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Finite-amplitude vibration of a bubble in water*

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The numerical results obtained by Rayleigh–Plesset (R-P) equation failed to agree with the experimental Mie scattering data of a bubble in water without inappropriately increasing the shear viscosity and decreasing the surface tension coefficient. In this paper, a new equation proposed by the present authors (Qian and Xiao) is solved. Numerical solutions obtained by using the symbolic computation program from both the R-P equation and the Qian-Xiao (Q-X) equation clearly demonstrate that Q-X equation yields best results matching the experimental data (in expansion phase). The numerical solutions of R-P equation also demonstrate the oscillation of a bubble in water depends strongly upon the surface tension and the shear viscosity coefficients as well as the amplitude of driving pressure, so that the uniqueness of the numerical solutions may be suspected if they are varied arbitrarily in order to fit the experimental data. If the bubble's vibration accompanies an energy loss such as the light radiation during the contract phase, the mechanism of the energy loss has to be taken into account. We suggest that by use of the bubble's vibration to investigate the state equations of aqueous solutions seem to be possible. We also believe that if one uses this equation instead of R-P equation to deal with the relevant problems such as the 'phase diagrams for sonoluminescing bubbles', etc., some different results may be expected.

Keywords: bubble, sonoluminescence, finite-amplitude vibration of a bubble
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1. Introduction

When a bubble is trapped in liquid by a strong sound field it undergoes finite-amplitude oscillations. During the collapse when its volume approaches to the minimum, the energy density of the gas inside the bubble becomes extremely high so that light emission is produced, which was first observed by Gaitan *et al.*[1] In order to investigate the mechanism of the sonoluminescence a variety of physical processes were suggested,[2−7] all of which have something in common that they have to resort to a equation to describe the vibration of the bubble. Moreover, in many practical cases the bubble's vibration has been involved, for example, in the uses of high intensity focused ultrasound (HIFU) many phenomena relevant to bubbles were observed, which would have tremendous influence upon the HIFU operation, and thus upon the physical and medical therapy.[8,9] Recently, the authors of Ref.[10] asserted that they observed the cavitation fusion, which up to now has been debated about hotly. In order to examine theoretically whether this phenomenon is true, an equation to exactly describe bubble's vibration becomes absolutely necessary.

During the rarefaction half-cycle the radius of the bubble will increase from its ambient value to the maximum $R_{\max}$, then decreases to its minimum value $R_{\min}$ during the compression half-cycle, which spans almost two degrees of magnitude so that its instantaneous radius obeys a nonlinear equation. Rayleigh, the pioneer to explore the subject of the nonlinear vibration of a bubble, investigated the vibration of a void in an incompressible liquid.[11] Plesset,[12] Nolting and Neppiras,[13] and Poritsky[14] studied a similar case, where the void was filled with gas or vapour, with the effects of the shear viscosity and the surface tension being taken into account. In order to investigate the vibration of a bubble in the compressible liquid, Trilling,[15] Herring[16] and Flynn[17] made an assumption that the vibrating bubble emits acoustic radiation, in which $r\phi$ propagates toward the liquid with the sound speed of infinitesimal-amplitude wave c_0, where r is the distance from the bubble's centre to a point in liquid, and ϕ is the velocity potential. Based on the Kirkwood–Bethe (KB) assumption that the kinetic enthalpy $h + u^2/2$ multiplied by the distance r is a Riemann–Earnshaw simple wave, for an isentropic process Gilmore[18] obtained the KBCG equation, where h and u are the enthalpy and the particle velocity in the liquid, respectively. Moreover,

