Vol 17 No 10, October 2008
1674-1056/2008/17(10)/3739-05

Chinese Physics B

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Generation and classification of robust remote symmetric Dicke states

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(Received 26 March 2008; revised manuscript received 23 April 2008)

In this paper, we present an approach to generating arbitrary symmetric Dicke states with distant trapped ions and linear optics. Distant trapped ions can be prepared in the symmetric Dicke states by using two photon-number-resolving detectors and a polarization beam splitter. The atomic symmetric Dicke states are robust against decoherence, for atoms are in a metastable level. We discuss the experimental feasibility of our scheme with current technology. Finally, we discuss the classification of arbitrary n-qubit symmetric Dicke states under statistical local operation and classical communication and prove the existence of $[n/2]$ inequivalent classes of genuine entanglement of n-qubit symmetric Dicke states.

Keywords: linear optics, ion trap, symmetric Dicke states
PACC: 4250, 0367

1. Introduction

Entanglement is a unique resource in quantum information processing (QIP). Two parties that share an entanglement can use it to perform quantum cryptography,[1−5] or quantum secret sharing[6] and quantum teleportation.[7] The creation of multipartite entanglement between distant locations is thus an important goal. At the moment, the characterization of multipartite entanglement is not fully understood, however. Fortunately, these drawbacks have not prevented the rapid progress of generating and classifying multiqubit entangled states.

Recently, many proposals have been presented for robust bipartite entanglement distribution[8−12] based on joint measurement on photons. These distributed bipartite entanglement can be used to distribute a secure key in quantum communication. Furthermore, it may be used to prepare a quantum repeater in long-distance quantum communication. With the development of efficient and scalable entanglement distribution, several experiments have successfully shown genuine entangled multiphoton states and entangled atomic states.[13,14] Recently, a method is presented of generating symmetric Dicke states either in distant matter or in photon polarization qubits by using a multifold detection technique,[15] which uses explicitly the difference in geometrical phase between possible quantum paths.

In this paper, we present an approach to generating any symmetric Dicke state in distant atoms by using projective measurement on emitted photons from atoms with a polarization beam splitter (PBS) and photon number resolving (PNR) detectors. Each atom is first prepared in an excited state and then jumps to a metastable state via spontaneous decay. The emitted photons will interfere at a PBS, and then they are measured by the PNR detectors to collapse into a symmetric Dicke state. In consequence, the atoms also collapse into a symmetric Dicke state which is a genuine multipartite entangled state. Since the entangled atoms are in a metastable state, the remote atomic symmetric Dicke states are long robust against decoherence. Another interesting property of symmetric Dicke states we want to discuss is that they are highly robust against particle loss. We discuss the classification of symmetric Dicke states under statistical local operation and classical communication.

2. Atom–photon entanglement

Now we describe our proposal in more detail, starting with exciting ions. Suppose that each of the n parties starts out with an ion that contains a lambda system of levels. Here we select $^{40}Ca^{+}$ as the target ion, which has been considered and demonstrated in some recent QIP experiments. We can use one of the

$P_{3/2}$ levels, say, $m = -3/2$, to serve as the excited level e, which can be prepared by preparing the state $|S_{1/2}, m = -1/2\rangle$ first and then by optically pumping and applying σ_- light at a wavelength of 393 nm.[16] Here we choose $m = -5/2$ and $m = -1/2$ sublevels of $D_{5/2}$ as states $|0\rangle_{\text{atom}}$ and $|1\rangle_{\text{atom}}$, respectively. The states $|0\rangle$ and $|1\rangle$ can be coupled to the excited state $|e\rangle$ ($m = -3/2$) of $P_{3/2}$ by σ_- light and σ_+ light at a wavelength of 854 nm, see Fig.1. Emission of a photon from an ion leads to the state

$$\frac{1}{\sqrt{2}}[|0\rangle_{\text{atom}}\sigma_- + |1\rangle_{\text{atom}}\sigma_+].$$

