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Magnon energy gap in a four-layer ferromagnetic superlattice*

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The magnon energy band in a four-layer ferromagnetic superlattice is studied by using the linear spin-wave approach and Green's function technique. It is found that three modulated energy gaps exist in the magnon energy band along K_x direction perpendicular to the superlattice plane. The spin quantum numbers and the interlayer exchange couplings all affect the three energy gaps. The magnon energy gaps of the four-layer ferromagnetic superlattice are different from those of the three-layer one. For the four-layer ferromagnetic superlattice, the disappearance of the magnon energy gaps $\Delta\omega_{12}$, $\Delta\omega_{23}$ and $\Delta\omega_{34}$ all correlates with the symmetry of this system. The zero energy gap $\Delta\omega_{23}$ correlates with the symmetry of interlayer exchange couplings, while the vanishing of the magnon energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ corresponds to a translational symmetry of x-direction in the lattice. When the parameters of the system deviate from these symmetries, the three energy gaps will increase.

Keywords: layered ferromagnetic superlattice, magnon energy gap, spin quantum number, interlayer exchange couplings

PACC: 7510J, 7530D, 7570F

1. Introduction

The interesting phenomena and practical benefits discovered in the fields of photonics[1] and semiconductor superlattices[2] have led to a renewed attention to other materials with a spatial periodicity. Investigations of layered magnetic systems have been a subject of growing interests,[3−5] because artificial multilayer/superlattice materials are realizable in laboratories, whose characteristics may be much different from those of its component materials.[6−8] For multilayered magnetic materials, either the addition of a single layer to a film or the variety of structural and/or magnetic parameters (for instance, anisotropy, spin quantum numbers, interlayer and intralayer exchange couplings) of a material would lead to new features in the band spectrum and magnetic properties of the material.

The band spectrum, which is the signature of the periodicity, has been intensively studied in plasmonic crystals,[9] phononic crystals,[10] periodic ferroelectric media.[11] The materials with periodic magnetic structure can be refereed to as magnonic crystals. In magnetic superlattice systems, the magnon as a kind of spin excitations is an interesting subject.[12−27,30] The spin-wave spectra were investigated of a superlattice composed of two alternating ferromagnetic films with antiferromagnetic coupling at the interfaces.[16] A theory was presented for bulk and surface spin waves in a triangular antiferromagnet with ferromagnetic interlayer coupling, and developed for spin-wave dispersion relations of a semi-infinite frustrated system as well as for thin films.[17] Interface spin waves in a bilayer of a two-sublattice Heisenberg ferrimagnet with the nearest-neighbour exchange interaction were studied.[18] The spin-wave spectrum and the ferromagnetic resonance spectra of a magnetic thin film were studied by van Stapele *et al*,[19] Barnas[20] analysed the spin-wave spectra of infinite, semi-infinite, and finite ferromagnetic superlattices with arbitrary elementary units by the method of the transfer-matrix formalism. It was found that the magnon energy gap

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strongly affects the physical properties of layered magnetic systems.[21] However, little attention has been paid on the systemic theoretical study of magnon energy gap of a superlattice. The temperature dependence of magnetization and optical magnon gap in bilayer antiferromagnetic $YBa_2Cu_3O_6$ was obtained by employing the Green's function technique and the Callen decoupling approximation.[22] Deng *et al*[23] studied the magnon energy gap of a two - layer magnetic superlattice with a periodic anisotropy, by using local coordinates and a spin-Bose transformation quantum approach. The magnon dispersion relation with two ferromagnetic layers in the pure exchange limit, the pure dipolar limit, as well as both exchange and dipolar interactions, was calculated, and it was concluded that the magnon energy gap can be derived mainly from the exchange interaction.[24,25] In our previous work,[26,27] the magnon energy band was studied for a ferrimagnetic[26] and ferromagnetic[27] superlattices with three layers in a unit cell. The results showed that there are two modulated energy gaps, and the absence of the energy gap at a certain condition means that the system has a high magnetically structural symmetry.[26,27]

In order to understand further the magnon gaps of multi-layer superlattices, in this paper, we study the magnon energy gaps in a superlattice system constructed by four kinds of ferromagnetic materials that are coupled ferromagnetically. It has been observed that there are four energy spectra branches along K_x direction in the four-layer ferromagnetic superlattice, one is an acoustic branch and other three are optic. The magnon energy gaps $\Delta\omega_{12}$, $\Delta\omega_{23}$ and $\Delta\omega_{34}$ exist in the magnon energy band of the system, and the zero energy gap $\Delta\omega_{23}$ correlates with the symmetry of interlayer exchange couplings, and the disappearance of the magnon energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ corresponds to a translational symmetry of x-direction in a lattice. When the parameters of the system deviate from these symmetries, the three energy gaps will increase.

