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Photon–photon interactions with inner coupled double-cavity *

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This paper describes the interaction between two spatial modes of the optical fields with a single atom trapped inner coupled double-cavity. Theoretical derivation and numerical simulation with the experimental available parameters show that photon–photon switching and π phase shift of single photons may be achieved with current experimental technology. As the probe and control fields are in different spatial modes, the system is superior for implementing cavity QED-based photonic quantum networks.

Keywords: interaction, photon–photon switching, phase shift, inner coupled double-cavity

PACC: 4250, 3280, 0365

1. Introduction

The interaction of photons has attracted great attention due to its application in optical quantum information processing.[1−10] Since the strength of direct interaction between photons is extremely weak, some assistant systems such as atom, cavity or other optical devices are always applied to act as medium for achieving an effective interaction between photons. Up to now, there are mainly two ways to induce an effective interaction between photons. One way is the linear quantum optics approach.[10,11] However, as the optical quantum gate constructed with this technique is essentially probabilistic, it is difficult to realize scalable quantum computation with high probability of success.[10] Furthermore, in order to increase the probability of success of the quantum gates the sources of large entanglement chains are needed[12−14] and special error correction is also necessary for the probabilistic gates.[10] Another way to achieve effective interaction of photons is the Kerr nonlinearity approach.[1−8,15] Based on this technique, deterministic CNOT gate can be created with photonic qubits.[16]Recent researches on the nonlinearities of photons focus on the cavity QED schemes and these works predicted large phase shift[2,5−7] and single-photon switching.[8] The nonlinearities of photons with cavity QED system are observed in the experiments.[17,18] However, the previous work merely considered one dimensional cavity-emitter system. With one dimensional treatment, both the probe field and the control field propagate in the same spatial mode when they reflect and transmit through one of the two cavity mirrors. Thus to deal with the probe field and the pump field at the same time in experiment, one has to combine or split the photon current with beam splitter,[17] and it causes photon loss when these photons reflect and transmit on the beam splitter. So it is a significant obstacle for implementing scalable quantum networks with photonic qubits.

In this paper, we introduce inner coupled double-cavity (ICD-cavity) which consists of two perpendicularly inner coupled Fabry–Perot cavities (see Fig.1). On one side, photon–photon interaction in our scheme is based on the Kerr nonlinearity approach,[5,7,15] therefore the quantum logic gate constructed with this system can be determinant;[16] On the other side, analogous to the case that optical fields propagate through a beam splitter in linear quantum optics, probe field and control field propagate in two spatial modes of different directions in our system. Different spatial modes of the optical fields are extremely important for realization of photonic quantum algorithm[19] and quantum networks.[20,21]

This paper is organized as follows. We first present the structure of our optical system and the

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Hamiltonian of this system in Section 2. In Section 3, equations for the motion of the system are derived and the solutions for the equations are given. Using the solutions we analyse the transmission properties of the probe field and the control field. In Section 4, the phase variation of the probe field is described. In Section 5, we compare the number of photons in the cavity with the number of photons in the input output fields. In Section 6, we discuss the feasibility and the application of the present optical system. Finally, the paper is summarized in Section 7.

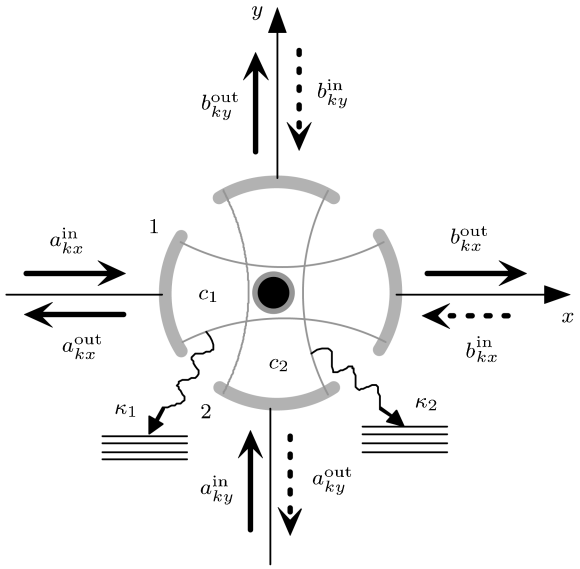


Fig.1. The ICD-cavity is made up of two one-dimensional cavities which are perpendicularly inner coupled and named cavity 1 and cavity 2, respectively. A single atom is trapped in the centre of the ICD-cavity. Both cavity 1 and cavity 2 are two sided and interact with the input output fields. κ_1, κ_2 are the intrinsic decay rates corresponding to the cavity 1 and the cavity 2.

2. The optical system and the Hamiltonian

The two cavities of the ICD-cavity are named as cavity 1 and cavity 2 respectively with corresponding central frequencies ν_1 and ν_2. The probe field is input from one side of the cavity 1 and reflected or transmitted by the cavity 1 ; the control field is input from one side of the cavity 2 and reflected or transmitted by the cavity 2 (see Fig.1). Each cavity holds a single mode. The probe field is resonant with the cavity 1, its central frequency is ν_{kx}; the control field is resonant with the cavity 2, its central frequency is ν_{ky}. An ordinary three-level atom (see Fig.2) is trapped at the centre of the ICD-cavity. The cavity 1 is resonantly coupled to the transition $|1\rangle - |2\rangle$ of the atom and the cavity 2 is coupled to the transition $|2\rangle - |3\rangle$ with a detuning Δ.