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it is noteworthy that in these previous theories the viscosity occurring in the kinetic equation was neglected, the influences of which were taken into account in the boundary conditions.[14] By taking the shear viscosity into account, an equation was given by Keller and Miksis (KM),[19] where the shear viscosity is included in the inertial term. If the viscosity is neglected, one can obtain the Lastman and Wentzell (LW) equation.[20] Recently, by a suggestion of one-parameter family of equations in the compressible liquids Prosperetti[21] attempted to combine the various theoretical equations into a comprehensively unitary expression, which seems to be difficult. For convenience of formulating the problem, in this paper we suggest an equation by a "dull" way, which can be described as follows:

$$\begin{aligned}&\left[n_1\frac{4\eta}{\rho cR}+(1-n_2M)\right]R\ddot{R}+\frac{3}{2}\left(1-\frac{n_3}{3}M\right)\dot{R}^2\\&=(1+n_6M)h+\frac{R}{\dot{R}}(M-n_4M^2+n_5M^3)\dot{h}-\frac{4\eta}{\rho R}\dot{R}\\&\quad-\frac{2\sigma}{\rho R}+n_7f(R,\dot{R};t),\end{aligned}\tag{1}$$

where ρ is the density, c is the sound speed, η is the shear viscosity coefficient, σ is the surface tension coefficient of the liquid surrounding the bubble, R is the instantaneous radius of the bubble, and $M=\dot{R}/c$. The function f indicates some additional terms, and

$$h=\int_{P(\infty)}^{P(R)}\frac{\mathrm{d}p}{\rho},\tag{2}$$

where the pressure of the liquid on the surface of the bubble can be denoted by

$$P(R)=(P_0+2\sigma/R)[(R_0^3-a^3)/(R_0^3-a^3)]^\gamma-2\sigma/R$$

and $P(\infty)=P_0-p_\mathrm{a}\sin\omega t$, where R_0 is the ambient radius of the bubble, a is the van der Waals hard-core radius, P_0 is the static pressure, and p_a is the amplitude of driving pressure. Thus, one has following results: $n_1=0,\ n_2=2,\ n_3=4,\ n_4=1,\ n_5=1,\ n_6=0,\ n_7=0$ from equation by Trilling; $n_1=0, n_2=2, n_3=0,\ n_4=0,\ n_5=0,\ n_6=0,\ n_7=0$ from the equation by Flynn; $n_1=1,\ n_2=1,\ n_3=1,\ n_4=0,\ n_5=0,\ n_6=1,\ n_7=1$ from the equation by Keller–Miksis; $\eta=0,\ n_2=1,\ n_3=1,\ n_4=0,\ n_5=0,\ n_6=1, n_7=1$ from the equation by Lastman–Wentzell; $n_1=0,\ n_2=0, n_3=0,\ n_4=0,\ n_5=0,\ n_6=0,\ n_7=0$, from the Rayleigh–Plesset equation. In the above-mentioned theories the approximation

$$h\approx[P(R)-P(\infty)]/\rho_0$$

is taken; and one has $n_1=0,\ n_2=1,\ n_3=1,\ n_4=1,\ n_5=0,\ n_6=1,\ n_7=0$ from equation by KBCG, where h can be obtained by substituting Tait's state equation into Eq.(2). It is worth noticing that when $M>1$, some theoretical equations present the problem that the effective mass becomes negative (negative inertia).[22]

2. Q (Qian)-X (Xiao) equation

Obviously, there are several defects in the R-P equation. (1) It can only be used in the situation where $M\ll 1$. However, the numerical computation shows that when the radius of the bubble approaches to its minimum value, we have $M>1$ even $M\gg 1$. (2) The volume viscosity ξ is neglected, which cannot be valid in liquids such as water. As pointed out by Leonard Hall the structure relaxation exists in water,[23] which causes a non-zero volume viscosity ξ. The sound absorption experiment carried out by Liebermann showed that only if the volume viscosity ($\xi\approx 8\eta/3$[24–26]) is taken into account, can the theoretical result agree with the measurements. (3) It is assumed that the bubble emits an infinitesimal-amplitude radiation, which disagrees with the experimental results obtained in Ref.[27], where the shock waves in water near the bubble's surface were observed. (4) In the process of mathematical derivation of R-P equation and the numerical computation, all of them did not consider the change of the sound speed as well as the density etc. with the time t.