Spin waves in the layered magnetic systems have been investigated by use of various classical and quantum theories. The classical approaches ignore quantum fluctuations that might significantly affect the magnon excitation. Therefore, we shall use the quantum linear spin - wave method. However, the linear spin-wave approach is appropriate only to the region of zero temperature or low temperatures.[28] At high temperatures, this approach is insufficient and some kinds of self-consistent treatment[29,30] or spin Green's function method[22,31,32] are necessary. In this paper, we employ the linear spin-wave approach and Green's function technique only for zero temperature, which is good enough for the problem we are dealing with.

This paper is organized as follows: In Section 2, we describe the model, the Hamiltonian of the system and calculation procedure. Section 3 gives the effect of spin quantum numbers and interlayer exchange couplings on the energy gaps of the system. Section 4 presents a conclusion.

2. Model and calculation procedure

We consider a Heisenberg model for a four-layer ferromagnetic superlattice with a simple cubic lattice. A schematic diagram of the four-layer ferromagnetic superlattice model is illustrated in Fig.1. A unit cell of the superlattice consists of four layers, 1, 2, 3 and 4, where spins are denoted by S_l (l=1, 2, 3, 4) for each layer. The nearest neighbouring spins within each sublayer are coupled ferromagnetically by the intralayer exchange couplings J_l (l=1, 2, 3, 4), respectively. The ferromagnetic interlayer exchange couplings $J_{l,l+1}$ exist between the spins in the nearest neighbouring layers. The superlattice structure is stacked periodically along x-direction that is normal to layers (yz-planes). The Hamiltonian is:

$$H = -\frac{1}{2}\sum_{l=1}^{4}\sum_{\rho,\delta_{//}} J_l S_{l,\rho} S_{l,\rho+\delta_{//}} - \sum_{l=1}^{4}\sum_{\rho} J_{l,l+1} S_{l,\rho} S_{l+1,\rho}, \quad (1)$$

where l is the number of sublayers, $\delta_{//}$ represents that only the exchanges between the nearest neighbours in yz-planes are taken into account. The direction of spins of the initial state in the four sublayers is along the positive z-direction. Therefore, all the interlayer exchange couplings J_{12}, J_{23}, J_{34} and J_{41} and the intralayer exchange coupling J_1, J_2, J_3 and J_4 are positive. There are N sites on each layer-lattice, and total $4N$ sites in the system. By use of the Holstein–Primakoff transform[33] and the linear spin-wave approximation,[34] when introducing the spin-wave operators $b_{lk}(b_{lk}^{+})$ (l=1, 2, 3, 4), we can rewrite Eq.(1) as follows:

$$H = -N\left(\frac{Z_{yz}}{2}\sum_{l=1}^{4}J_l S_l^2 + \sum_{l=1}^{4}J_{l,l+1}S_l S_{l+1}\right) + \sum_{l=1}^{4}(J_l S_l Z_{yz} + J_{l,l+1}S_{l+1} + J_{l-1,l}S_{l-1})\sum_k b_{lk}^+ b_{lk}$$
$$-\sum_{l=1}^{4}\sqrt{S_l S_{l+1}}J_{l,l+1}\sum_k(\gamma_{-kx}b_{l,k}b_{l+1,k}^+ + \gamma_{kx}b_{l,k}^+ b_{l+1,k}) - \frac{1}{2}Z_{yz}\sum_{l=1}^{4}J_l S_l\sum_k\gamma_{k_{//}}(2b_{lk}b_{lk}^+ - 1). \quad (2)$$

Here if l=1, $l-1$=4; if l=4, $l+1$=1. And Z_{yz}=4 represents the number of nearest neighbours in the yz-planes that are the same. Note that

$$\gamma_{k_{//}} = \frac{1}{Z_{yz}}\sum_{\delta_{//}} e^{ik\cdot\delta_{//}}, \quad (3a)$$

$$\gamma_{\pm k_x} = \sum_{\delta_x} e^{\pm ik\cdot\delta_x}. \quad (3b)$$

Here $\gamma_{kx} \neq \gamma_{-kx}$; γ_{kx} and γ_{-kx} are complex because δ_x has only one value, but $\gamma_{k_{//}}$ is real because $\delta_{//}$ can take four different values within the layer.

We first define the four-order matrix retarded Green functions as

$$G(k,\omega) = [G_{i,j}]_{4\times4}, \quad (4)$$

where $G_{i,j} = \langle\langle b_{ik};\ b_{jk}^+\rangle\rangle_\omega$ (i=1–4; j=1–4).