If each of two ions emits a photon, the combined wave function can be described as,

$$\begin{aligned}&\frac{1}{2}[|0\rangle_{\text{atom}}\sigma_- + |1\rangle_{\text{atom}}\sigma_+] \otimes [|0\rangle_{\text{atom}}\sigma_- + |1\rangle_{\text{atom}}\sigma_+]\\ =\ &\frac{1}{2}[|\Phi^+\rangle_{\text{atom}}|\Phi^+\rangle_{\text{photon}} + |\Phi^-\rangle_{\text{atom}}|\Phi^-\rangle_{\text{photon}}\\ &-|\Psi^+\rangle_{\text{atom}}|\Psi^+\rangle_{\text{photon}} - |\Psi^-\rangle_{\text{atom}}|\Psi^-\rangle_{\text{photon}}], \quad (1)\end{aligned}$$

where $|\Phi^\pm\rangle_{\text{atom}} = (|0\rangle_{\text{atom}}|0\rangle_{\text{atom}} \pm |1\rangle_{\text{atom}}|1\rangle_{\text{atom}})/\sqrt{2}$ and $|\Psi^\pm\rangle_{\text{atom}} = (|0\rangle_{\text{atom}}|1\rangle_{\text{atom}} \pm |1\rangle_{\text{atom}}|0\rangle_{\text{atom}})/\sqrt{2}$ are the maximally entangled Bell states for ions, with corresponding definitions for photons. This indicates that these two ions will collapse into a Bell state as soon as the two emitted photons are in the Bell state. In experiment, we can perform a Bell basis measurement on the two emitted photons so that these two atoms will also collapse into a Bell state.

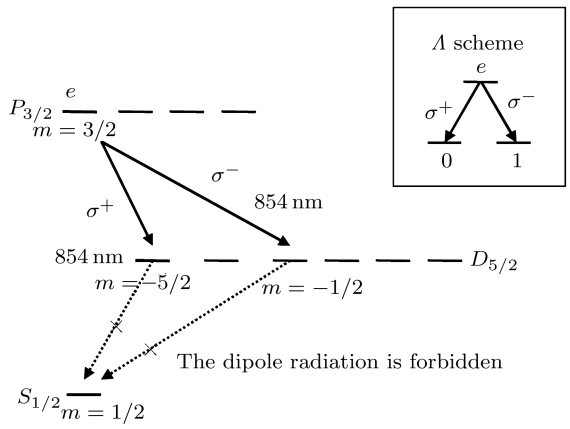


Fig.1. Sketch of the $^{40}Ca^+$ levels used to generate the symmetric Dicke states.

3. Generation arbitrary n atoms symmetric Dicke states

Along the line above, we now present our scheme for arbitrary symmetric Dicke states with trapped ions and linear optics. Introduce the symmetric Dicke states first. In an n-qubit system, the Dicke state, usually denoted by $|n, m\rangle$, is in general written as

$$\begin{aligned}|n, m\rangle = &\binom{n}{m}^{-(1/2)} \sum Q_k\\ &\times|0_1, 0_2, ..., 0_m, 1_1, 1_2, ..., 1_{n-m}\rangle,\end{aligned}$$

where $\{Q_k\}$ are the complete set of all possible distinct permutations of the qubits. In order to explicitly explain the symmetric Dicke states, we present all four symmetric Dicke states of three qubits here

$$\begin{aligned}|3, 0\rangle &= |0, 0, 0\rangle,\\ |3, 1\rangle &= \frac{1}{\sqrt{3}}(|0, 1, 1\rangle + |1, 0, 1\rangle + |1, 1, 0\rangle),\\ |3, 2\rangle &= \frac{1}{\sqrt{3}}(|0, 0, 1\rangle + |1, 0, 0\rangle + |0, 1, 0\rangle),\end{aligned}$$

and

$$|3, 3\rangle = |1, 1, 1\rangle.$$

We now discuss how to generate three atoms symmetric Dicke states. We will use the photon-number-resolving detectors[17] and linear optical elements in our scheme to generate symmetric Dicke states $|3, m\rangle$. First, three trapped ions are prepared in the excited state $|e\rangle$ and then each of them emits a photon in the superposition state $\phi_{\text{photon}} = (\sigma_+ + \sigma_-)/\sqrt{2}$. The photons have the same optical paths to the polarization beam splitter (PBS). If these three photons arrive at the PBS at the same time (In fact, it only needs these three photons to be interferential when they arrive at the PBS.), they will interfere at the PBS and then they are detected at the photon-number-resolving detectors. These three photons will collapse into one of the four symmetric Dicke states and then the three remote ions will also collapse into the corresponding symmetric Dicke states.