Based on the generalized hydrodynamic equation including the volume viscosity and the assumption that $r\phi$ is a Riemann–Earnshaw radiation, we have obtained an equation (Q-X)[28] as follows:

$$\begin{aligned}&\left(1+M+\frac{1}{2}M^2+\frac{h(0,t)}{c^2}+\frac{b}{\rho^2c^4}\frac{\partial p}{\partial t}\right)R\ddot{R}+\frac{3}{2}\left[(1+M)\left(1+\frac{2}{3}M\right)+\frac{R}{2c^2}\frac{\partial c}{\partial t}\right]\dot{R}^2\\&=\left[(1+M)(1+2M)-\frac{R}{c^2}\frac{\partial c}{\partial t}\right]h(0,\tau)+R(1+M)\frac{1}{c}\frac{\partial h(0,t)}{\partial t}\end{aligned}$$

$$-\frac{b}{\rho^2 c^2}\left\{\left[\frac{R}{c^2}\frac{\partial c}{\partial t}-M(1+M)-(1+M)^2\right]\frac{\partial p}{\partial t}-\rho\, c^2\frac{R}{c}(1+M)\frac{\partial}{\partial t}\left(\frac{1}{\rho\, c^2}\frac{\partial p}{\partial t}\right)\right\}, \tag{3}$$

where $M=\dot{R}/c$, $\dot{X}=\partial X/\partial t$, $\ddot{X}=\partial^2 X/\partial t^2$, $b=\xi+4\eta/3$, ξ and η are the volume and shear viscosities, respectively, ($b=4\eta$)[25,26] for water. In the numerical computation, the Tait equation $[P(R)+B]/[P(\infty)+B]=[\rho(R)/\rho(\infty)]^n$ will be used to calculate ρ, c etc., where $B=3074\,\mathrm{atm}$. ($1\,\mathrm{atm}\simeq 10^5\,\mathrm{Pa}$), $n=7.14$ for water.

3. Numerical computation

Long time ago, the R-P equation was always used to explain sonoluminescence phenomena, during which, for a given ambient size of the bubble, the ratio R_m/R_0 is best match with the experimental data by varying p_a.[2] Substituting the values of R_0 and p_a into R-P equation, one obtains its numerical solution. However, the theoretical results given by this procedure usually do not agree with the Mie scattering data without enlarging the shear viscosity and reducing the surface tension coefficient.[2,3,22,29,30] In this paper, Eq.(3) and the R-P equation are solved numerically, using the symbolic computation program with Runge-Kutta method. Figure 1 shows the results for the frequency $f=26.5\,\mathrm{kHz}$, $p_\mathrm{a}=1.325\,\mathrm{atm.}$, $R_0=4.5\,\mu\mathrm{m}$, $c_0=1481\,\mathrm{m/s}$, where the solid line indicates the Q-X result with $b=0.004\,\mathrm{Pa\cdot s}$, $\sigma=0.072\,\mathrm{N/s}$; the dashed line indicates R-P result with $\eta=0.007\,\mathrm{Pa\cdot s}$ (enlarged 7 times),[2] $\sigma=0\,\mathrm{N/s}$; and the dotted line is the R-P result with $\eta=0.001\,\mathrm{Pa\cdot s}$, $\sigma=0.072\,\mathrm{N/s}$ (both of them are the normal values); the symbols (circles) represent the Mie scattering experimental data (taken from Ref.[2]), which shows that during the period from the expansion of the bubble to its minimum size, only the theoretical result by Q-X equation (where the normal values of the viscosity and surface tension coefficient are used) shows a good fit to the experimental data. Figure 2 shows the results for $f=26.5\,\mathrm{kHz}$, $p_\mathrm{a}=1.075\,\mathrm{atm.}$, $R_0=10.5\,\mu\mathrm{m}$, $c_0=1481\,\mathrm{m/s}$, where the solid line indicates the Q-X result with $b=0.004\,\mathrm{Pa\cdot s}$, $\sigma=0.072\,\mathrm{N/s}$; the dashed line indicates the R-P result with $\eta=0.007\,\mathrm{Pa\cdot s}$ (enlarged 7 times),[2] $\sigma=0\,\mathrm{N/s}$; and the dotted line is the R-P result with $\eta=0.001\,\mathrm{Pa\cdot s}$, $\sigma=0.072\,\mathrm{N/s}$ (both of them are the normal values), the symbols (circles)