By using the equation of the Green functions, we obtain the solution of the Green function as follows:

$$G(k,\omega) = \frac{1}{\det(D(\omega))}[M_{i,j}]_{4\times4}, \quad (5)$$

where

$$D(\omega) = [W_{i,j}]_{4\times4} + [H_{i,j}]_{4\times4}. \quad (6)$$

Here in the four-order matrix W, only the diagonal matrix elements W_{ii} (i=1–4) are non-zero and equal to ω, which represents the energy spectrum of the system. The matrix elements H_{ij} (i,j=1, 2, 3, 4) in the four-order matrix H are given in Appendix. The matrix elements M_{ii} (i=1–4) in Eq.(5) are same as those Appendix B in Ref.[35].

Setting the determinant to zero, i.e., $\det(D(\omega))$=0, we obtain four positive numerical solutions for the spin wave spectra of the ferromagnetic four-layer superlattice.

In the following, we shall discuss whether the energy gaps exist in the energy band structure along the K_x direction (perpendicular to the superlattice plane) of the present system, and how the spin quantum number, the interlayer exchange couplings affect the energy gap along the K_x direction. In this paper, we set the intralayer exchange couplings a unit ($J_1 = J_2 = J_3 = J_4 = 1.0$).

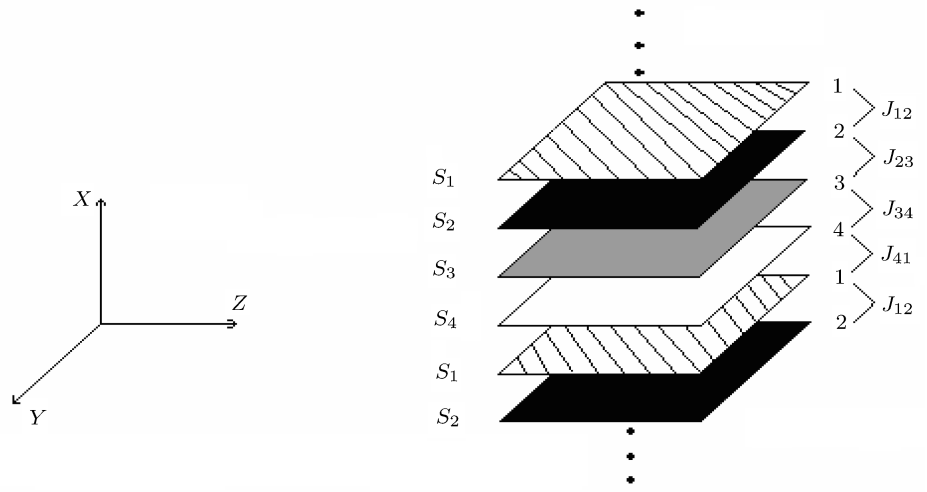


Fig.1. A schematic diagram for the four-layer ferromagnetic superlattice. The interlayer exchange couplings and spin quantum numbers are illustrated.

3. Results and discussion

Figure 2 shows the K_x dependence of the energy spectra of the four-layer ferromagnetic superlattice. From Fig.2, four energy spectra branches ω_1, ω_2, ω_3 and ω_4 exist in the energy band, ω_1 is an acoustic branch and ω_2, ω_3 and ω_4 are optic branches. Furthermore, there are three magnon energy gaps $\Delta\omega_{12}$, $\Delta\omega_{23}$ and $\Delta\omega_{34}$ in the energy band. The magnon energy gap $\Delta\omega_{12}$ is defined as the gap between the top of the branch ω_1 and the bottom of the branch ω_2, which is different from the optical magnon gap at K_x=0. The magnon energy gap $\Delta\omega_{23}$ lies between the bottom of the branch ω_3 and the top of the branch ω_2, while the magnon energy gap $\Delta\omega_{34}$ is between the bottom of the branch ω_4 and the top of the branch ω_3. As in the cases of photon, electron and phonon, the magnon energy gap should strongly affect the physical properties of the magnetic system. We then focus our attention on the effects of the spin quantum numbers and interlayer exchange couplings on the magnon energy gaps of the system.

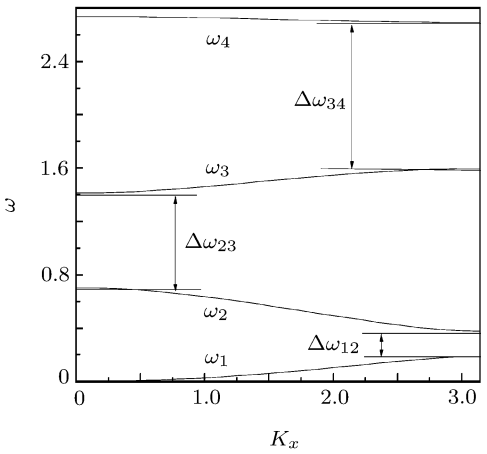


Fig.2. K_x dependence of the energy spectra ω of the four-layer ferromagnetic superlattice with S_1=0.5, S_2=1.0, S_3=0.5, S_4=1.5, J_{12}=1.0, J_{23}=0.5, J_{34}=0.5, J_{41}=0.8, $J_1 = J_2 = J_3 = J_4$=1.0 and $K_y = K_z$=0. Here, $\Delta\omega_{ij}$ is the energy gap between the energy spectra ω_i and ω_j.