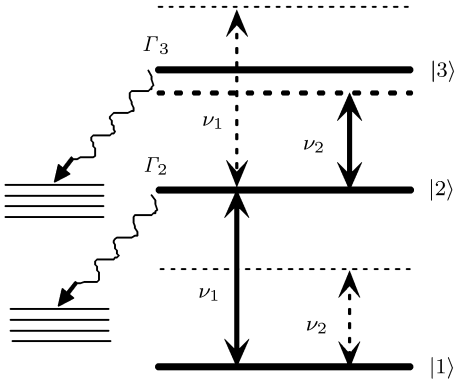


Fig.2. An ordinary three-level atom coupled to two cavities and the reservoir is illustrated. The transition $|1\rangle \leftrightarrow |2\rangle$ is coupled to the Cavity 1. The centre frequency ν_1 of the cavity 1 is far off-resonant with the transition $|2\rangle \leftrightarrow |3\rangle$. The transition $|2\rangle \leftrightarrow |3\rangle$ is coupled to the cavity 2 with a detuning $\Delta = \omega_{32} - \nu_2$. The cavity 2 is also far off-resonant with the transition $|1\rangle \leftrightarrow |2\rangle$. The decay rates of the 2nd and 3rd levels of the atom are Γ_2 and Γ_3, respectively.

We describe the dynamics of the opened atom–cavity system in Heisenberg picture by using input output field theory.[22] The Hamiltonian for the system is

$$H = H_0 + H_1, \tag{1}$$

$$\begin{aligned} H_0 =& \sum_{i=1}^{3} E_i \sigma_{ii} + \sum_{j=1}^{2} \hbar\nu_j c_j^\dagger c_j \\ &+ \int_{-\infty}^{\infty} \hbar\nu a_{kx}^\dagger(\nu) a_{kx}(\nu) \mathrm{d}\nu \\ &+ \int_{-\infty}^{\infty} \hbar\nu b_{kx}^\dagger(\nu) b_{kx}(\nu) \mathrm{d}\nu \\ &+ \int_{-\infty}^{\infty} \hbar\nu a_{ky}^\dagger(\nu) a_{ky}(\nu) \mathrm{d}\nu \\ &+ \int_{-\infty}^{\infty} \hbar\nu b_{ky}^\dagger(\nu) b_{ky}(\nu) \mathrm{d}\nu, \end{aligned}$$

$$\begin{aligned} H_1 =& \hbar g_{12}(c_1^\dagger \sigma_{12} + \sigma_{21} c_1) + \hbar g_{23}(c_2^\dagger \sigma_{23} + \sigma_{32} c_2) \\ &+ \mathrm{i} \int_{-\infty}^{\infty} \hbar N_1(\nu)[(a_{kx}^\dagger(\nu) + b_{kx}^\dagger(\nu)) c_1 \\ &- c_1^\dagger (a_{kx}(\nu) + b_{kx}(\nu))] \mathrm{d}\nu \\ &+ \mathrm{i} \int_{-\infty}^{\infty} \hbar N_2(\nu)[(a_{ky}^\dagger(\nu) + b_{ky}^\dagger(\nu)) c_2 \\ &- c_2^\dagger (a_{ky}(\nu) + b_{ky}(\nu))] \mathrm{d}\nu, \end{aligned}$$

where H_0 is the free evolution energy of the atom, the cavities and the input output fields. H_1 is the interaction energy. In the Hamiltonian, E_i represents eigenvalue of the atomic energy level i and $\sigma_{ij} = |i\rangle\langle j|$ is the atom transition operator. c_j is the annihilation operator of the cavity mode in the cavity j. The probe fields are described by the annihilation operators $a_{kx}(\nu)$ and $b_{kx}(\nu)$, and the control fields are described by the annihilation operators $a_{ky}(\nu)$ and $b_{ky}(\nu)$ (see Fig.1). These operators satisfy the commutation relations $[a_{kq}(\nu), a^{\dagger}_{kq'}(\nu')] = \delta_{qq'}\delta(\nu-\nu')$ and $[b_{kq}(\nu),\ b^{\dagger}_{kq'}(\nu')] = \delta_{qq'}\delta(\nu-\nu')$. Here $q, q' = x, y$ indicate spatial modes of the fields. The wave numbers k_x, k_y satisfy the relations $k_x = \nu_{kx}/c$ and $k_y = \nu_{ky}/c$ respectively, where c is the velocity of light in vacuum space. $N_1(\nu)$ and $N_2(\nu)$ denote coupling coefficients between the cavity modes and input output fields.

3. Cavity QED-based electromagnetic induced transparency

In this section, we present cavity QED-based electromagnetic induced transparency with ICD-cavity. When the strength of the control field is zero, the resonant coupling between the cavity 1 and the atom transition $|1\rangle - |2\rangle$ leads the probe field to reflect on the cavity 1.[5,23] When the power of the control field is increased to be strong enough, it drives the cavity 2 and causes energy shift of the atomic level E_2, and makes the atom detuning from the cavity 1. Then the probe field which is resonant with the cavity 1 would be transmitted.[24,25] The detailed descriptions are as follows.

According to the Heisenberg equation $\dfrac{\mathrm{d}O}{\mathrm{d}t} = \dfrac{1}{\mathrm{i}\hbar}[O, H]$ (O is any operator), the equations of motion for the considered system can be expressed as follows:

$$\frac{\mathrm{d}\sigma_{12}}{\mathrm{d}t} = -\left(\mathrm{i}\omega_{21} + \frac{\Gamma_2}{2}\right)\sigma_{12} + \mathrm{i}g_{12}\sigma^z_{21}c_1 - \mathrm{i}g_{23}c^{\dagger}_2\sigma_{12}\sigma_{23}, \tag{2}$$

$$\frac{\mathrm{d}\sigma_{23}}{\mathrm{d}t} = -\left(\mathrm{i}\omega_{32} + \frac{\Gamma_3}{2}\right)\sigma_{23} + \mathrm{i}g_{12}c^{\dagger}_1\sigma_{12}\sigma_{23} + \mathrm{i}g_{23}\sigma^z_{32}c_2, \tag{3}$$

