Vol 17 No 10, October 2008
1674-1056/2008/17(10)/3841-06

Chinese Physics B

© 2008 Chin. Phys. Soc.
and IOP Publishing Ltd

Bright and dark small amplitude nonlinear localized modes in a quantum one-dimensional Klein–Gordon chain*

Liu Yang(刘 洋) and Tang Yi(唐 翌)†

Department of Physics, Xiangtan University, Xiangtan 411105, China

(Received 14 December 2007; revised manuscript received 2 March 2008)

By means of the Glauber's coherent state method combined with multiple-scale method, this paper investigates the localized modes in a quantum one-dimensional Klein–Gordon chain and finds that the equation of motion of annihilation operator is reduced to the nonlinear Schrödinger equation. Interestingly, the model can support both bright and dark small amplitude travelling and non-travelling nonlinear localized modes in different parameter spaces.

Keywords: nonlinear localized modes, coherent state, Klein–Gordon chain
PACC: 6320P, 6320R

1. Introduction

Over the past several decades, nonlinear localized modes (also called discrete breathers) have become a popular investigating field in one-dimensional anharmonic systems because some important properties of solids are more or less determined by the nonlinearity and discreteness. Particularly, in the classical one-dimensional lattice chain, the nonlinear localized modes, which are time-periodic and localized in space, have been intensively investigated by both analytical and numerical methods and great progress has been made.[1−28]

In searching for experimental means to verify nonlinear localized modes in real microscopic systems, recent attention has been paid to quantum nonlinear excitations. Generally, the quantum nonlinear excitations are studied by both numerical and analytical methods. Several important properties of the quantum nonlinear excitations are exhibited,[5−14] which improve our understanding of the nonlinear excitations. For example, adopting 'pseudofield' operators, Konotop and Takeno[12] investigated the quantum one-dimensional FPU chain and quantum envelope soliton solution has been found. In addition, by means of Glauber's coherent method combined with continuum approximation, Shi *et al*[9−11] investigated the quantum one-dimensional monoatomic chain and obtained the nonlinear phonon localized solutions for the expectation value of the displacement. However, in Refs.[9] and [10], in order to solve the equation of motion of annihilation operator, they introduced continuum approximation. So the results are valid for small wave vector. Actually, the one-dimensional lattice chain is discrete. However, the application of the continuum approximation would lead to the higher symmetry of the system. Therefore, one may lose some solutions when the system has higher symmetry. In the contrary, the application of the multiple-scale method, lowering the symmetry, enriches the class of solutions because fewer restrictions are imposed.[1] In fact, by using exact numerical diagonalization of the Hamiltonian, the multi-phonon bound states[5] (or biphonons bound states[6−8]) exist as eigenstates, which are natural counterparts of nonlinear localized modes of classical nonlinear microscopic systems. The numerical results not only give us much interesting information of the anharmonic lattice, but also imply that breathers can exist in quantum anharmonic lattice.

In this paper, by means of Glauber's coherent method combined with a simplified version of the multiple-scale method,[15] we investigated nonlinear localized modes in a quantum one-dimensional Klein–Gordon chain. Fortunately, we find that the equation of motion of annihilation operator can be solved ana-

*Project supported by the Key Project of Hunan Provincial Educational Department of China (Grant No 04A058).
†Corresponding author. E-mail: tangyii-phys@sohu.com

lytically without adopting continuum approximation. Analytical nonlinear localized mode solutions for both bright and dark localized modes are demonstrated. This paper is organized as follows. In Section 2 the Klein–Gordon chain is formulated by means of the coherent state presentation combined with multiple-scale method. The nonlinear Schrödinger equation is obtained. In Section 3 we present the results and discussions. All possible nonlinear localized modes in Klein–Gordon chain are given in different parameter space. The paper is concluded in Section 4.