Figure 3(a) shows the dependence of the magnon energy gap $\Delta\omega_{12}(\Delta\omega_{34})$ on the first- and the second-layer spin quantum numbers S_1 and S_2 of the four-layer ferromagnetic superlattice, for S_3=1.0, S_4= 1.5, $J_{12} = J_{23} = J_{34} = J_{41}$=1.0, and $J_1 = J_2 = J_3 = J_4$=1.0. From Fig.3(a), the energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ are the same. The energy gap $\Delta\omega_{12}(= \Delta\omega_{34})$ approaches to zero only when the parameters of the system are close to a point with S_1=1.0 and S_2=1.5, at that point the parameters of the system are $S_1 = S_3$=1.0, $S_2 = S_4$= 1.5, $J_{12} = J_{23} = J_{34} = J_{41}$=1.0, and $J_1 = J_2 = J_3 = J_4$=1.0.

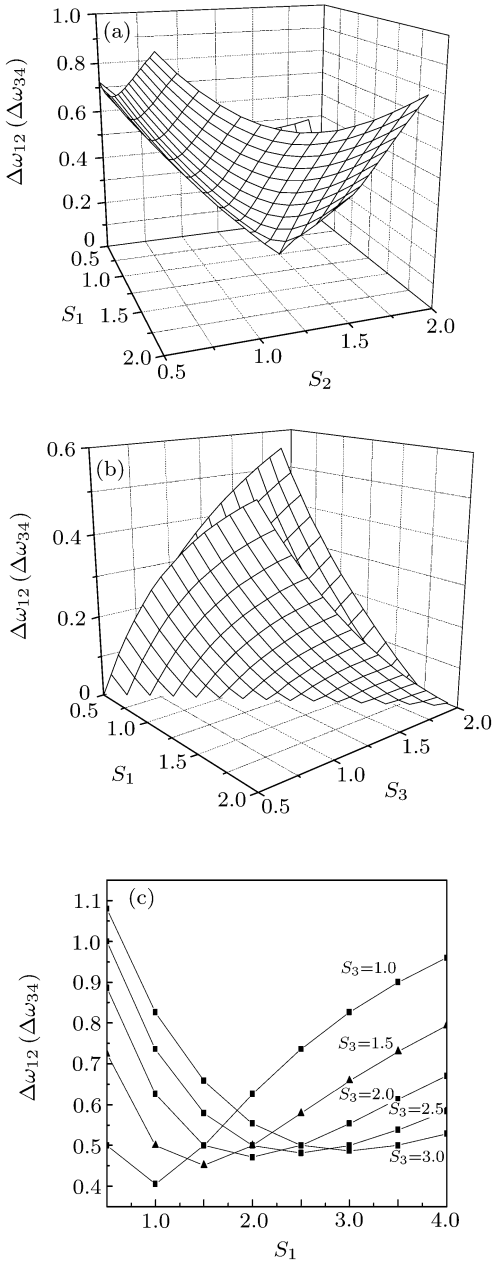


Fig.3. (a) The dependence of the magnon energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ on S_1 and S_2; (b) and (c) the dependence of the magnon energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ on S_1 and S_3 of the four-layer ferromagnetic superlattice. The parameters are $J_{12} = J_{23} = J_{34} = J_{41}$=1.0 and $J_1 = J_2 = J_3 = J_4$=1.0, (a) S_3=1.0, S_4=1.5 (b) $S_2 = S_4$=0.5 and (c) S_2=0.5, S_4=1.0.

Figure 3(b) illustrates the dependence of the magnon energy gap $\Delta\omega_{12}(\Delta\omega_{34})$ on the first- and the third-layer spin quantum numbers S_1 and S_3 of the four-layer ferromagnetic superlattice, with $S_2 = S_4$=0.5, $J_{12} = J_{23} = J_{34} = J_{41}$=1.0, and $J_1 = J_2 = J_3 = J_4$=1.0. In Fig.3(b), also the energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ are the same. When the first-layer spin quantum number S_1 is close to the third-layer spin quantum number S_3, the energy gap $\Delta\omega_{12}$ (= $\Delta\omega_{34}$) approaches to zero.

