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Phase modulation of propagation effect with Doppler broadening*

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This paper studies the propagation effect in a closed lambda-type three-level atomic system with Doppler broadening. It is shown that, Doppler broadening due to atomic motion and propagation effect associated with driving field depletion along the active medium decreases obviously the gain and output of the lasing without inversion (LWI); the relative phase between the probe and driving fields has a remarkable modulation role to the propagation effect on LWI when Doppler broadening presents; by choosing suitable value of the relative phase, we can get the largest gain and output of LWI.

Keywords: closed system, Doppler broadening, propagation effect, lasing without inversion
PACC: 4250

1. Introduction

Recently, lasing without inversion (LWI) has attracted much attention[1−5] as it can provide a new tool in the pursuit of "tabletop" ultraviolet and γ-ray laser and has other interesting statistical properties. LWI is the result of quantum coherence and interference. There are many ways to generate quantum coherence and interference in an atomic system. Generally, they are produced by a coherent driving field or by initial coherence. A kind of coherence can be created by interference of spontaneous emission in the atomic systems with near-degenerate levels, usually called spontaneously generated coherence (SGC). The existence of SGC effect depends on the nonorthogonality of dipole matrix elements. Effect of SGC on the electromagnetically induced transparency, LWI and optical bistability etc. have been widely investigated.[6−14] Moreover, it has been shown that atomic systems with SGC are sensitive to the relative phase of the applied fields.[15−24] The propagation effect associated with driving field depletion along the active medium is an important factor reducing LWI gain.[25−27] In addition, in LWI experiments, usually the active medium is an atomic gas, and Doppler broadening is prominent. Doppler broadening also produces an obvious influence on LWI gain.[28−31] Hence, in order to get larger LWI gain and output, investigating modulation role of the relative phase on propagation effect is necessary under the condition of Doppler broadening presenting. So far, to our knowledge, there is no work about this topic. In this paper, we will do this work. In order to see the damage of Doppler broadening on LWI gain, we will discuss two cases with and without Doppler broadening, respectively.

2. Equations of motion

We consider a closed lambda-type three-level atomic system, as illustrated in Fig.1. Transition

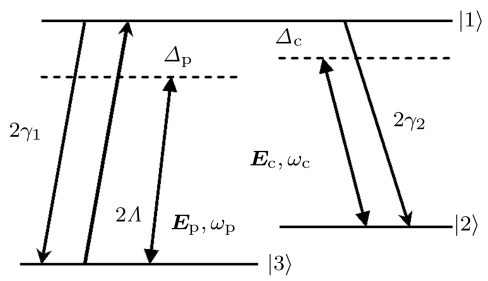


Fig.1. Closed lambda-type three-level system.

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between the levels $|1\rangle$ and $|2\rangle$ is driven by a strong coherent driving field with the amplitude $\boldsymbol{E}_{\rm c} = \boldsymbol{\varepsilon}_{\rm c}{\rm e}^{-{\rm i}\omega_{\rm c}t} + {\rm c.c.}$, frequency $\omega_{\rm c}$ and Rabi frequency $\Omega_{\rm c} = \boldsymbol{\varepsilon}_{\rm c}\cdot\boldsymbol{d}_{21}/2\hbar$. An incoherent pump with a pumping rate 2Λ and a weak coherent probe field with the amplitude $\boldsymbol{E}_{\rm p} = \boldsymbol{\varepsilon}_{\rm p}{\rm e}^{-{\rm i}\omega_{\rm p}t} + {\rm c.c.}$, frequency $\omega_{\rm p}$ and Rabi frequency $\Omega_{\rm p} = \boldsymbol{\varepsilon}_{\rm p}\cdot\boldsymbol{d}_{31}/2\hbar$ are applied between levels $|1\rangle$ and $|3\rangle$. $2\gamma_1$ and $2\gamma_2$ denote the spontaneous emission rates from level $|1\rangle$ to level $|3\rangle$ and $|2\rangle$, respectively. Using the rotating-wave and the electric dipole approximations, we can write the density matrix equations of motion of this system as:

$$\dot{\sigma}_{11} = -2(\gamma_1+\gamma_2)\sigma_{11} + {\rm i}(\Omega_{\rm p}\sigma_{31} - \Omega_{\rm p}^*\sigma_{13}) + 2\Lambda\sigma_{33} + {\rm i}(\Omega_{\rm c}\sigma_{21} - \Omega_{\rm c}^*\sigma_{12}), \tag{1a}$$