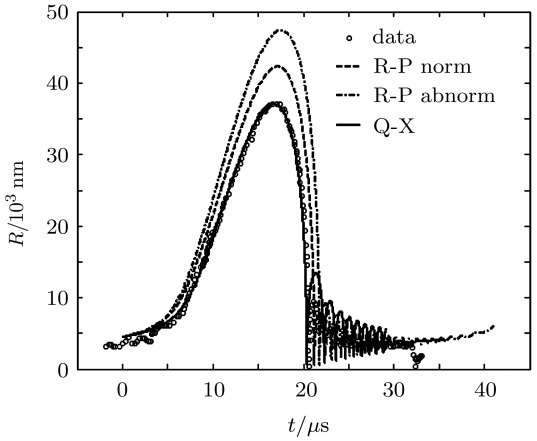


Fig.1. The history of an air bubble vibration in water, for frequency 26.5 kHz, sound pressure $p_\mathrm{a}=1.325\,\mathrm{atm.}$, $R_0=4.5\,\mu\mathrm{m}$, $c_0=1481\,\mathrm{m/s}$, $\gamma=1.4$; symbols (circles): Mie scattering data (taken from the Fig.3(a) in Ref.[2]); solid line: from Q-X equation, for total viscosity $b=0.004\,\mathrm{Pa\cdot s}$, surface tension coefficient $\sigma=0.072\,\mathrm{N/m}$; dashed line: from R-P equation, for shear viscosity $\eta=0.007\,\mathrm{Pa\cdot s}$, $\sigma=0\,\mathrm{N/m}$; dotted line: from R-P equation, for $\eta=0.001\,\mathrm{Pa\cdot s}$, $\sigma=0.072\,\mathrm{N/m}$.

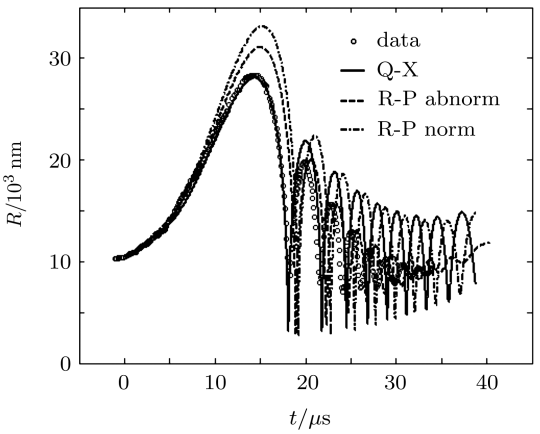


Fig.2. The history of an air bubble vibration in water, for frequency 26.5 kHz, sound pressure $p_\mathrm{a}=1.075\,\mathrm{atm.}$, $R_0=10.5\,\mu\mathrm{m}$, $c_0=1481\,\mathrm{m/s}$, $\gamma=1.4$; symbols (circles): Mie scattering data (taken from Fig.3(b) in Ref.[2]); solid line: from Q-X equation, for total viscosity $b=0.004\,\mathrm{Pa\cdot s}$, surface tension coefficient $\sigma=0.072\,\mathrm{N/m}$; dashed line: from R-P equation, for shear viscosity $\eta=0.007\,\mathrm{Pa\cdot s}$, $\sigma=0\,\mathrm{N/m}$; dotted line: from R-P equation, for $\eta=0.01\,\mathrm{Pa\cdot s}$, $\sigma=0.072\,\mathrm{N/m}$.

are the Mie scattering experimental data (taken from Ref.[2]). The Q-X results are in excellent agreement with the Mie scattering data except those in the con-