If $S_2 \neq S_4$, the dependence of the magnon energy gap $\Delta\omega_{12}(\Delta\omega_{34})$ on the first- and the third-layer spin quantum numbers S_1 and S_3 of the four-layer ferromagnetic superlattice is shown in Fig.3(c). It is clear that, the energy gap $\Delta\omega_{12}$ is equal to the energy gap $\Delta\omega_{34}$, and when $S_1 = S_3$, the energy gap $\Delta\omega_{12}$ (= $\Delta\omega_{34}$) becomes minimum with the change of the spin quantum number S_1.

By numerical calculation, it is found that when $J_{12} = J_{23} = J_{34} = J_{41}$, the energy gap $\Delta\omega_{12}$ is always equal to the energy gap $\Delta\omega_{34}$; otherwise, the two energy gaps are unequal. This is because the energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ all derive from the point $K_x = \pi$, and therefore, they have some similar properties.

Next, we will study the effects of the interlayer exchange couplings on the magnon energy gap $\Delta\omega_{12}$. The effects of the interlayer exchange couplings J_{12}, J_{23} and J_{34} on the energy gap $\Delta\omega_{12}$ are shown in Figs.4(a) and 4(b). From Fig.4(a), the energy gap $\Delta\omega_{12}$ approaches to zero only at a J_{12} close to 0.5 and J_{23} close to 1.0, with J_{34}=0.5 and J_{41}=1.0. From Fig.4(b), when J_{12} is close to J_{34}, the energy gap $\Delta\omega_{12}$ approaches to zero, with $J_{23} = J_{41}$=1.0. Namely, the interlayer exchange couplings also affect the energy gap $\Delta\omega_{12}$. When J_{12} is close to J_{34} and J_{23} is close to J_{41}, the energy gap $\Delta\omega_{12}$ approaches to zero.

By numerical calculation, we find that when the parameters of the system are close to the condition of $S_1 = S_3$, $S_2 = S_4$, $J_{12} = J_{34}$ and $J_{23} = J_{41}$, both the energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ approach to zero; as the parameters of the system deviates from the condition, the energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ will increase. It is known that, at the point where $S_1 = S_3$, $S_2 = S_4$, $J_{12} = J_{34}$ and $J_{23} = J_{41}$, $J_1 = J_2 = J_3 = J_4$, there is a translational symmetry in a period of x-direction of the four-layer ferromagnetic superlattice, and the system is not a four-layer superlattice, but a two-layer one, and then there will be not three magnon energy gaps. However, when the parameters of the system are close (not yet equal) to this point, the system is still a four-layer superlattice. Therefore, the above results have physical signification.

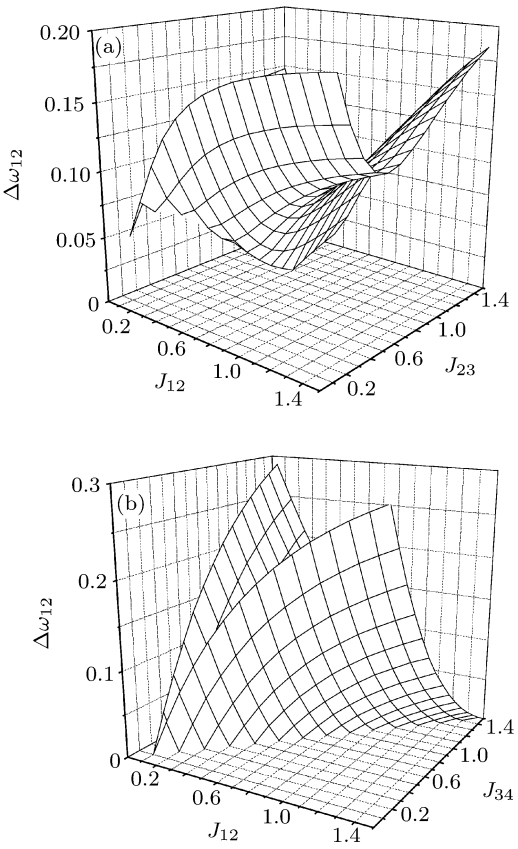


Fig.4. (a) The dependence of the magnon energy gap $\Delta\omega_{12}$ on J_{12} and J_{23}; and (b) the dependence of the magnon energy gaps $\Delta\omega_{12}$ on J_{12} and J_{34} of the four-layer ferromagnetic superlattice. The parameters are $S_1 = S_2 = S_3 = S_4$=0.5, J_{41}=1.0 and $J_1 = J_2 = J_3 = J_4$=1.0, (a) J_{34}=0.5 and (b) J_{23}=1.0.

Figures 5(a) and 5(b) show the spin quantum number dependence of the energy gap $\Delta\omega_{23}$. It is clear that there is a line with zero energy gaps. From Fig.5(a), when the spin quantum numbers satisfy the condition of $S_1 - S_2$=0.5 (=$S_4 - S_3$), the energy gap $\Delta\omega_{23}$ is zero. From Fig.5(b), the vanishing of the magnon energy gap corresponds to $S_1 + S_3$=2.0 (=$S_2 + S_4$). This means that, the disappearance of the energy gap $\Delta\omega_{23}$ corresponds to the condition of $S_1 + S_3 = S_2 + S_4$ with $J_{12} = J_{23} = J_{34} = J_{41}$=1.0.