$$\frac{\mathrm{d}c_1}{\mathrm{d}t} = -\left(\mathrm{i}\nu_1 + \frac{\kappa_1}{2} + \gamma_1\right)c_1 - \mathrm{i}g_{12}\sigma_{12} - \sqrt{\gamma_1}(a^{\mathrm{in}}_{kx} + b^{\mathrm{in}}_{kx}), \tag{4}$$

$$\frac{\mathrm{d}c_2}{\mathrm{d}t} = -\left(\mathrm{i}\nu_2 + \frac{\kappa_2}{2} + \gamma_2\right)c_2 - \mathrm{i}g_{23}\sigma_{23} - \sqrt{\gamma_2}(a^{\mathrm{in}}_{ky} + b^{\mathrm{in}}_{ky}), \tag{5}$$

where $\omega_{32} = (E_3 - E_2)/\hbar$ and $\omega_{21} = (E_2 - E_1)/\hbar$ are the atom transition frequencies. The decay rates of the atom, Γ_2, Γ_3, and the cavities, κ_1, κ_2, are phenomenologically introduced in the equations. $\sigma^z_{32} = \sigma_{33} - \sigma_{22}$ and $\sigma^z_{21} = \sigma_{22} - \sigma_{11}$ are the inversion operators of the atom. The first Markov approximation[22] is considered in the equations as

$$N_j(\nu) = \sqrt{\gamma_j/2\pi}, \quad j = 1, 2. \tag{6}$$

Here γ_j is the radiation decay rate of the cavity j, which is associated with the radiation quality factor of the cavity. While κ_j is the intrinsic decay rate of the cavity j, which is related to the intrinsic quality factor of the cavity describing the field losses by mirror absorption and scattering due to the mirror imperfections.[26]

In operators and out operators of the input output fields are defined by

$$\begin{aligned}
a^{\mathrm{in}}_{kq}(t) &= \sqrt{1/2\pi}\int_{-\infty}^{\infty} a_{kq}(\nu, t_0)\mathrm{e}^{-\mathrm{i}\nu(t-t_0)}\mathrm{d}\nu,\\
a^{\mathrm{out}}_{kq}(t) &= \sqrt{1/2\pi}\int_{-\infty}^{\infty} a_{kq}(\nu, t_1)\mathrm{e}^{-\mathrm{i}\nu(t-t_1)}\mathrm{d}\nu,\\
b^{\mathrm{in}}_{kq}(t) &= \sqrt{1/2\pi}\int_{-\infty}^{\infty} b_{kq}(\nu, t_0)\mathrm{e}^{-\mathrm{i}\nu(t-t_0)}\mathrm{d}\nu,\\
b^{\mathrm{out}}_{kq}(t) &= \sqrt{1/2\pi}\int_{-\infty}^{\infty} b_{kq}(\nu, t_1)\mathrm{e}^{-\mathrm{i}\nu(t-t_1)}\mathrm{d}\nu,
\end{aligned} \tag{7}$$

with commutation relations

$$\begin{aligned}
&[a^{\mathrm{in}}_{kq}(t), a^{\mathrm{in}\dagger}_{kq'}(t')] = \delta_{qq'}\delta(t-t'),\\
&[a^{\mathrm{out}}_{kq}(t), a^{\mathrm{out}\dagger}_{kq'}(t')] = \delta_{qq'}\delta(t-t'),\\
&[b^{\mathrm{in}}_{kq}(t), b^{\mathrm{in}\dagger}_{kq'}(t')] = \delta_{qq'}\delta(t-t'),\\
&[b^{\mathrm{out}}_{kq}(t), b^{\mathrm{out}\dagger}_{kq'}(t')] = \delta_{qq'}\delta(t-t'),\\
&q, q' = x, y.
\end{aligned} \tag{8}$$

In operators are related to out operators by the boundary conditions of the cavity 1 and the cavity 2 as

$$\begin{aligned}
a^{\mathrm{out}}_{kx}(t) - a^{\mathrm{in}}_{kx}(t) &= \sqrt{\gamma_1}c_1(t),\\
b^{\mathrm{out}}_{kx}(t) - b^{\mathrm{in}}_{kx}(t) &= \sqrt{\gamma_1}c_1(t),\\
a^{\mathrm{out}}_{ky}(t) - a^{\mathrm{in}}_{ky}(t) &= \sqrt{\gamma_2}c_2(t),\\
b^{\mathrm{out}}_{ky}(t) - b^{\mathrm{in}}_{ky}(t) &= \sqrt{\gamma_2}c_2(t).
\end{aligned} \tag{9}$$

The above formulas (1)–(9) are valid for the optical fields with narrow band of frequencies.[24]

To obtain the solutions of Eqs.(2–5), a few approximations are made. Initially, the atom is supposed to be in its ground state and both the cavity 1 and the cavity 2 are in the vacuum states. Thus, the atom and the cavities are predominantly driven by the input fields. Therefore, the operators for the atom and the cavities can be separated into two components by

$$\begin{aligned}\sigma_{12}(t) &= \tilde{\sigma}_{12}(t)\mathrm{e}^{-\mathrm{i}\nu_{kx}t},\\ \sigma_{23}(t) &= \tilde{\sigma}_{23}(t)\mathrm{e}^{-\mathrm{i}\nu_{ky}t},\\ c_1(t) &= \tilde{c}_1(t)\mathrm{e}^{-\mathrm{i}\nu_{kx}t},\\ c_2(t) &= \tilde{c}_2(t)\mathrm{e}^{-\mathrm{i}\nu_{ky}t}.\end{aligned} \tag{10}$$