tract phase of the bubble (see Section 4). Figure 3(a) shows the results $f = 26.5\,\mathrm{kHz}$, $p_\mathrm{a} = 1.35\,\mathrm{atm.}$, $R_0 = 4.5\,\mu\mathrm{m}$, $c_0 = 1481\,\mathrm{m/s}$, where the solid line indicates the Q-X result with $b = 0.004\,\mathrm{Pa\cdot s}$, $\sigma = 0.072\,\mathrm{N/s}$; the dashed line indicates the R-P result with $\eta = 0.003\,\mathrm{Pa\cdot s}$ (enlarged 3 times),[3] $\sigma = 0.03\,\mathrm{N/s}$; and the dotted line is the R-P result with $\eta = 0.001\,\mathrm{Pa\cdot s}$, $\sigma = 0.072\,\mathrm{N/s}$ (both of them are the normal values), the symbols (dots) represent the Mie scattering data (taken from Ref.[3]). It is worthy to notice that the theoretical result by Q-X equation is closest to the experimental data and the result (dashed line with $\eta = 0.003\,\mathrm{Pa\cdot s}$ and $\sigma = 0.03\,\mathrm{N/s}$) is different from that (Fig.1 in Ref.[3]) given in Ref.[3]. It is worth noticing that the numerical results obtained by Q-X equation are still lower than the Mie scattering data, where the values p_a and R_0 in Ref.[3] have been continued to be used. However, they are the fitting parameters, by which an optimal fitness between the solution of R-P equation and the data could be achieved only by varying the surface tension and viscosity abnormally. On the other hand, if the Q-X numerical solutions with the normal values of the surface tension and viscosity are used to fit the data, the fitness should be appropriate. Figure 3(b) shows the results for a given size of bubble $R_0 = 4.5\,\mu\mathrm{m}$, p_a should be $1.4\,\mathrm{atm.}$ in order to fit the numerical solution of Eq.(3) with the data. Figure 4(a) shows the vibration of a

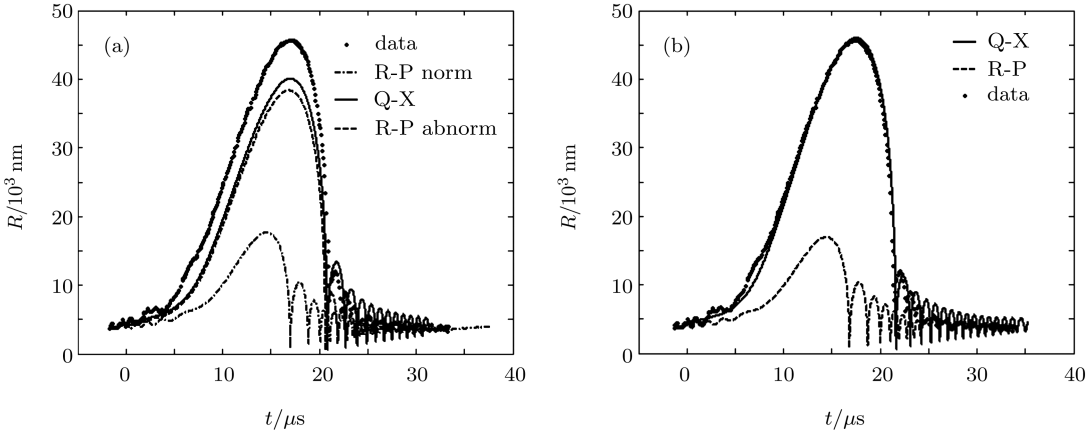


Fig.3. (a) The history of an air bubble vibration in water, for frequency $26.5\,\mathrm{kHz}$, sound pressure $p_\mathrm{a} = 1.35\,\mathrm{atm.}$, $R_0 = 4.5\,\mu\mathrm{m}$, $c_0 = 1481\,\mathrm{m/s}$, $\gamma = 1.4$; symbols (circles): Mie scattering data (taken from Fig.1 in Ref.[3]); solid line: from Q-X equation, for total viscosity $b = 0.004\,\mathrm{Pa\cdot s}$, surface tension coefficient $\sigma = 0.072$; dashed line: from R-P equation, for shear viscosity $\eta = 0.003\,\mathrm{Pa\cdot s}$, $\sigma = 0.03\,\mathrm{N/m}$; dotted line: from R-P equation, for $\eta = 0.001\,\mathrm{Pa\cdot s}$, $\sigma = 0.072\,\mathrm{N/m}$. (b) The history of an air bubble vibration in water, for frequency $26.5\,\mathrm{kHz}$, sound pressure $p_\mathrm{a} = 1.4\,\mathrm{atm.}$, $R_0 = 4.5\,\mu\mathrm{m}$, $c_0 = 1481\,\mathrm{m/s}$, $\gamma = 1.4$; symbols (circles): Mie scattering data (taken from the Fig.1 in Ref.[3]); solid line: from Q-X equation, for total viscosity $b = 0.004\,\mathrm{Pa\cdot s}$, surface tension coefficient $\sigma = 0.072\,\mathrm{N/m}$; dashed line: from R-P equation, for shear viscosity $\eta = 0.003\,\mathrm{Pa\cdot s}$, $\sigma = 0.03\,\mathrm{N/m}$; dotted line: from R-P equation, for $\eta = 0.001\,\mathrm{Pa\cdot s}$, $\sigma = 0.072\,\mathrm{N/m}$.