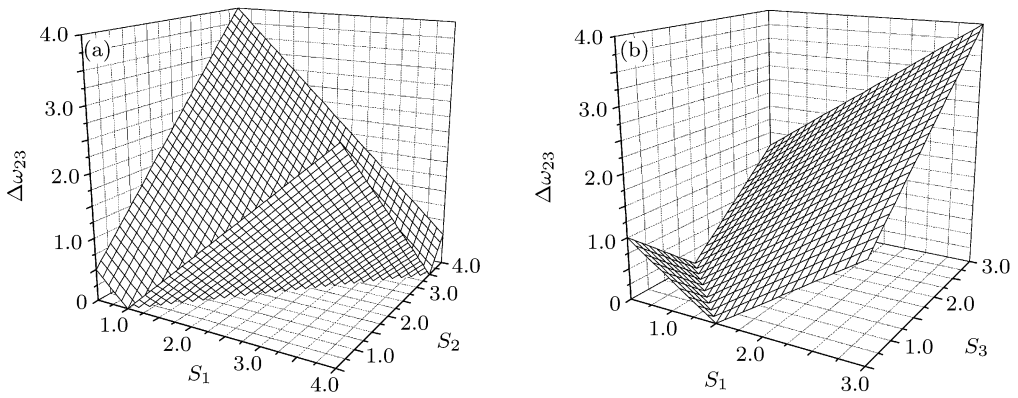


Fig.5. The dependence of the magnon energy gap $\Delta\omega_{23}$ on (a) S_1 and S_2 (b) S_1 and S_3 of the four-layer ferromagnetic superlattice, with (a) S_3=1.0 and (b) S_2=0.5, other parameters are S_4=1.5, $J_{12} = J_{23} = J_{34} = J_{41}$=1.0 and $J_1 = J_2 = J_3 = J_4$=1.0.

Figure 6 shows the dependence of the energy gap $\Delta\omega_{23}$ on interlayer exchange couplings J_{12} and J_{23} for S_1=1.5, S_2=0.5, S_3=1.0, S_4=2.0 and $J_{34} = J_{41}$=0.5. When interlayer exchange couplings $J_{12} = J_{23} = J_{34} = J_{41}$=0.5, the energy gap $\Delta\omega_{23}$ is zero. If interlayer exchange couplings satisfy the condition of $J_{34} \neq J_{41}$ with the same spin quantum numbers in Fig.6, there is no zero magnon energy gap. It is noted that the spin quantum numbers satisfy the condition of $S_1 + S_3 = S_2 + S_4$ in Fig.6. By numerical calculation we find that, when the parameters of the system satisfy the condition of $S_1 + S_3 = S_2 + S_4$ and $J_{12} = J_{23} = J_{34} = J_{41}$, the magnon energy gap $\Delta\omega_{23}$ is zero.

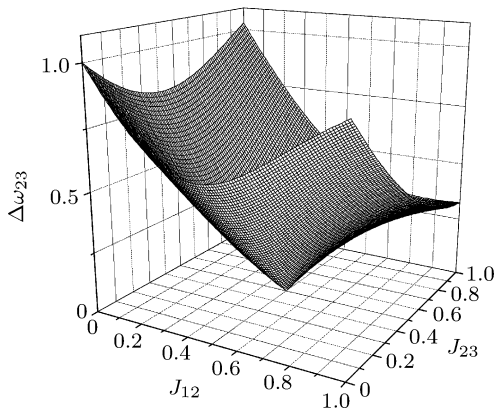


Fig.6. The dependence of the magnon energy gap $\Delta\omega_{23}$ on J_{12} and J_{23} of the four-layer ferromagnetic superlattice. The parameters for calculation are S_1=1.5, S_2=0.5, S_3=1.0, S_4=2.0, $J_{34} = J_{41}$= 0.5 and $J_1 = J_2 = J_3 = J_4$=1.0.