2. Method

Consider the Klein–Gordon chain with nearest neighbours interactions, whose Hamiltonian is given by

$$H=\sum_n \frac{p_n^2}{2M}+\frac{1}{2}c\left(u_n-u_{n-1}\right)^2+V\left(u_n\right). \quad (1)$$

Where $V\left(u_n\right)=\frac{1}{2}k_2u_n^2+\frac{1}{4}k_4u_n^4$ is a substrate potential at the n-th particle. p_n and u_n are momentum and displacement of the n-th particle respectively, c is the harmonic force constant, k_2 and$k_4$are on-site potential parameters and we set $k_2>0$ in this paper. We will show later that the parameter k_4 plays a crucial role for the emergence of bright or dark nonlinear localized modes. The lattice model has been used as a prototype for analysing the effects of nonlinearity and spatial discreteness on localization of vibrational energy and energy transport (see Ref.[1] for a review). If the on-site potential $V\left(u_n\right)$ is chosen as a Morse potential, Eq.(1) is the Peyrard–Bishop model for DNA denaturation.[16]

Substituting $V\left(u_n\right)=\frac{1}{2}k_2u_n^2+\frac{1}{4}k_4u_n^4$ into Eq.(1), one get

$$H=\sum_n \frac{p_n^2}{2M}+\frac{1}{2}\left(c+k_2\right)u_n^2 +\frac{1}{2}c\left(u_{n-1}^2-2u_nu_{n-1}\right)+\frac{1}{4}k_4u_n^4. \quad (2)$$

In this paper we consider that anharmonic and interparticle effects are relatively small,[17] i.e.

$$c\ll k_2,\quad \left|k_4\bar{u}^2\right|\ll k_2. \quad (3)$$

Where $\bar{u}$ is a root-mean-square value of any of the u_n. The condition (3) is reasonable in some real systems.[18]

We may quantize the Hamiltonian through the substitutions[10,11,17]

$$u_n\to\left(\frac{\hbar}{2M\omega_0}\right)^{\frac{1}{2}}\left(b_n^+ + b_n\right),$$
$$p_n\to\left(\frac{\hbar M\omega_0}{2}\right)^{\frac{1}{2}}\mathrm{i}\left(b_n^+ - b_n\right). \quad (4)$$

Here $\omega_0=\sqrt{(k_2+c)/M}$. The Bose operator b_n^+ and b_n obey the Bose commutation relation $\left[b_n,b_{n'}^+\right]=\delta_{nn'}$. The quantized Hamiltonian of the model can be written as follows.

$$H=H_{\mathrm{eff}}+H' \quad (5)$$

with

$$H_{\mathrm{eff}}=\sum_n\frac{\hbar\omega_0}{4}\left(b_n^+b_n+b_nb_n^+\right) +\sum_n\frac{c\hbar}{4M\omega_0}\left[\left(b_n^+b_n+b_nb_n^+ +b_{n-1}^+b_{n-1}+b_{n-1}b_{n-1}^+-2b_n^+b_{n-1}-2b_nb_{n-1}^+\right)\right] +\sum_n\frac{k_2\hbar}{4M\omega_0}\left(b_n^+b_n+b_nb_n^+\right)+\sum_n\frac{k_4}{4}\left(\frac{\hbar}{2M\omega_0}\right)^2\left(6b_n^+b_n^+b_nb_n+12b_n^+b_n+3\right), \quad (6)$$

$$H'=-\frac{\hbar\omega_0}{4}\left(b_n^+b_n^++b_nb_n\right)+\frac{\hbar k_2}{4M\omega_0}\left(b_n^+b_n^++b_nb_n\right) +\frac{k_4}{4}\left(\frac{\hbar}{2M\omega_0}\right)^2(b_n^+b_n^+b_n^+b_n^+ +b_n^+b_n^+b_n^+b_n+b_n^+b_n^+b_nb_n^+ +b_nb_nb_nb_n+b_nb_nb_n^+b_n +b_nb_nb_nb_n^+ +b_n^+b_nb_n^+b_n^+ +b_n^+b_nb_nb_n+b_nb_n^+b_n^+b_n^+ +b_nb_n^+b_nb_n) +\frac{\hbar c}{4M\omega_0}\left(b_n^+b_n^++b_nb_n+b_{n-1}^+b_{n-1}^++b_{n-1}b_{n-1}-b_{n-1}^+b_n^+-b_{n-1}b_n-b_n^+b_{n-1}^+-b_nb_{n-1}\right). \quad (7)$$