The slow time variable terms $\tilde{\sigma}_{12}(t)$, $\tilde{\sigma}_{23}(t)$, $\tilde{c}_1(t)$ and $\tilde{c}_2(t)$ are regarded as constant, which is an usual approximation in quantum optics.[27−29] Then from Eqs.(3) and (10), we have

$$\sigma_{23}(t) = \frac{\mathrm{i}g_{12}c_1^{\dagger}\sigma_{12}\sigma_{23} + \mathrm{i}g_{23}\sigma_{23}^{z}c_2}{\mathrm{i}\Delta + \Gamma_3/2}, \tag{11}$$

where $\Delta = \omega_{32} - \nu_{ky}$ and $\nu_{ky} = \nu_2$. Defining the photon number in the cavity 2 by $n_2 = c_2^{\dagger}c_2$, from Eqs.(2), (10) and (11) we obtain

$$\sigma_{12}(t) = \frac{\mathrm{i}g_{12}\sigma_{21}^{z}c_1}{g_{23}^2\bar{n}_2/(\mathrm{i}\Delta + \Gamma_3/2) + \Gamma_2/2}, \tag{12}$$

where $\bar{n}_2 = \langle c_2^{\dagger}c_2\rangle$.

Analogous to the case of electromagnetic induced transparency,[15,30] when an atom is initially in its ground state and the exciting field is weak enough, the atom would be predominantly in its ground state during the whole interacting process, which is called weak excitation limit. Considering this approximation we have

$$\langle\sigma_{21}^{z}(t)\rangle \approx -1. \tag{13}$$

From Eqs.(4), (9), (10), (12) and (13) we derive the relations of in operators and out operators for the probe fields

$$\begin{pmatrix} a_{kx}^{\mathrm{out}} \\ b_{kx}^{\mathrm{out}} \end{pmatrix} = \begin{pmatrix} S_{11}^1 & S_{12}^1 \\ S_{21}^1 & S_{22}^1 \end{pmatrix}\begin{pmatrix} a_{kx}^{\mathrm{in}} \\ b_{kx}^{\mathrm{in}} \end{pmatrix}, \tag{14}$$

where the matrix elements are

$$\begin{aligned} S_{11}^1 = S_{22}^1 &= \frac{\kappa_1/2 + \delta}{\kappa_1/2 + \gamma_1 + \delta},\\ S_{12}^1 = S_{21}^1 &= \frac{-\gamma_1}{\kappa_1/2 + \gamma_1 + \delta}\end{aligned}$$

and

$$\delta = \frac{g_{12}^2}{g_{23}^2\bar{n}_2/(\mathrm{i}\Delta + \Gamma_3/2) + \Gamma_2/2}. \tag{15}$$

We assume that the probe field is input from just one side of the cavity 1, giving $b_{kx}^{\mathrm{in}} = 0$ in Eq.(14). The input probe field a_{kx}^{in} reflects into the mode a_{kx}^{out} or transmits into the mode b_{kx}^{out} (see Fig.1). The corresponding reflection probability P_{R} and the transmission probability P_{T} are determined by the matrix elements as

$$P_{\mathrm{R}} = |S_{11}^1|^2, \quad P_{\mathrm{T}} = |S_{21}^1|^2. \tag{16}$$

It can be easily demonstrated that $P_{\mathrm{R}} + P_{\mathrm{T}} \approx 1$, but the sum is a little less than 1 due to the energy loss of the field. Equations (14) and (15) imply that the propagation of the probe field is related to the mean number $\bar{n}_2$ of photons in the cavity 2.

In Fig.3, based on Eq.(15) the reflection probability P_{R} and the transmission probability P_{T} of the probe field are plotted versus mean number $\bar{n}_2$ of the photons in the cavity 2. Figure 3 shows that when there is no photon in the cavity 2, the probe field in mode k_x is fully reflected, and when only about 1 photon presents in the cavity 2, the probe field is almost fully transmitted through the cavity 1. The probabilities are not exactly come to 1 due to the presence of the atom decay and the cavity decay. Figure 4 shows that when the atom decay and the cavity decay are negligible the probe field can be absolutely reflected or transmitted by the field in the cavity 2. In other words, the cavity 1 becomes transparent when the control field transmits into the cavity 2 and interacts with the atom. The basic parameters of the atom–cavity system $(\Omega_1, \Gamma, \kappa)/2\pi = (220, 2.6, 14.2)$ MHz are reported in Ref.[31]. Here Ω_1 is one-photon Rabi frequency which is two times of the atom cavity coupling strength g, Γ is the atom decay rate and κ is the cavity intrinsic decay rate. In our system, the atom has decay rates $\Gamma_2 = \Gamma$ and $\Gamma_3 = 2\Gamma$ for the second and third levels, respectively. The cavity 1 and the cavity 2 are supposed to be coupled to the atom with the same coupling strength, i.e. $g_{12} = g_{23} = g$. The intrinsic decay rates of the cavity 1 and the cavity 2 are also set to be equal, namely $\kappa_1 = \kappa_2 = \kappa$. Besides, the radiation decay rates of the cavity 1 and the cavity 2 are taken to be $\gamma_1 = \gamma_2 = 120\Gamma$ which is in the bad cavity limit,[23,29] referring to Ref.[32] we set the detuning to be $\Delta = 4\Gamma_2$.

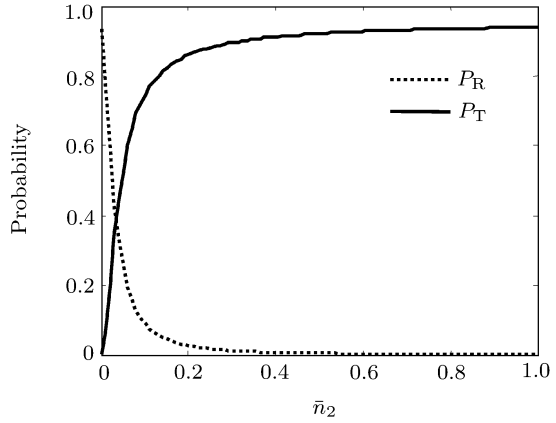


Fig.3. The reflection probability and the transmission probability of the probe fields. When the mean number $\bar{n}_2$ of photons in the cavity 2 is 0, the input probe field is reflected with a probability near to 1. When $\bar{n}_2$ approaches 1, the input probe field is transmitted at about the same probability. The corresponding parameters are $g_{12} = g_{23} = 110\,\mathrm{MHz}$, $\kappa_1 = \kappa_2 = 14.2\,\mathrm{MHz}$, $\Gamma_2 = \Gamma_3/2 = 2.6\,\mathrm{MHz}$, $\gamma_1 = \gamma_2 = 120\Gamma_2$ and $\Delta = 4\Gamma_2$.