Next, we compare our results with the previous results for a three-layer ferromagnetic superlattice in Ref.[27]. The zero energy gap $\Delta\omega_{23}$ of the three-layer ferromagnetic superlattice corresponds only to the case of $J_{12} = J_{23} = J_{13}$. Namely, the disappearance of the energy gap $\Delta\omega_{23}$ means the highest symmetry among the three interlayer couplings J_{12}, J_{23} and J_{13} of the three-layer ferromagnetic superlattice. The zero energy gap $\Delta\omega_{23}$ of the four-layer ferromagnetic superlattice corresponds not only to the case of $J_{12} = J_{23} = J_{34} = J_{41}$, but also the case of $S_1 + S_3 = S_2 + S_4$. This is due to the complex lattice structure of the four-layer ferromagnetic superlattice. For the three-layer ferromagnetic superlattice, the parameters of the system for the zero energy gap $\Delta\omega_{12}$ have a C_2 symmetry operation around the z-axis and a θC_2 symmetry operation around the y-axis, where θ is time-reversal operator. For the four-layer ferromagnetic superlattice, when the parameters of the system are close to the condition of $S_1 = S_3$, $S_2 = S_4$, $J_{12} = J_{34}$ and $J_{23} = J_{41}$, the energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ approach to zero. Under this condition, in a period of x-direction of the four-layer ferromagnetic superlattice, there is a translational symmetry. When the parameters of the system deviate from the condition, the energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ will increase. Namely, for the four-layer ferromagnetic superlattice, the vanishing of the magnon energy gaps $\Delta\omega_{12}$, $\Delta\omega_{23}$ and $\Delta\omega_{34}$ all correlate with the symmetry of this system. The zero energy gap $\Delta\omega_{23}$ correlates with the symmetry of interlayer exchange couplings, and the disappearance of the magnon energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ corresponds to a translational symmetry in x-direction. The viewpoint that the vanishing of the magnon energy gap of a magnetic superlattice corresponds to a symmetry, agrees with the results in Refs.[23–27].

4. Conclusion

In conclusion, we have studied systematically the magnon energy band gaps in the four-layer ferromagnetic superlattice, by using the linear-spin wave approach and Green's function technique. We have discussed the effects of the spin quantum number and the interlayer exchange couplings on the magnon energy band gaps of the system.

There are four energy spectra branches along K_x direction in the four-layer ferromagnetic superlattice. The three energy gaps $\Delta\omega_{12}$, $\Delta\omega_{23}$ and $\Delta\omega_{34}$ exist in the magnon energy band of the system. When the parameters of the system are close to the condition of $S_1 = S_3$, $S_2 = S_4$, $J_{12} = J_{34}$ and $J_{23} = J_{41}$, the energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ approach to zero; when they deviate from this condition, the energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ will increase. When $J_{12} = J_{23} = J_{34} = J_{41}$, the energy gap $\Delta\omega_{12}$ is always equal to the energy gap $\Delta\omega_{34}$; otherwise, the two energy gaps are unequal. This may be because both the energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ are derived from the point $K_x = \pi$ in the Brillouin zone. When the parameters of the system satisfy the condition of $S_1 + S_3 = S_2 + S_4$ and $J_{12} = J_{23} = J_{34} = J_{41}$, the magnon energy gap $\Delta\omega_{23}$ is zero. Of course, there are other parameters that also correspond to the disappearance of the magnon energy gaps $\Delta\omega_{12}$, $\Delta\omega_{23}$ or $\Delta\omega_{34}$ (because their variation rule is complex, we have not discussed in this paper). For the four-layer ferromagnetic superlattice, the vanishing of the magnon energy gaps $\Delta\omega_{12}$, $\Delta\omega_{23}$ and $\Delta\omega_{34}$ all correlate with the symmetry of this system. The zero energy gap $\Delta\omega_{23}$ correlates with the symmetry of interlayer exchange couplings, and the disappearance of the magnon energy gaps $\Delta\omega_{12}$ and $\Delta\omega_{34}$ corresponds to a translational symmetry in x-direction in a lattice. The results about the magnon energy gap of the four-layer ferromagnetic superlattice may be applicable for absorbing electromagnetic wave in the magnetic thin films.

Appendix

The parameters H_{ij} are described as follows:

$$H_{11} = J_1 S_1 Z_{yz}(1 - \gamma_{k_{//}}) + S_2 J_{12} + S_4 J_{41},$$

$$H_{12} = \sqrt{S_1 S_2} J_{12} \gamma_{kx},$$

$$H_{13} = H_{24} = H_{31} = H_{42} = 0,$$

$$H_{14} = \sqrt{S_1 S_4} J_{14} \gamma_{-kx},$$

$$H_{21} = \sqrt{S_1 S_2} J_{12} \gamma_{-kx},$$

$$H_{22} = S_2 J_2 Z_{yz}(1 - \gamma_{k_{//}}) + S_1 J_{12} + S_3 J_{23},$$

$$H_{23} = \sqrt{S_2 S_3} J_{23} \gamma_{kx},$$

$$H_{32} = \sqrt{S_2 S_3} J_{23} \gamma_{-kx},$$

$$H_{33} = S_3 J_3 Z_{yz}(1 - \gamma_{k_{//}}) + S_2 J_{23} + S_4 J_{34},$$

$$H_{34} = \sqrt{S_3 S_4} J_{34} \gamma_{kx},$$

$$H_{41} = \sqrt{S_4 S_1} J_{41} \gamma_{kx},$$

$$H_{43} = \sqrt{S_3 S_4} J_{34} \gamma_{-kx},$$

$$H_{44} = S_4 J_4 Z_{yz}(1 - \gamma_{k_{//}}) + S_3 J_{34} + S_1 J_{41}.$$

References

[1] Joannopulos J D, Meade R D and Win J N 1995 *Photonic Crystals* (Princeton: Princeton University Press)