deuterium bubble, with a frequency of $32.5\,\mathrm{kHz}$, sound pressure $p_\mathrm{a} = 1.4\,\mathrm{atm.}$, $R_0 = 3\,\mu\mathrm{m}$, $c_0 = 1481\,\mathrm{m/s}$, and $\gamma = 1.4$. In this figure symbols (circle) denote Mie scattering data (taken from Fig.3(a) in Ref.[29]); solid line denotes the result obtained from Q-X equation with the total viscosity $b = 0.004\,\mathrm{Pa\cdot s}$, surface tension coefficient $\sigma = 0.072\,\mathrm{N/s}$; dashed line (R-P abnorm) the result obtained from R-P equation with the shear viscosity $\eta = 0.003\,\mathrm{Pa\cdot s}$, $\sigma = 0.03\,\mathrm{N/s}$; and dotted line the result obtained from R-P equation with $\eta = 0.001\,\mathrm{Pa\cdot s}$ and $\sigma = 0.072\,\mathrm{N/s}$. In order to fit the data normally with the Q-X numerical solutions, Fig.4(b) shows similar results to those in Fig.4(a) only with $p_\mathrm{a} = 1.45\,\mathrm{atm.}$ instead of $p_\mathrm{a} = 1.4\,\mathrm{atm.}$ Figure 5 shows the similar results for ethane bubble in water.

In order to inspect the dependence of the vibration of the bubble upon the shear viscosity and surface tension, R-P equation was solved numerically for various shear viscosities and surface tensions. Figure 6 demonstrates the relationship between the vibration of a deuterium bubble and the coefficients of surface tension ($\sigma = 0.0$, 0.01, 0.03, 0.05, $0.072\,\mathrm{N/m}$), where the coefficient of shear viscosity $\eta = 0.001\,\mathrm{Pa\cdot s}$, ambient radius $R_0 = 3.0\ \mu\mathrm{m}$, frequency $f = 32.5\,\mathrm{kHz}$, $p_\mathrm{a} = 1.4\,\mathrm{atm.}$ and $\gamma = 1.4$. Figure 7 shows the relationship between the vibration of a deuterium bubble