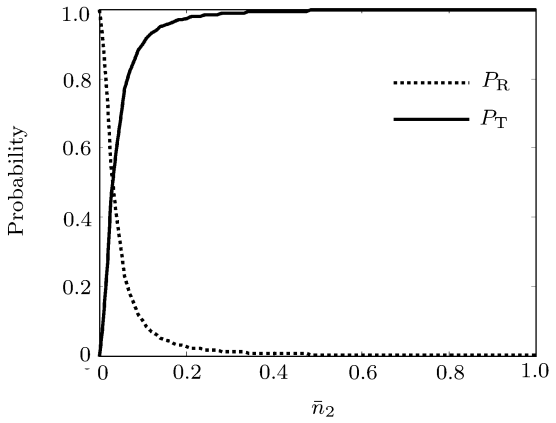


Fig.4. Noise of the atom and the cavity is neglected in this figure. The probability of the reflection and the transmission of the probe field is exactly come to 1 when the mean number of photons in the cavity 2 is 0 and 1, respectively. The parameters are $g_{12} = g_{23} = 110\,\mathrm{MHz}$, $\gamma_1 = \gamma_2 = 120\Gamma_2$, $\Delta = 4\Gamma_2$, $\kappa_1 = \kappa_2 = 0$ and $\Gamma_2 = \Gamma_3/2 = 0$.

Consequently, we derive the relations of in operators and out operators for the control field and show how the mean photon number of the cavity 2 is determined by the control field. With analogous approaches as above, combining Eqs.(2) and (3) we have

$$\sigma_{23}(t) \propto \sigma_{32}^{z}(t). \tag{17}$$

From Eq.(14) we know $\langle\sigma_{11}(t)\rangle \approx 1$, therefore the expectation value of $\sigma_{32}^{z}(t)$ is close to zero. Considering relation (17) we neglect the term containing the transition operator $\sigma_{23}(t)$ in Eq.(5). This approximation can be seen in the description of electromagnetic induced transparency.[30] Using Eqs.(9), (10) and the simplified Eq.(5), we obtain

$$\begin{pmatrix} a_{ky}^{\mathrm{out}} \\ b_{ky}^{\mathrm{out}} \end{pmatrix} = \begin{pmatrix} S_{11}^2 & S_{12}^2 \\ S_{21}^2 & S_{22}^2 \end{pmatrix} \begin{pmatrix} a_{ky}^{\mathrm{in}} \\ b_{ky}^{\mathrm{in}} \end{pmatrix}, \tag{18}$$

where the matrix elements are

$$\begin{aligned} S_{11}^2 = S_{22}^2 &= \frac{\kappa_2/2}{\kappa_2/2+\gamma_2}, \\ S_{12}^2 = S_{21}^2 &= \frac{-\gamma_2}{\kappa_2/2+\gamma_2}. \end{aligned} \tag{19}$$

With the matrix elements S_{11}^2, S_{12}^2, S_{21}^2 and S_{22}^2, the transmission and the reflection probabilities of the control fields can be computed. From Eq.(19) it is known that, for $\gamma_2 >> \kappa_2$, Eq.(18) becomes

$$\begin{pmatrix} a_{ky}^{\mathrm{out}} \\ b_{ky}^{\mathrm{out}} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} a_{ky}^{\mathrm{in}} \\ b_{ky}^{\mathrm{in}} \end{pmatrix}. \tag{20}$$

Analogous to the case of cavity 1, if the control field is input from one side of the cavity 2, i.e. $b_{ky}^{\mathrm{in}} = 0$, then from Eqs.(9) and (20), we have

$$\begin{aligned} n_2 &= (a_{ky}^{\mathrm{in}})^{\dagger} a_{ky}^{\mathrm{in}}/\gamma_2, \\ n_2 &= (b_{ky}^{\mathrm{out}})^{\dagger} b_{ky}^{\mathrm{out}}/\gamma_2. \end{aligned} \tag{21}$$

Equation (21) denotes the mean number of photons in the cavity 2 is proportional to the strength of the control field. Therefore, the reflection and the transmission of the probe field are remarkably affected by the control field.

4. Phase shift of the probe field

In Section 3, we discussed the amplitude of the optical fields which interact with the atom–cavity system. Now we turn our attention to the phase of the optical field. For this purpose we consider an alternative ICD-cavity in which the cavity 1 is just one sided. The input probe field a_{kx}^{in} is reflected by the cavity 1 and evolves into a_{kx}^{out} (see Fig.5).