Here phonon conserving term H_{eff} is the effective Hamiltonian. It is obvious that the phonon non-conserving term H' will cause complexity in this problem. However, under the condition of Eq.(3), it has been proved that non-conserving term H' is unimportant and can be omitted in Ref.[18]. Therefore, only $H_{\text{eff}}$is used in this paper. As is known, the equation of motion of annihilation operator in Heisenberg picture may be written as $\mathrm{i}\hbar\dfrac{\partial}{\partial t}b_m = [b_m, H_{\text{eff}}]$. Subsequently, we get

$$\begin{aligned}
\mathrm{i}\frac{\partial}{\partial t}b_m =&\frac{\omega_0}{2}b_m + \frac{c}{2M\omega_0}b_m + \frac{c}{2M\omega_0}b_m - \frac{c}{2M\omega_0}b_{m-1} - \frac{c}{2M\omega_0}b_{m+1}\\
&+ \frac{k_2}{2M\omega_0}b_m + 3k_4\hbar\left(\frac{1}{2M\omega_0}\right)^2 b_m^+ b_m b_m + 3k_4\hbar\left(\frac{1}{2M\omega_0}\right)^2 b_m\\
=&\frac{\omega_0}{2}b_m + \frac{c}{2M\omega_0}b_m + \frac{c}{2M\omega_0}b_m - \frac{c}{2M\omega_0}b_{m-1} - \frac{c}{2M\omega_0}b_{m+1}\\
&+ \frac{k_2}{2M\omega_0}b_m + 3k_4\hbar\left(\frac{1}{2M\omega_0}\right)^2 b_m^+ b_m b_m + 3k_4\hbar\left(\frac{1}{2M\omega_0}\right)^2 b_m\\
=&\left(\frac{\omega_0}{2} + \frac{c}{M\omega_0} + \frac{k_2}{2M\omega_0} + 3k_4\hbar\left(\frac{1}{2M\omega_0}\right)^2\right) b_m - \frac{c}{2M\omega_0}\left(b_{m-1} + b_{m+1}\right)\\
&+ 3k_4\hbar\left(\frac{1}{2M\omega_0}\right)^2 b_m^+ b_m b_m. \qquad (8)
\end{aligned}$$

Although Eq.(8) is complex, it can be solved analytically. Following Refs.[9], [10] and [11], we introduce Glauber's coherent states to Eq.(8)

$$\begin{aligned}
b_m\left|\beta\right\rangle &= \beta_m\left|\beta\right\rangle,\\
\left|\beta\right\rangle &= \prod_i \left|\beta_i\right\rangle. \qquad (9)
\end{aligned}$$

In this representation, Eq.(8) takes the form

$$\begin{aligned}
&\mathrm{i}\frac{\partial}{\partial t}\beta_m\\
=&\left(\frac{\omega_0}{2} + \frac{c}{M\omega_0} + \frac{k_2}{2M\omega_0} + 3k_4\hbar\left(\frac{1}{2M\omega_0}\right)^2\right)\beta_m\\
&- \frac{c}{2M\omega_0}\left(\beta_{m-1} + \beta_{m+1}\right)\\
&+ 3k_4\hbar\left(\frac{1}{2M\omega_0}\right)^2 \beta_m^+\beta_m\beta_m\\
=&A\beta_m - B\left(\beta_{m-1} + \beta_{m+1}\right) + C\left|\beta_m\right|^2\beta_m, \qquad (10)
\end{aligned}$$

with

$$A = \left(\frac{\omega_0}{2} + \frac{c}{M\omega_0} + \frac{k_2}{2M\omega_0} + 3k_4\hbar\left(\frac{1}{2M\omega_0}\right)^2\right), \qquad (11)$$

$$B = \frac{c}{2M\omega_0}, \qquad (12)$$

$$C = 3k_4\hbar\left(\frac{1}{2M\omega_0}\right)^2. \qquad (13)$$

Now, we adopt a simplified version of the multiple-scale method.[15] Since we look for small amplitude nonlinear localized modes, so we make transformation $\beta_m \to \varepsilon\beta_m$, where ε is a small parameter. Then Eq.(10) reduces to

$$\begin{aligned}
\mathrm{i}\frac{\partial}{\partial t}\beta_m =&A\beta_m - B\left(\beta_{m-1} + \beta_{m+1}\right)\\
&+ C\varepsilon^2\left|\beta_m\right|^2\beta_m. \qquad (14)
\end{aligned}$$