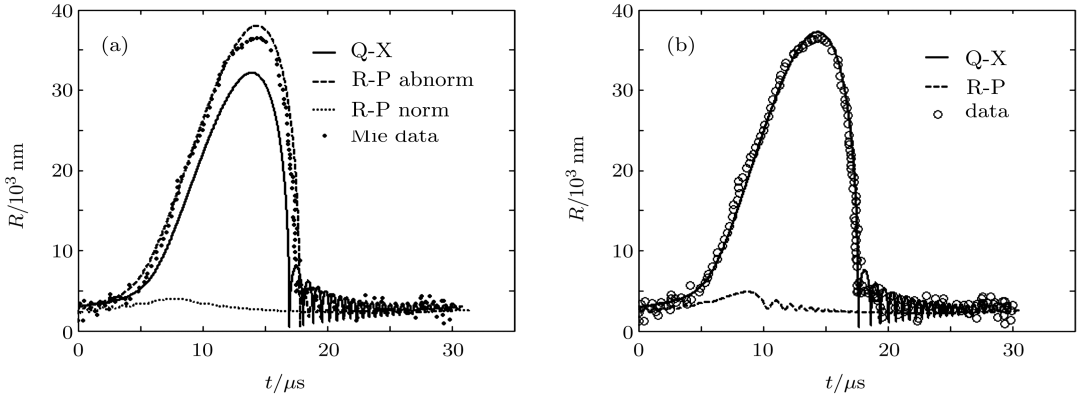


Fig.4. (a) The history of a deuterium bubble vibration in water, for frequency 32.5 kHz, sound pressure $p_a = 1.4$ atm., $R_0 = 3\,\mu$m, $c_0 = 1481$ m/s, $\gamma = 1.4$; symbols (circles): Mie scattering data (taken from Fig.3(a) in Ref.[29]); solid line: from Q-X equation, for total viscosity $b = 0.004\,\mathrm{Pa\cdot s}$, surface tension coefficient $\sigma = 0.072\,\mathrm{N/m}$; dashed line: from R-P equation, for shear viscosity $\eta = 0.003\,\mathrm{Pa\cdot s}$, $\sigma = 0.03\,\mathrm{N/m}$; dotted line: from R-P equation, for $\eta = 0.01\,\mathrm{g/(cm\ s)}$, $\sigma = 0.072\,\mathrm{N/m}$. (b) The history of a deuterium bubble vibration in water, for frequency 32.5 kHz, sound pressure $p_a = 1.45$ atm., $R_0 = 3\,\mu$m, $c_0 = 1481$ m/s, $\gamma = 1.4$; symbols (circles): Mie scattering data (taken from Fig.3(a) in Ref.[29]); solid line: from Q-X equation, for total viscosity $b = 0.004\,\mathrm{Pa\cdot s}$, surface tension coefficient $\sigma = 0.072\,\mathrm{N/m}$; dashed line: from R-P equation, for $\eta = 0.001\,\mathrm{Pa\cdot s}$, $\sigma = 0.072\,\mathrm{N/m}$.

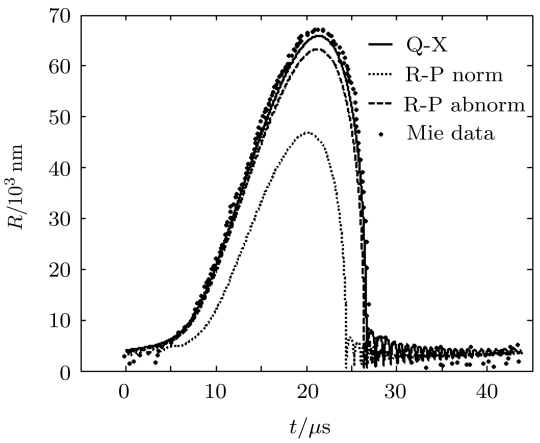


Fig.5. The history of an ethane bubble vibration in water, for frequency 23 kHz, sound pressure $p_a = 1.5$ atm., $R_0 = 4\,\mu$m, $c_0 = 1481$ m/s, $\gamma = 1.2$; symbols (circles): Mie scattering data (taken from Fig.6(a) in Ref.[30]); solid line: from Q-X equation, for total viscosity $b = 0.004\,\mathrm{Pa\cdot s}$, surface tension coefficient $\sigma = 0.072\,\mathrm{N/m}$; dashed line: from R-P equation, for shear viscosity $\eta = 0.003\,\mathrm{Pa\cdot s}$, $\sigma = 30$ dyne/cm; dotted line: from R-P equation, for $\eta = 0.001\,\mathrm{Pa\cdot s}$, $\sigma = 0.072\,\mathrm{N/m}$.

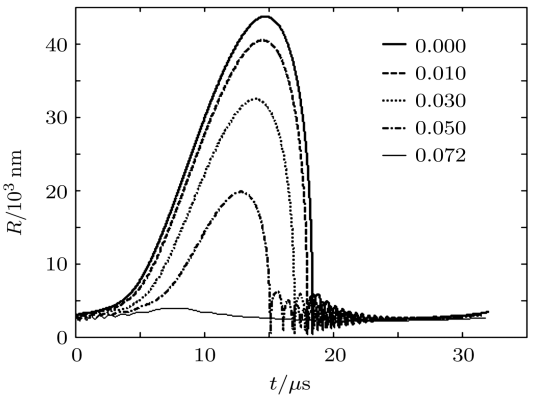


Fig.6. Vibration of a deuterium bubble for various coefficients of surface tension: $\sigma = 0.0$, 0.01, 0.03, 0.05, 0.072 N/m. Coefficient of shear viscosity $\eta = 0.001\,\mathrm{Pa\cdot s}$; ambient radius $R_0 = 3.0\,\mu$m; frequency $f = 32.5$ kHz; $p_a = 1.4$ atm.; $\gamma = 1.4$.