We first explain how the photons collapse into symmetric Dicke states when they are detected by the photon-number-resolving detectors, see Fig.2. When a photon is in the state σ_+ ($|0\rangle$), it will pass through the PBS and be detected by the photon-number-resolving detector PD1. Otherwise, if the photon is in the state σ_- ($|0\rangle$), it will be reflected and then detected by PD2. In the case of three photons, there may be four different distributions in the photon-number-resolving detectors. (1) The three photons were all detected by photon-number-resolving detector (PD1); (2) One photon is detected by PD1 and the other two are

both detected by PD2. (3) Two photons are both detected by PD1 and the other one is detected by PD2. (4) The three photons are all detected by PD2. Although we can detect the photons, we cannot distinguish one of them from others. For example, if one photon is detected in PD1 and two photons are both detected in PD2, then these three photons may be in the state $|0,1,1\rangle$, $|1,0,1\rangle$, or $|1,1,0\rangle$. Since all of three states are possible, these three photons are in the superposition state $|3,1\rangle_{\rm photon} = (|0,1,1\rangle + |1,0,1\rangle + |1,1,0\rangle)/\sqrt{3}$ after the PBS. Consequently, the three corresponding ions will also collapse into state $|3,1\rangle_{\rm atom} = (|0,1,1\rangle + |1,0,1\rangle + |1,1,0\rangle)/\sqrt{3}$ as soon as the emitted photons collapse into $|3,1\rangle_{\rm photon}$. If all of three photons are detected in PD1, then the three ions will collapse into $|3,0\rangle_{\rm atom} = |0,0,0\rangle$. Likewise, when three photons are all detected in PD2, they will collapse into $|3,3\rangle_{\rm atom} = |1,1,1\rangle$. And when two photons are both detected in PD1 and one photon is detected in PD2, the ions will collapse into $|3,1\rangle_{\rm atom} = (|0,0,1\rangle + |1,0,0\rangle + |1,0,1\rangle)/\sqrt{3}$. In this way, we can prepare all four symmetric Dicke states of three qubits.

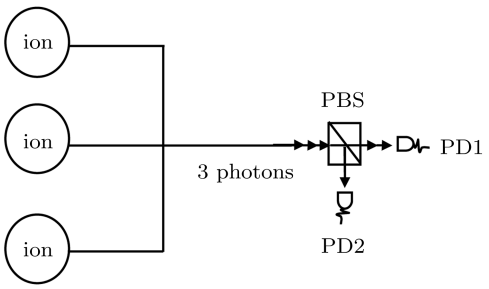


Fig.2. Three trapped ions are in the excited state $|e\rangle$ first. They have the same optical paths to the PBS. Thus these three emitted photons will interfere at the PBS. After the PBS, these three photons will collapse into one of the symmetric Dicke states $|n,m\rangle_{\rm photon}$ according to the photon number distribution in PD1 and PD2 because of the identity of the photons. Consequently, the three atoms will also collapse into the symmetric Dicke state $|n,m\rangle_{\rm photon}$. These atoms are in a metastable level now, so the symmetric Dicke states of these atoms are robust against the decoherence.

Now we generalize the approach above to generating arbitrary n-qubit symmetric Dicke states. Suppose that there are n trapped ions in the excited state $|e\rangle$, and they have the same optical paths to the PBS. Then the n emitted photons interfere at the PBS. Suppose that m photons are detected in PD1 and $n-m$ photons are detected in PD2. Then after the PBS, there are m photons in state $|0\rangle$ and $n-m$ photons in state $|1\rangle$. Since we cannot identify which m photons are in the state $|0\rangle$ or $|1\rangle$, these n photons are then in the symmetric Dicke state $|n,m\rangle_{\rm photon} = \binom{n}{m}^{-(1/2)} \sum Q_k |0_1,0_2,...,0_m,1_1,1_2,...,1_{n-m}\rangle$ for the symmetry. Accordingly, these n ions will also collapse into $|n,m\rangle_{\rm atom}$. Here m is $0 \le m \le n$, so that we can generate all the symmetric Dicke states of n atoms probabilistically.

4. Experimental feasibility

We now discuss the practical implementation of the generation of arbitrary n-atom symmetric Dicke states. As it has been discussed above, we can select $^{40}Ca^{+}$ as the candidate. The life of the excited state $|e\rangle$ is on the order of 10^{-8} s and the life of the metastable state is about one second. Therefore, the entanglement of the remote atoms are robust for the long-lived atomic metastable states. The photon-number-resolving detector has been demonstrated experimentally.[17] It has been shown that the photon-number-resolving detector can identify about 10 photons each pulse with a detection efficiency of about 88%. Therefore, the efficiency of our scheme with photon-number-resolving detectors is about 0.88^n. For example, the efficiency η is about 0.28 when $n = 10$, that is to say, we have a successful probability of 0.28 to generate 10 atoms in the symmetric Dicke states in each run if the ion traps and optical channels are perfectly.