We now look for modulated solution of the form

$$\beta_m = \phi\left(t_1, t_2, x_1\right)\mathrm{e}^{\mathrm{i}(kma-\omega t)}, \qquad (15)$$

which consists of a slowly varying envelope modulating a carrier wave. Where $t_1 = \varepsilon t, t_2 = \varepsilon^2 t, x_1 = \varepsilon x$. The derivative operator $\partial/\partial t$ and $\partial/\partial x$ are expanded as

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t} + \varepsilon\frac{\partial}{\partial t_1} + \varepsilon^2\frac{\partial}{\partial t_2}, \quad \frac{\partial}{\partial x} = \varepsilon\frac{\partial}{\partial x_1}. \qquad (16)$$

Consequently, from Eq.(14) to Eq.(16) we get

$$\begin{aligned}
&\left[\omega - A + 2\cos\left(ka\right)B\right]\phi_m\\
&+ \varepsilon\left[\mathrm{i}\frac{\partial\phi_m}{\partial t_1} + 2\mathrm{i}aB\sin\left(ka\right)\frac{\partial\phi_m}{\partial x_1}\right]\\
&+ \varepsilon^2\left[\mathrm{i}\frac{\partial\phi_m}{\partial t_2} + a^2B\cos\left(ka\right)\frac{\partial^2\phi_m}{\partial x_1^2} + C\left|\phi_m\right|^2\phi_m\right]\\
=&0. \qquad (17)
\end{aligned}$$

Equate coefficient at each order of ε, we get

$$\omega = A - 2B\cos(ka), \tag{18}$$

$$\frac{\partial \phi_m}{\partial t_1} = -2aB\sin(ka)\frac{\partial \phi_m}{\partial x_1}, \tag{19}$$

$$\mathrm{i}\frac{\partial \phi_m}{\partial t_2} + a^2 B\cos(ka)\frac{\partial^2 \phi_m}{\partial x_1^2} + C|\phi_m|^2\phi_m = 0. \tag{20}$$

Equation (18) gives the dispersion relation. Therefore, we can get the group velocity

$$v_{\mathrm{g}} = \frac{\mathrm{d}\omega}{\mathrm{d}k} = 2Ba\sin(ka). \tag{21}$$

From Eq.(19), we can see that

$$\phi_m = \phi_m(t_2, z), \tag{22}$$

where $z = x_1 - 2aB\sin(ka)\,t_1$. Then Eq.(20) transformed into

$$\mathrm{i}\frac{\partial \phi_m}{\partial t_2} + B'\frac{\partial^2 \phi_m}{\partial z^2} + C|\phi_m|^2\phi_m = 0, \tag{23}$$

where $B' = a^2 B\cos(ka)$. Eq.(23) is the famous nonlinear Schrödinger equation.

3. Results and Discussions

The solutions of Eq.(23) depend on the sign of the dispersive coefficient B' and the nonlinear coefficient C.[19] If $B'C > 0$, Eq.(23) has a bright envelope soliton solution which has a vanishing amplitude at$|z| \to \infty$. If $B'C < 0$, Eq.(23) has a dark envelope soliton solution.

A. $k \in [-\pi/2a, \pi/2a]$

From Eq.(12), we can see that $B > 0 (c > 0)$, so, while $k \in [-\pi/2a, \pi/2a]$, $B' > 0$. From Eq.(13), we find that k_4 determines the sign of the nonlinear coefficient C. When $k_4 > 0$, the nonlinear coefficient $C > 0$. Then the bright nonlinear localized mode of Eq.(23) is given by

$$\phi_m = \sqrt{\frac{2B'}{C}}\gamma\,\mathrm{sech}\left[\gamma(z - z_0)\right]\mathrm{e}^{\mathrm{i}\gamma^2 B' t_2}, \tag{24}$$

where the soliton amplitude γ is a free parameter. Combining Eqs.(15), (21) and (23), we can get

$$\begin{aligned}\beta_m =& \sqrt{\frac{2B'}{C}}\gamma\,\mathrm{sech}\,\varepsilon\gamma\left[(m - m_0)a - v_{\mathrm{g}}t\right] \\ &\times \mathrm{e}^{\mathrm{i}(kma - \omega t) + \mathrm{i}\varepsilon^2\gamma^2 B' t}.\end{aligned} \tag{25}$$

