solution in Fig.2(b) to the following equations

$$n(x) = [\phi_0(x) + \phi_1(x)]^2, \tag{26}$$

and

$$v(x) = \frac{J_t}{[\phi_0(x) + \phi_1(x)]^2}, \tag{27}$$

we obtain the spatial evolutions of the atomic number density $n(x)$ and the superfluid velocity $v(x)$ as shown in Figs.3(a) and 3(b), respectively. From Fig.3 we can see that the spatial population and the motion of the condensed atoms are all stable (except at position $x = 0$), which indicates that the transport of BECs in the system is a stable nonlinear transport. If we set the $\varepsilon[n(x)] = g_0 n(x)$ in Eq.(13), then we can plot spatial evolutions of the stress tensor $T(x)$ as indicated in Fig.4 by applying the numerical solution in Fig.2(b) to Eq.(13). From Fig.4, we see that the spatial evolution of $T(x)$ depends on the chemical potential μ sensitively. For example, when the chemical potential μ is smaller, the spatial evolutional curves of $T(x)$ are the nonlinear and non-periodic curves (see Figs.4(a) and 4(b)), and there appears a rapid change at $x = 0$, which means that the stress tensor is a nonlinear and non-periodic stress tensor. However, with the chemical potential μ increasing, the stress tensor $T(x)$ turns periodic. For example, when the chemical potentialμ increases to 4.45, i.e. $\mu = 4.45$, the spatial evolutions of the tress tensor becomes approximately periodic (see Fig.4(c)), showing that the transport of BECs is approximately a bounded periodic transport in this case, which implies that we can control the non-periodic behaviour of the system by adjusting the chemical potential μ. Similarly, we can also adjust other parameters of the system (such as the $g_0, V_0, \ldots$) to control the transport of BECs in experiment.

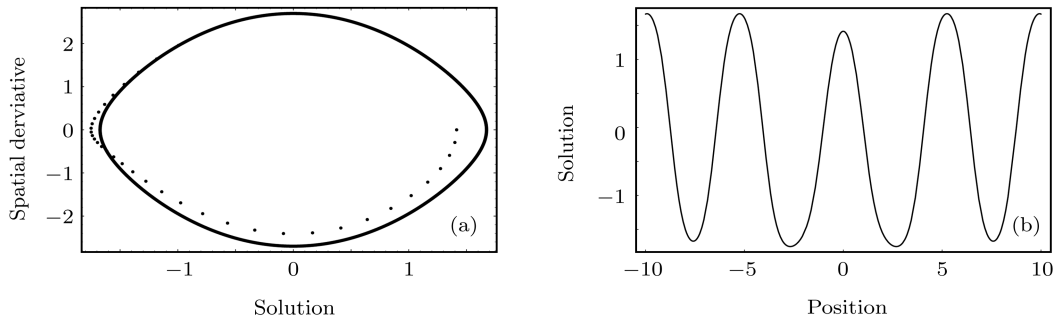


Fig.2. The chaotic phase orbit in the equivalent phase space of the ϕ versus $\mathrm{d}\phi/\mathrm{d}x$ (a) and the spatial evolution from the corrected solution of Eq.(17) (b), each with parameters: $V_0 = 0.35$, $g_0 = 0.5$, $\mu = 2$, $J_t = 0.01$, $L = 4$ and $\sigma = 1$.

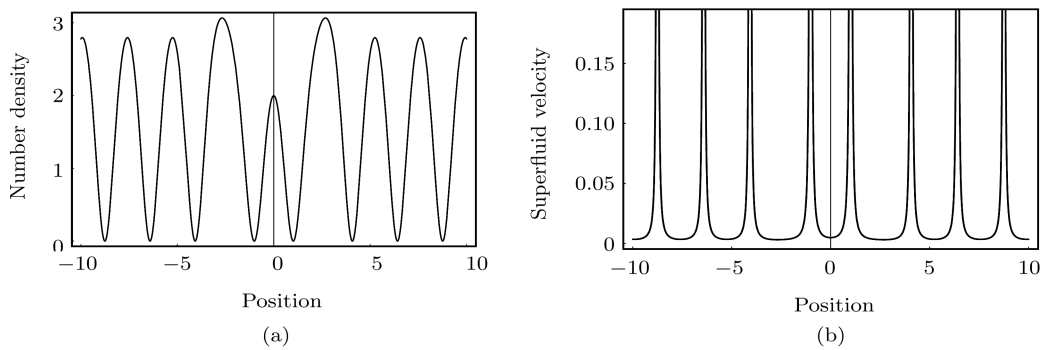


Fig.3. The spatial evolutions of atomic number density (a) and superfluid velocity (b) of the system with parameters the same as those in Fig.2.

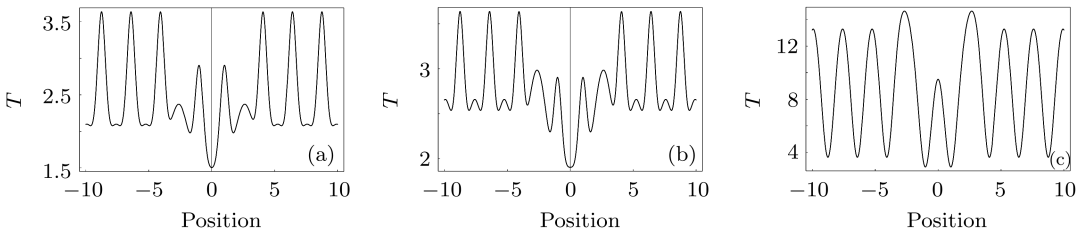


Fig.4. The spatial evolutions of the stress tensor $T(x)$ in this system with parameters the same as those in Fig.2, with the values of chemical potential μ being 1.25 (a), 1.45 (b), and 4.45 (c).

5. Conclusions

We have studied the nonlinear transport of weakly interacting BECs through a symmetric double barrier potential in a magnetic waveguide. By using the mean-field method and the macroscopic one-body wave function, we investigate the Gross–Pitaevskii equation. We obtain the analytical perturbation solution of BEC system by using the direct perturbation technique. Owing to the double barrier potential $V(x)$ that tends to zero as $x \to \pm\infty$, we find the first correction (23) to be the periodic one, thereby leading spatial evolutions of the phase orbit, the spatial population of BEC atoms and the superfluid velocity to be all periodic, and there exists no chaotic behaviour in the BEC system, indicating that the transport of BECs through a double barrier potential in a magnetic waveguide is a stable nonlinear and periodic transport. Corresponding numerical simulations are in good agreement with the theoretical analytical results. We can control the transport of BECs by adjusting the system parameters.

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