and the shear viscosity coefficients ($\eta = 0.001$, 0.003, 0.004, 0.005, 0.007 Pa·s), where the coefficient of surface tension $\sigma = 0.05\,\mathrm{N/m}$, ambient radius $R_0 = 3.0\,\mu m$, frequency $f = 32.5$ kHz, $p_a = 1.4$ atm. and $\gamma = 1.4$. The results from Figs.6 and 7 show that both the surface tension and shear viscosity have great influence upon the bubble's vibration; for example, the amplitude of the bubble's vibration will increase more than twice if the surface tension is neglected.

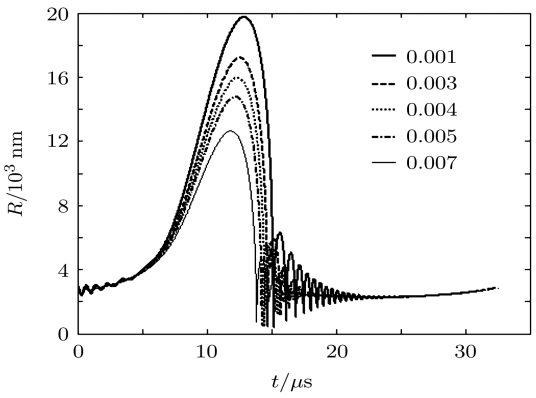


Fig.7. Vibration of a deuterium bubble for various shear viscosity coefficients: $\eta = 0.001$, 0.03, 0.004, 0.05, 0.07 Pa·s. Coefficient of surface tension $\sigma = 0.05\,\mathrm{N/m}$; ambient radius $R_0 = 3.0\,\mu$m; frequency $f = 32.5$ kHz; $p_a = 1.4$ atm.; $\gamma = 1.4$.

4. Discussions

As is already known, R-P equation is only valid for Mach number M much less than 1. Unfortunately, the Mach number of liquid near the surface of the bubble is close to, even larger than, 1 when the bubble contracts to its minimum radius. Moreover, the numerical solutions for Eqs.(1) (R-P equation) and (3) demonstrate that the history of a bubble depends upon surface tension coefficient and viscosity (see Figs.6 and 7) as well as the driving pressure (see Figs.3(a), 3(b), 4(a), 4(b)), so an arbitrary variation of them (particularly, for degas-distilled water) to match the numerical results with the data will violate the physical circumstance; furthermore, the uniqueness of the numerical solutions may even be suspected. However, Eq.(3), where the Tait state equation was used to exactly describe the function h as well as the sound speed, can be used to the situation for $M > 1$, so that its numerical solutions agree with the experimental data very well without necessarily varying surface tension coefficient and viscosity (see Figs.1–5). Thus, the present authors suggest that using the bubble's vibration to investigate the state equations of aqueous solutions seem to be possible. They also believe that if one uses this equation instead of R-P equation to deal with the relevant problems such as the 'phase diagrams for sonoluminescing bubbles', etc., some different results may be obtained.

From Figs.1–3, it can be seen that in bubble's contract phase the calculated instantaneous radius of the bubble is larger than the Mie scattering data. A possible explanation can be stated as follows. During the collapse of a bubble, the temperature inside the bubble becomes so high that the transport process, the heat radiation, even the light emission occur inevitably, which will increase its effective damping and decrease its bounces, in other words, a reasonable result should be that the theoretical bounces of both the R-P and Q-X numerical solutions are always larger than the experimental ones. Thus, the effective damping generated by the heat process and light emission should be taken into account.

5. Conclusions

The numerical results demonstrate that the equation describing the finite-amplitude vibration of a bubble revised by the present authors (Q-X) yields best results to match the experimental data. It is worth noticing that a best fit of the numerical solutions of Q-X equation with the experimental data is in the bubble's expansion phase rather than in contract phase. If the bubble's vibration accompanies an energy loss such as the light radiation during the contract phase, the mechanism of the energy loss has to be taken into account.[4−6]

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