Finally, expectation value of the displacement of lattice sites is given by

$$\begin{aligned}u_m =& \sqrt{\frac{\hbar}{2m\omega_0}}\left(\beta_m + \beta_m^*\right) \\ =& \varepsilon\sqrt{\frac{\hbar}{2m\omega_0}}\left\{2\sqrt{\frac{2B'}{C}}\gamma\,\mathrm{sech}\,\varepsilon\gamma\left[(m - m_0)a - v_{\mathrm{g}}t\right]\right. \\ &\left.\times\cos\left(kma - \omega t + \varepsilon^2\gamma^2 B' t\right)\right\}.\end{aligned} \tag{26}$$

Equation (26) represents a bright envelope soliton with group velocity $v_{\mathrm{g}} = \dfrac{\mathrm{d}\omega}{\mathrm{d}k} = 2Ba\sin(ka)$. At the centre of the Brillouin zone, i.e., at $k = 0$, the group velocity $v_{\mathrm{g}} = 0$, then Eq.(26) represents a non-travelling breather.

$$\begin{aligned}u_m =& \varepsilon\sqrt{\frac{\hbar}{2M\omega_0}}\left\{2\sqrt{\frac{2B'}{C}}\gamma\,\mathrm{sech}\,\left[\varepsilon\gamma(m - m_0)a\right]\right. \\ &\left.\times\cos(kma + \Omega t)\right\},\end{aligned} \tag{27}$$

where the oscillation frequency $\Omega = \left(\varepsilon^2\gamma^2 Ba^2 - \omega\right)$. Nonlinear localized modes with similar properties are often called intrinsic localized modes.[20–22]

When $k_4 < 0$, the nonlinear coefficient $C < 0$. Then the dark localized mode of Eq.(23) is given by[21,23]

$$\phi_m = D(z, t_2)\,\mathrm{e}^{\mathrm{i}\mu(z, t_2)}, \tag{28}$$

where

$$D(z, t_2) = D_0\left[1 - \rho^2\,\mathrm{sech}^2\left(\rho D_0\sqrt{\frac{-C}{2B'}}z\right)\right]^{\frac{1}{2}}, \tag{29}$$

$$\begin{aligned}\mu(z, t_2) =& \sqrt{\frac{-C}{2B'}}\left[\sqrt{1 - \rho^2}D_0 z + \tan^{-1}\left\{\frac{\rho}{\sqrt{1 - \rho^2}} \times \tanh\left(\rho D_0\sqrt{\frac{-C}{2B'}}z\right)\right\}\right] \\ &- \frac{D_0 C}{2}\left(3 - \rho^2\right)t_2.\end{aligned} \tag{30}$$

Here, D_0 is a free parameter and ρ $(0 \leq \rho \leq 1)$ is a parameter which controls the depth of the modulation of amplitude. Finally, we get

$$\begin{aligned}\beta_m &= \phi_m \mathrm{e}^{\mathrm{i}(kma-\omega t)} \\ &= D\left(z,t_2\right)\mathrm{e}^{\mathrm{i}\mu(z,t_2)} \times \mathrm{e}^{\mathrm{i}(kma-\omega t)} \\ &= D_0\left[1-\rho^2\,\mathrm{sech}^{\,2}\left(\rho D_0\varepsilon\sqrt{\frac{-C}{2B'}}\left(ma-v_{\mathrm{g}}t\right)\right)\right]^{\frac{1}{2}} \\ &\quad\times \mathrm{e}^{\mathrm{i}\sqrt{\frac{-C}{2B'}}\left[\sqrt{1-\rho^2}D_0\varepsilon(ma-v_{\mathrm{g}}t)+\tan^{-1}\left\{\frac{\rho}{\sqrt{1-\rho^2}}\times\tanh\left(\rho D_0\varepsilon\sqrt{\frac{-C}{2B'}}(ma-v_{\mathrm{g}}t)\right)\right\}\right]-\frac{D_0C}{2}\varepsilon^2\left(3-\rho^2\right)t+\mathrm{i}(kma-\omega t)}. \end{aligned} \tag{31}$$