[2] Grahn H T (Ed.) 1995 *Semiconductor Superlattices:Growth & Electronic Properties* (Singapore: World Scientific)

[3] Grünberg P, Schreiber R, Peng Y, Brodsky M B and Sower H 1986 *Phys. Rev. Lett.* **57** 2442

[4] Baibich M N, Broto J M, Fert A, Nguyen Van Dau F, Petroff F, Etienne P, Creuzet G, Friederich A and Chazelas J 1988 *Phys. Rev. Lett.* **61** 2472

[5] Parkin S S P, More N and Roche K P 1990 *Phys. Rev. Lett.* **64** 2304

[6] Zhang Z Q, Falco C M, Ketterson J B and Schuller I K 1981 *Appl. Phys. Lett.* **38** 424

[7] Jarlborg T and Freeman A J 1982 *J. Appl. Phys.* **53** 8041

[8] Binasch G, Grunberg P, Saurenbach F and Zinn W 1989 *Phys. Rev.* B **39** 4828

[9] Barnes W L, Dereux A and Ebbesen T W 2003 *Nature* (London) **424** 824

[10] Akjouj A, Al-Wahsh H, Sylla B, Djafari-Rouhani B and Dobrzynski L 2004 *J. Phys.: Condens. Matter* **16** 37

[11] Zhu S N, Zhu Y Y, Li Y Q and Ming N B 2000 *Phase Transit.* **72** 239

[12] Dobrzynski L and Djafari–Rouhani B 1986 *Phys. Rev.* B **33** 3251

[13] Vasseur J O, Dobrzynski L, Djafari–Rouhani B and Puszkarski H 1996 *Phys. Rev.* B **54** 1043

[14] Krawczyk M, Jean-Claude Levy, Daniel Mercier and Puszkarski H 2001 *Phys. Lett.* A **282** 186

[15] Zhang Z D 1997 *Phys. Rev.* B **55** 12408

[16] LePage J G and Camley R E 1989 *Phys. Rev.* **B 40** 9113

[17] Meloche E, Pinciuc C M and Plumer M L 2006 *Phys. Rev.* B **74** 094424

[18] Che H, Xia Y, Lin D L, Qiu X M and Zheng H 1991 *Phys. Rev.* B **44** 9444

[19] van Stapele R P, Greidanus F J M and Smits J W 1985 *J. Appl. Phys.* **57** 1282
[20] Barnas J 1992 *Phys. Rev.* B **45** 10427
[21] Lopez J and de Lima O F 2006 *Solid State Communications* **139** 273
[22] Amit Pratap, Govind, Ajay and Joshi S K 2001 *Physica* C **355** 31
[23] Deng D S, Jin X F and Tao R B 2002 *Phys. Rev.* B **66** 104435
[24] Schwenk D, Fishman F and Schwabl F 1988 *Phys. Rev.* B **38** 11618
[25] Fishman F, Schwabl F and Schwenk D 1987 *Phys. Lett.* A **121** 192
[26] Qiu R K, Zhang Z D, Guo L Q, Ying C H and Liang J 2006 *Phys. Stat. Sol.* (b) **243** 1983
[27] Qiu R K, Zhang Z D, Han L, Ma G K and Zhang X D 2007 *Phys. Stat. Sol.* (b) **244** 4678
[28] Wei G Z, Qiu R K and Du A 1995 *Phys. Lett.* A **205** 335
[29] Irkhin V Yu, Katanin A A and Katsnelson M I 1996 *J. Magn. Magn. Mater.* **164** 66
[30] Irkhin V Yu, Katanin A A and Katsnelson M I 1999 *Phys. Rev.* B **60** 1082
[31] Wei G Z, Du A and Qiu R K 1996 *J. Phys.: Condens. Matter* **8** 5039
[32] Schwieger S, Kienert J and Nolting W 2005 *Phys. Rev.* B **71** 024428
[33] Holstein T and Primakoff H 1940 *Phys. Rev.* **58** 1098
[34] Anderson P W 1952 *Phys. Rev.* **86** 694
[35] Qiu R K, Zhang Z D and Guo L Q 2008 *J. Magn. Magn. Mater.* DOI information:10.1016/j.jmmm. 2008. 03. 036