For the ICD-cavity in which the cavity 1 is one sided, the equations of motion for the system (2), (3), and (5) are the same as the original. We only need to change the Eq. (4) as follows:

$$\frac{\mathrm{d}c_1}{\mathrm{d}t} = -\left(\mathrm{i}\nu_1 + \frac{\kappa_1}{2} + \frac{\gamma_1}{2}\right)c_1 - \mathrm{i}g_{12}\sigma_{12} - \sqrt{\gamma_1}a_{kx}^{\mathrm{in}}. \tag{22}$$

For the cavity 1 there is only one boundary condition, i.e. $a_{kx}^{\mathrm{out}}(t) - a_{kx}^{\mathrm{in}}(t) = \sqrt{\gamma_1}c_1(t)$, for the cavity 2 there

are still two boundary conditions. The solutions of the Eqs.(2), (3), (5) and (22) can be obtained by the analogous derivation in the Section 3. For the control field, the relations of in operators and out operators are not changed, also the same as Eq.(18). While, for the probe field it becomes

$$a_{kx}^{\text{out}} = (X + \mathrm{i}Y)a_{kx}^{\text{in}} = A\mathrm{e}^{\mathrm{i}\Phi}a_{kx}^{\text{in}}, \tag{23}$$

where $X + \mathrm{i}Y = \dfrac{\kappa_1 - \gamma_1 + 2\delta}{\kappa_1 + \gamma_1 + 2\delta}$, $A = \sqrt{X^2 + Y^2}$, $\Phi = \arccos\left(\dfrac{X}{\sqrt{X^2+Y^2}}\right)$, and δ is given in Eq.(15).

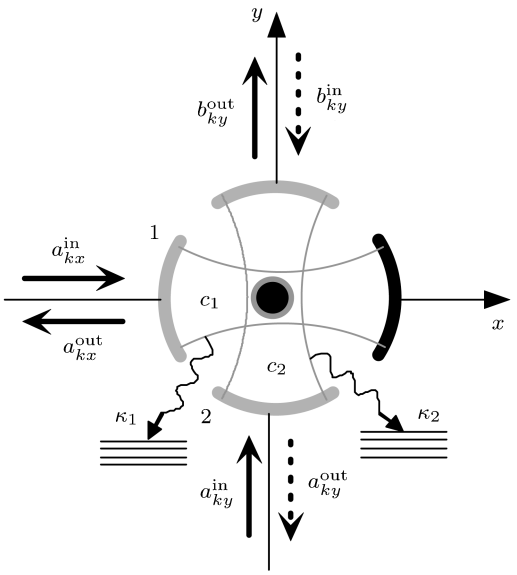


Fig.5. The ICD-cavity illustrated in this figure is the same as the system described in Fig.1 except the cavity 1 here is just one-sided.

In Fig.6, the reflection probability A^2 and the phase shift Φ of the probe field are plotted versus the mean number $\bar{n}_2$ of the photons in the cavity 2. When the photon number is zero in the cavity 2, the probe field is reflected at a high probability close to 90% without any phase shift. When the control field drives the cavity 2 and the mean number of the photons in the cavity 2 is about equal to 1, the probe field is reflected at about the same probability as above case but with a phase shift near π. The phase of the probe field can be continuously shifted from 0 to π. Analogous to the Fig.3, the phase shift of the field in Fig.6 does not exactly increase to π and the reflection probability is also not come to 1 due to the energy loss by the decay of the atom–cavity system. However, it is strange that when $\Delta = 0$, namely, the control field, the cavity 2 and the atom transition $|2\rangle \leftrightarrow |3\rangle$ are all resonant with each other, the phase of the probe field is sharply and absolutely shifted from 0 to π when the average photon number in the cavity 2 increases from 0.008 to 0.009 (see Fig.7). The radiation decay rates of the cavity 1 and the cavity 2 are set to be $\gamma_1 = \gamma_2 = 240\Gamma$ in Figs.6 and 7 which satisfy the bad cavity limit.

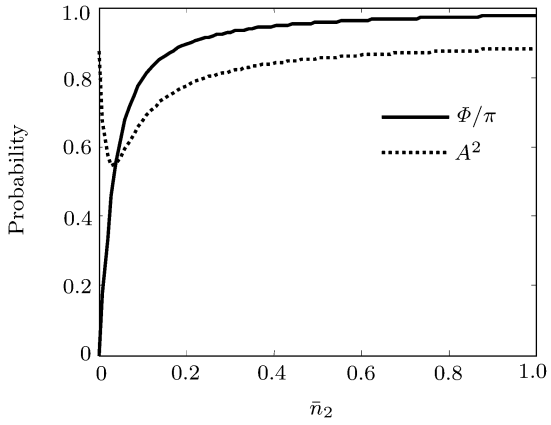


Fig.6. If the mean number of photons in the cavity 2 is 0, the reflection probability of the probe field is close to 90% and the phase shift is0. While, if the mean number of photons in cavity 2 comes to 1, the reflection probability of the probe field is about the same as the above case, but its phase shift is close to π. The parameters are $g_{12} = g_{23} = 110\,\text{MHz}$, $\kappa_1 = \kappa_2 = 14.2\,\text{MHz}$, $\Gamma_2 = \Gamma_3/2 = 2.6\,\text{MHz}$, $\gamma_1 = \gamma_2 = 240\Gamma_2$ and $\Delta = 4\Gamma_2$.

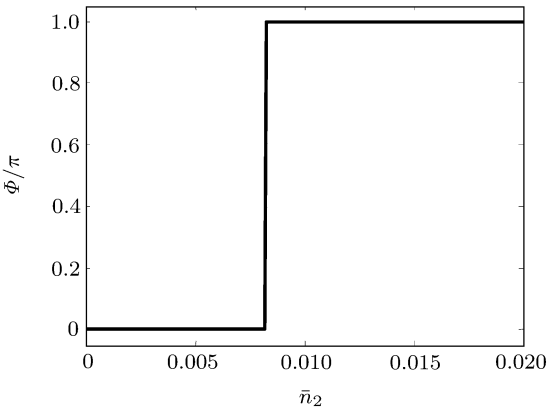


Fig.7. The detuning between the cavity 2 and the atom transition$|2\rangle \leftrightarrow |3\rangle$ is set to be Zero, i.e. $\Delta = \omega_{32} - \nu_2 = 0$. The phase shift of the probe field sharply increases from 0 to π with the increasing of the mean number of photons in the cavity 2 from 0.008 to 0.009. The corresponding parameters are $g_{12} = g_{23} = 110\,\text{MHz}$, $\kappa_1 = \kappa_2 = 14.2\,\text{MHz}$, $\Gamma_2 = \Gamma_3/2 = 2.6\,\text{MHz}$, and $\gamma_1 = \gamma_2 = 240\Gamma_2$.