From Eq.(31), we finally obtain expectation value of the displacement of lattice sites

$$\begin{aligned}u_m =& \sqrt{\frac{\hbar}{2M\omega_0}}\left(\beta_m+\beta_m^*\right) \\ =& \varepsilon\sqrt{\frac{\hbar}{2M\omega_0}}\left\{2D_0\left[1-\rho^2\,\mathrm{sech}^{\,2}\left(\rho D_0\varepsilon\sqrt{\frac{-C}{2B'}}\left(ma-v_{\mathrm{g}}t\right)\right)\right]^{\frac{1}{2}}\right\} \\ &\times\cos\left\{\sqrt{\frac{-C}{2B'}}\left[\sqrt{1-\rho^2}D_0\varepsilon\left(ma-v_{\mathrm{g}}t\right)+\tan^{-1}\left\{\frac{\rho}{\sqrt{1-\rho^2}}\times\tanh\left(\rho D_0\varepsilon\sqrt{\frac{-C}{2B'}}\left(ma-v_{\mathrm{g}}t\right)\right)\right\}\right]\right. \\ &\left.-\frac{D_0C}{2}\varepsilon^2\left(3-\rho^2\right)t+kma-\omega t\right\}. \end{aligned} \tag{32}$$

As we discussed above, Eq.(32) represents a dark envelope soliton with group velocity $v_{\mathrm{g}}=\dfrac{\mathrm{d}\omega}{\mathrm{d}k}=2Ba\sin\left(ka\right)$. At the centre of the Brillouin zone, i.e., at $k=0$, the group velocity $v_{\mathrm{g}}=0$, which represents non-travelling breather

$$\begin{aligned}u_m =& \varepsilon\sqrt{\frac{\hbar}{2M\omega_0}}\left\{2D_0\left[1-\rho^2\,\mathrm{sech}^{\,2}\left(\rho D_0\varepsilon\sqrt{\frac{-C}{2B'}}ma\right)\right]^{\frac{1}{2}}\right\} \\ &\times\cos\left\{\sqrt{\frac{-C}{2B'}}\left[\sqrt{1-\rho^2}D_0\varepsilon ma+\tan^{-1}\left\{\frac{\rho}{\sqrt{1-\rho^2}}\times\tanh\left(\rho D_0\varepsilon\sqrt{\frac{-C}{2B'}}ma\right)\right\}\right]kma-\Omega t\right\}, \end{aligned} \tag{33}$$

where the oscillation frequency $\Omega=\left[\dfrac{D_0C}{2}\varepsilon^2\left(3-\rho^2\right)+\omega\right]$.

B. $k\in[-\pi/a,-\pi/2a]$ or $k\in[\pi/2a,\pi/a]$

While $k\in[-\pi/a,-\pi/2a]$ or $k\in[\pi/2a,\pi/a]$, $B'<0$. When $k_4>0$, we get dark travelling localized mode

$$\begin{aligned}u_m =& \varepsilon\sqrt{\frac{\hbar}{2M\omega_0}}\left\{2D_0\left[1-\rho^2\,\mathrm{sech}^{\,2}\left(\rho D_0\varepsilon\sqrt{\frac{-C}{2B'}}\left(ma-v_{\mathrm{g}}t\right)\right)\right]^{\frac{1}{2}}\right\} \\ &\times\cos\left\{\sqrt{\frac{-C}{2B'}}\left[\sqrt{1-\rho^2}D_0\varepsilon\left(ma-v_{\mathrm{g}}t\right)+\tan^{-1}\left\{\frac{\rho}{\sqrt{1-\rho^2}}\times\tanh\left(\rho D_0\varepsilon\sqrt{\frac{-C}{2B'}}\left(ma-v_{\mathrm{g}}t\right)\right)\right\}\right]\right. \\ &\left.-\frac{D_0C}{2}\varepsilon^2\left(3-\rho^2\right)t+kma-\omega t\right\}, \end{aligned} \tag{34}$$

where $v_{\mathrm{g}}=\dfrac{\mathrm{d}\omega}{\mathrm{d}k}=2Ba\sin\left(ka\right)$ is the group velocity. Different from part A, at the boundary of the Brillouin zone, i.e., at $k=\pi/a$ or $k=-\pi/a$, the group velocity $v_{\mathrm{g}}=0$.