Another linear optical scheme can be used to generate symmetric Dicke states when there are not any photon-number-resolving detectors in the lab. In fact, we can use a group of beam splitters and single-photon detectors to realize the scheme instead. In Fig.3, we present the scheme to generate n-qubit symmetric Dicke states. Suppose there are n photons in the arriving pulse. If the n single-photon detectors each have detected a photon and m photons are in the $|0\rangle$, then the n photons have been prepared in the $|n,m\rangle_{\rm photon}$ state. Accordingly, the n ions have been prepared in the $|n,m\rangle_{\rm atom}$ states, too.

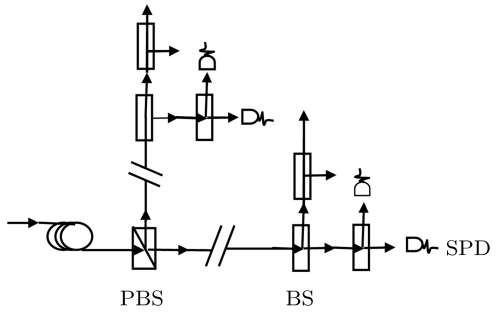


Fig.3. The experimental scheme for probabilistically generating symmetric Dicke states. There are two groups of the BSs after the two outports of the PBS. At the each outport of every final BS, there is a single-photon detector (SPD) used to detect the incoming photons and then the total number of the SPD is n. If each SPD has detected one photon, then the n photons have been prepared in one of the symmetric Dicke states. Accordingly, the n atoms have also been prepared in the n-qubit symmetric Dicke states. However, such a scheme is not deterministic because a photon pulse cannot be split deterministically by the BS.

5. Classification symmetric Dicke states

One of charming characters for symmetric Dicke states is its robustness against qubit loss. For three-qubit symmetric Dicke states $|3,1\rangle$ and $|3,2\rangle$, the loss of one qubit does not entirely break the entanglement between the other two qubits, for they have a probability of 2/3 in the maximally entangled Bell state. For the n-qubit symmetric state $|n,m\rangle$, when one qubit is traced out, the other $n-1$ qubits will be in the state $|n-1,m-1\rangle$ or in the state $|n-1,m\rangle$ with the probabilities of $\binom{n-1}{m-1}\Big/\binom{n}{m}$ and $\binom{n-1}{m}\Big/\binom{n}{m}$, respectively.

The most important character of entanglement is its nonlocality. So, two entangled states can be classified as one kind of entanglement if they can be transformed into each other under statistical local operation and classical communication. It has been proved that the statistical local operation and classical communication is identical to local unitary operation on single copy.[18] We now discuss the genuine entangled states of n-qubit symmetric states. First, the states $|n,0\rangle$ and $|n,n\rangle$ are not genuine entangled states since both of them are product states. From the symmetry, we know that $|n,m\rangle$ and $|n,n-m\rangle$ can be transformed into each other by exchanging qubits $|0\rangle$ and $|1\rangle$. In fact, we have the relation between the states $|n,m\rangle$ and $|n,n-m\rangle$

$$|n,m\rangle = \sigma_x^1 \otimes \sigma_x^2 \otimes ... \otimes \sigma_x^n |n,n-m\rangle,$$

where $\sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|$ is the Pauli operator. Therefore, single copies of two different symmetric states $|n,m\rangle$ and $|n,n-m\rangle$ can be transformed into each other by local unitary operations.

Can the states $|n,m\rangle$ and $|n,k\rangle$ be transformed into each other under local unitary operations when $m < k \leq n/2$? The answer is negative. Here we give a brief discussion about this. First, from the discussion above, we know that each symmetric Dicke state corresponds to a photon number distribution. Second, each photon number distribution corresponds to a symmetric Dicke state. Thus, the n-qubit symmetric Dicke states are completely determined by the photon number distribution on the detectors. After the PBS, any local unitary operation cannot change the photon number distribution. Therefore, two symmetric Dicke states $|n,m\rangle$ and $|n,k\rangle$ cannot be transformed into each other under statistical local operations and classical communication when $m < k \leq n/2$. The reason why $|n,m\rangle$ and $|n,n-m\rangle$ can be transformed into each other is because of the symmetry between the qubits $|0\rangle$ and $|1\rangle$. Therefore, there may be $[n/2]$ classes of genuine entangled states of n-qubit symmetric Dicke states.

6. Summary

In this paper, we have presented an experimental scheme for generating arbitrary remote symmetric Dicke states. We have given remote symmetric Dicke states with one PBS and two photon-number-resolving detectors, which is different from the previous work by using the far-field method.[15] In our scheme, the entangled atoms are in a metastable level, so that the entangled states are long-lived states. We have also discussed the classification of the n-qubit symmetric Dicke states and pointed out that there are $[n/2]$ classes of genuine entangled states of n-qubit symmetric Dicke states.

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