5. The relations of the photon number inside and outside the cavity

For the opened atom–cavity system discussed above, the input output fields can be formulated as

$$\begin{pmatrix} a_{kq}^{\rm out} \\ b_{kq}^{\rm out} \end{pmatrix} = \begin{pmatrix} S_{11}^{j} & S_{12}^{j} \\ S_{21}^{j} & S_{22}^{j} \end{pmatrix} \begin{pmatrix} a_{kq}^{\rm in} \\ b_{kq}^{\rm in} \end{pmatrix}, \tag{24}$$

$$\begin{aligned} a_{kq}^{\rm out}(t) - a_{kq}^{\rm in}(t) &= \sqrt{\gamma_j} c_j(t), \\ b_{kq}^{\rm out}(t) - b_{kq}^{\rm in}(t) &= \sqrt{\gamma_j} c_j(t), \end{aligned} \tag{25}$$

where $q = x, y$, $j = 1, 2$. Assuming that the field is just input from one side of the cavity, i.e. $b_{kq}^{\rm in} = 0$, from Eqs.(24) and (25) we obtain the following relations:

$$\begin{aligned} \left\langle (a_{kq}^{\rm in})^{\dagger} a_{kq}^{\rm in} \right\rangle &= \gamma_j \langle c_j^{\dagger} c_j \rangle, \\ \left\langle (b_{kq}^{\rm out})^{\dagger} b_{kq}^{\rm out} \right\rangle &= \gamma_j \langle c_j^{\dagger} c_j \rangle, \end{aligned} \tag{26}$$

in case of $|S_{11}^{j}|^2 = 0$, $|S_{21}^{j}|^2 = 1$, and

$$\left\langle (a_{kq}^{\rm out})^{\dagger} a_{kq}^{\rm out} \right\rangle = \left\langle (a_{kq}^{\rm in})^{\dagger} a_{kq}^{\rm in} \right\rangle, \tag{27}$$

in case of $|S_{11}^{j}|^2 = 1$, $|S_{21}^{j}|^2 = 0$.

From Eq.(7), the first order time correlation function of the input field is given by $\left\langle (a_{kq}^{\rm in}(t))^{\dagger} a_{kq}^{\rm in}(t+\tau) \right\rangle = \dfrac{1}{2\pi} \displaystyle\int_{-\infty}^{\infty} \bar{n}_{kq}(\nu, t_0) {\rm e}^{-{\rm i}\nu\tau} {\rm d}\nu$. Then Fourier transform of the correlation function is

$$\begin{aligned} &\frac{1}{2\pi} \int_{-\infty}^{\infty} \left\langle (a_{kq}^{\rm in}(t))^{\dagger} a_{kq}^{\rm in}(t+\tau) \right\rangle {\rm e}^{{\rm i}\nu\tau} {\rm d}\tau \\ =&\frac{1}{2\pi} \bar{n}_{kq}(\nu, t_0). \end{aligned} \tag{28}$$

In the same way, for the output fields we have

$$\begin{aligned} &\frac{1}{2\pi} \int_{-\infty}^{\infty} \left\langle (a_{kq}^{\rm out}(t))^{\dagger} a_{kq}^{\rm out}(t+\tau) \right\rangle {\rm e}^{{\rm i}\nu\tau} {\rm d}\tau \\ =&\frac{1}{2\pi} \bar{n}_{kq}(\nu, t_1), \end{aligned} \tag{29}$$

$$\begin{aligned} &\frac{1}{2\pi} \int_{-\infty}^{\infty} \left\langle (b_{kq}^{\rm out}(t))^{\dagger} b_{kq}^{\rm out}(t+\tau) \right\rangle {\rm e}^{{\rm i}\nu\tau} {\rm d}\tau \\ =&\frac{1}{2\pi} \bar{m}_{kq}(\nu, t_1), \end{aligned} \tag{30}$$

where $n_{kq}(\nu, t_0) = a_{kq}^{\dagger}(\nu, t_0) a_{kq}(\nu, t_0)$ and $m_{kq}(\nu, t_0) = b_{kq}^{\dagger}(\nu, t_0) b_{kq}(\nu, t_0)$. $\bar{n}_{kq}(\nu, t_0)$ is the mean number of photons at the initial time t_0 in the mode $a_{kq}^{\rm in}$, $\bar{n}_{kq}(\nu, t_1)$ and $\bar{m}_{kq}(\nu, t_1)$ are the mean number of photons at the final time t_1 in the modes $a_{kq}^{\rm out}$ and $b_{kq}^{\rm out}$, respectively.

Now we derive the time correlation function of the cavity field. We first consider the cavity 1, then generalize it to either of the two cavities. From Eq.(4)

$$\begin{aligned} c_1(t+\tau) =& c_1(t) {\rm e}^{-({\rm i}\nu_1 + \kappa_1/2 + \gamma_1)\tau} \\ &- \int_t^{t+\tau} [{\rm i} g_{12} \sigma_{12}(t') + \sqrt{\gamma_1} (a_{kx}^{\rm in}(t') \\ &+ b_{kx}^{\rm in}(t'))] {\rm e}^{({\rm i}\nu_1 + \kappa_1/2 + \gamma_1)(t'-t-\tau)} {\rm d}t'. \end{aligned} \tag{31}$$

Considering $\left\langle c_1^{\dagger}(t) \sigma_{12}(t') \right\rangle = 0$, $\left\langle c_1^{\dagger}(t) a_{kx}^{\rm in}(t') \right\rangle = 0$, and $\left\langle c_1^{\dagger}(t) b_{kx}^{\rm in}(t') \right\rangle = 0$,[30] we have the correlation function of field for the cavity 1 as

$$\left\langle c_1^{\dagger}(t) c_1(t+\tau) \right\rangle = \left\langle c_1^{\dagger}(t) c_1(t) \right\rangle {\rm e}^{-({\rm i}\nu_1 + \kappa_1/2 + \gamma_1)\tau}. \tag{32}$$