When $k_4>0$, the bright travelling localized mode is obtained.

$$u_m=\varepsilon\sqrt{\frac{\hbar}{2M\omega_0}}\left\{2\sqrt{\frac{2B'}{C}}\gamma\,\mathrm{sech}\,\varepsilon\gamma\left[\left(m-m_0\right)a-v_{\mathrm{g}}t\right]\cos\left(kma-\omega t+\varepsilon^2\gamma^2B't\right)\right\}. \tag{35}$$

Similarly, at the boundary of the Brillouin zone, i.e., at $k=\pi/a$ or $k=-\pi/a$, the group velocity $v_{\mathrm{g}}=0$.

4. Conclusions

In this paper, by using Glauber's coherent state method combined with a multiple-scale method, we investigate analytically the quantum one-dimensional Klein–Gordon chain. We find that the equation of motion of annihilation operator can be reduced to the discrete nonlinear Schrödinger equation. Bright and dark nonlinear localized solutions for the expectation value of the displacements are found. Several interesting aspects emerged: the on-site potential parameter k_4 and wave vector k determine whether the bright localized modes or dark localized modes emerge. When $k_4 > 0$, the bright localized modes emerge with $k \in [-\pi/2a, \pi/2a]$ and the dark localized modes emerge with $k \in [-\pi/a, -\pi/2a]$ or $k \in [\pi/2a, \pi/a]$. When $k_4 < 0$, the result is just opposite: the dark localized modes emerge with $k \in [-\pi/2a, \pi/2a]$ and the bright localized modes emerge with$k \in [-\pi/a, -\pi/2a]$ or $k \in [\pi/2a, \pi/a]$. These phenomena, to the best of our knowledge, have not been reported previously. These results may improve our understanding of the simple anharmonic lattice.

References

[1] Flach S and Willis C 1998 *Phys. Rep.* **295** 181
[2] Braun Oleg M and Kivshar Yuri S 1998 *Phys. Rep.* **306** 1
[3] Zhou G H, Xia Q L, Pan L X and Yan J R 1996 *Phys. Rev.* B **53** 13977
[4] Zhou G H, Duan Y W and Yan J R 2001 *Chin. Phys.* **10** 689
[5] Wang W Z, Gammel J Tinka, Bishop A R and Salkola M I 1997 *Phys. Rev. Lett.* **76** 3598
[6] Proville L 2005 *Europhys. Lett.* **69** 763
[7] Proville L 2005 *Phys. Rev.* B **71** 104306
[8] Proville L 2006 *Physica* D **216** 191
[9] Huang G X, Shi Z P and Dai X X 1990 *Phys. Rev.* B **41** 12292
[10] Shi Z P and Huang G X 1991 *Phys. Rev.* B **44** 12601
[11] Huang G X 1992 *Chin. Phys. Lett.* **9** 151
[12] Konotop V V and Takeno S 2001 *Phys. Rev.* E **63** 066606
[13] Schulman L S and Tolkunov D 2006 *Phys. Rev. Lett.* **96** 065501
[14] Iivá Z and Tsironis G P 2006 *Physica* D **216** 200
[15] Remoissenet M 1986 *Phys. Rev.* B **33** 2386
[16] Peyrard M and Bishop A R 1989 *Phys. Rev. Lett.* **62** 2755
[17] Kimbal J C, Fong C Y and Shen Y R 1981 *Phys. Rev.* B **23** 4946
[18] Li D J, Mi X W, Deng K and Tang Y 2006 *Commun. Theor. Phys.* **45** 869
[19] Zakharov V E and Shabat A B 1971 *Zh. Eksp. Tero. Fiz.* **61** 118
Zakharov V E and Shabat A B 1972 *Sov. Phys. -JETP* **34** 62
[20] Sievers A J and Takeno S 1988 *Phys. Rev. Lett.* **61** 970
[21] Remoissenet M 1999 *Waves Called Solitons. Concepts and Experiments*, 2nd edn. (Berlin: Springer) pp210–212
[22] Page J B 1990 *Phys. Rev.* B **41** 7835
[23] Imran A, Jonathan A D and Wattis 2007 *Physica* D **231** 165
[24] Pan L X, Zhou G H, Xia Q L and Yan J R 2001 *Commun. Theor. Phys.* **35** 609
[25] Chen W Z 1994 *Phys. Rev.* B **49** 15063
[26] Xia Q L and Jiang D C 2004 *Commun. Theor. Phys.* **42** 295
[27] Tian Q and Li M S 2007 *Chin. Phys.* **16** 228
[28] Xu Q and Tian Q 2006 *Chin. Phys.* **15** 253