We narrate here that the operators are averaged over all the considered states including quantum states of the atom, the cavity mode, and the input output fields. Now we write Eq.(32) for any of the two cavities

$$\left\langle c_j^{\dagger}(t) c_j(t+\tau) \right\rangle = \bar{n}_j(t) {\rm e}^{-({\rm i}\nu_j + \kappa_j/2 + \gamma_j)\tau}, \tag{33}$$

where $n_j(t) = c_j^{\dagger}(t) c_j(t)$ is the photon number operator of the field in the cavity j. The Fourier transform of the correlation function (33) is given by

$$\begin{aligned} &\frac{1}{2\pi} \int_{-\infty}^{\infty} \left\langle c_j^{\dagger}(t) c_j(t+\tau) \right\rangle {\rm e}^{{\rm i}\nu\tau} {\rm d}\tau \\ =&\frac{\bar{n}_j(t)}{2\pi} \frac{\kappa_j/2 + \gamma_j + i(\nu - \nu_j)}{(\nu - \nu_j)^2 + (\kappa_j/2 + \gamma_j)^2}, \end{aligned} \tag{34}$$

where the real component represents the power spectrum of the cavity j. It is a Lorenzian distribution at $\nu = \nu_j$ with half-width $\kappa_j/2 + \gamma_j$.

We consider the Fourier transforms (28), (29), (30) and (31) of the correlation functions at the central frequencies of the input output fields and the cavity mode, respectively. Using Eqs.(26) and (27) additionally, we arrive at

$$\bar{n}_{kq}(\nu_{kq}, t_0) = \frac{\gamma_j \bar{n}_j(t)}{(\kappa_j/2 + \gamma_j)}, \tag{35}$$

$$\bar{m}_{kq}(\nu_{kq}, t_1) = \frac{\gamma_j \bar{n}_j(t)}{(\kappa_j/2 + \gamma_j)}, \tag{36}$$

$$\bar{n}_{kq}(\nu_{kq}, t_0) = \bar{n}_{kq}(\nu_{kq}, t_1), \quad \bar{n}_j(t) = 0. \tag{37}$$

On account of $\kappa_j << \gamma_j$ in the system, Eqs.(35) and (36) reveal conservation of photon number

$$\begin{aligned} &\bar{n}_{kq}(\nu_{kq}, t_0) \approx \bar{n}_j(t), \\ &\bar{m}_{kq}(\nu_{kq}, t_1) \approx \bar{n}_j(t), \\ &q = x, y, \quad j = 1, 2. \end{aligned} \tag{38}$$

Equations (37) and (38) indicate that mean number of the photons in the input field at an earlier time

t_0 is nearly equal to that of the photons in the cavity at a later time t ($t > t_0$), and the mean number of the photons in the cavity at an earlier time t is nearly equal to that of the photons in the output field at a later time t_1 ($t < t_1$). Additionally, a single photon in the cavity is sufficient to control both the amplitude and the phase of the probe field (see Figs.3, 4, 6 and 7). Therefore, both the π phase shift and the control of transmission reflection of an input single photon are available via the application of another input single photon. That means, with the cavity-emitter system described in the present paper, effective interaction between two distinct electromagnetic fields at the level of single photons may be realizable in the current experimental technology.

6. Discussion

We turn to discuss about the feasibility of the photon–photon interaction with the ICD-cavity system. First, based on cavity QED technique, phase shift of an optical field at the single photon level[17] and the switching effect by photons[18] are experimentally observed. Besides, designing a practical useful ICD-cavity may be possible, when we consider that a few unusual cavities are applied in an experimental research[33] and a theoretical work.[34] In fact, an atom in the Fabry-Perot cavity can be driven by a laser field from different direction except the cavity mode.[35−37] When the laser field is replaced by another cavity mode, the system would change into the form like the ICD-cavity presented here. For the technology of the trapping a single atom in the cavity, recent experiments report that single rubidium atom can be trapped in a cavity with storage time over 20 ms[38] or even up to 15 s.[37] The storage time duration is much longer than the decay time of the cavity $2\pi(\gamma_j + \kappa_j/2)^{-1} \sim 1\,\text{ns}$ ($j = 1, 2$) and the atom $2\pi\Gamma^{-1} \sim 400\,\text{ns}$ considered in our work. In a quite recent experiment,[39] it is demonstrated that a Fabry-Perot cavity can work with a single atom trapped in the cavity and this system is very useful for building optical cavity networks. Finally, we note that the basic parameters applied in the present paper are experimentally available.[31]

Photon–photon interactions with the ICD-cavity system may have great applications for the all-optical information processing. For the realization of practical quantum information processing and quantum computation, the first problem needed to solve is the scalability of the quantum qubit system. In the cavity QED schemes, to integrate a series of cavities is significant for the construction of an optical quantum networks. If the Fabry-Perot cavities are connected only in one dimension,[40] the scalability of the cavity system would be restricted. With the ICD-cavity system presented in our paper, cavities can be integrated in two dimensions and the control of the optical fields at the single photon level may be realized.

7. Conclusion

In Heisenberg picture, effective interaction between two optical fields based on the single atom trapped ICD-cavity system is demonstrated. Combining the theoretical derivation and numerical simulation by using the experimental available parameters, we obtain mainly two results. First, a single photon can almost fully control the transmission and the reflection of another single photon. Second, large phase shift of an optical field about π can be achieved by the interaction between two optical fields at the single photon level. In our system, the probe field and the control field propagate in different spatial modes. This system may be applied for the implementation of the quantum information processing.

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