$$\dot{\sigma}_{22} = 2\gamma_2\sigma_{11} + {\rm i}(\Omega_{\rm c}^*\sigma_{12} - \Omega_{\rm c}\sigma_{21}), \tag{1b}$$

$$\dot{\sigma}_{12} = -(\gamma_1+\gamma_2+{\rm i}\Delta_{\rm c})\sigma_{12} + {\rm i}\Omega_{\rm p}\sigma_{32} - {\rm i}\Omega_{\rm c}(\sigma_{11}-\sigma_{22}), \tag{1c}$$

$$\dot{\sigma}_{13} = -(\gamma_1+\gamma_2+\Lambda+{\rm i}\Delta_{\rm p})\sigma_{13} + {\rm i}\Omega_{\rm c}\sigma_{23} - {\rm i}\Omega_{\rm p}(\sigma_{11}-\sigma_{33}), \tag{1d}$$

$$\dot{\sigma}_{23} = -(\Lambda+{\rm i}\Delta_{\rm p}-{\rm i}\Delta_{\rm c})\sigma_{23} + 2(\gamma_1\gamma_2)^{1/2}\cos\theta\eta\sigma_{11} + {\rm i}\Omega_{\rm c}^*\sigma_{13} - {\rm i}\Omega_{\rm p}\sigma_{21}. \tag{1e}$$

The above equations are constrained by $\sigma_{ij} = \sigma_{ji}^*$ and $\sigma_{11}+\sigma_{22}+\sigma_{33} = 1$. Where $\sigma_{ii}(i = 1,2,3)$ is the population of the level $|i\rangle$, $\sigma_{ij}(i \neq j)$ is the atomic polarization between levels $|i\rangle$ and $|j\rangle$, $\Delta_{\rm p} = \omega_{13} - \omega_{\rm p}$ and $\Delta_{\rm c} = \omega_{12} - \omega_{\rm c}$ are the detunings of the probe and driving fields from their relevant atomic transitions, respectively. $2(\gamma_1\gamma_2)^{1/2}\cos\theta\eta\sigma_{11}$ represents the quantum interference effect resulting from the cross coupling between the spontaneous emission $|1\rangle \leftrightarrow |2\rangle$ and $|1\rangle \leftrightarrow |3\rangle$, i.e., the SGC effect. The dipole moments $\boldsymbol{d}_{12}$ and $\boldsymbol{d}_{13}$ are not orthogonal, this fact is necessary for the existence of the SGC effect. θ represents the angle between the two dipole moments. If levels $|2\rangle$ and $|3\rangle$ lie so closely that the SGC effect has to be taken into account, then $\eta = 1$, otherwise $\eta = 0$. When $\eta = 0$, Eqs.(1a)–(1e) reduce to the equations for a closed lambda three-level atomic system without SGC; when $\eta = 1$, Eqs.(1a)–(1e) describe a closed lambda three-level atomic system with SGC. Because of the existence of the SGC effect, the properties of the closed system depend greatly on relative phase of the applied fields. Hence the Rabi frequencies cannot be treated as real parameters. Redefining the system variables in Eqs.(1a)–(1e), $\rho_{ii} = \sigma_{ii}$, $\rho_{12} = \sigma_{12}\exp({\rm i}\Phi_{\rm c})$, $\rho_{13} = \sigma_{13}\exp({\rm i}\Phi_{\rm p})$, $\rho_{23} = \sigma_{23}\exp({\rm i}\Phi)$, $\Phi = \Phi_{\rm p} - \Phi_{\rm c}$, $\Omega_{\rm p} = G_{\rm p}\exp(-{\rm i}\Phi_{\rm p})$ and $\Omega_{\rm c} = G_{\rm c}\exp(-{\rm i}\Phi_{\rm c})$, we obtain the equations of motion for the redefined density matrix elements $\rho_{ij}(i,j = 1,2,3)$, which are found to be identical with Eqs.(1a)–(1e) except that σ_{ij}, η, $\Omega_{\rm p}$ and $\Omega_{\rm c}$ are replaced by ρ_{ij}, $\eta\exp({\rm i}\Phi)$, $G_{\rm p}$ and $G_{\rm c}$, respectively. Where $G_{\rm p}$ and $G_{\rm c}$ are real.

Using the slowly varying envelope and the rotating-wave approximations, we can write the propagation equations of the driving and probe fields as

$$\frac{\partial}{\partial z}G_{\rm c}(z) = \frac{3N\gamma_2\lambda_{\rm c}^2}{8\pi}{\rm Im}\rho_{21}(z), \tag{2a}$$

$$\frac{\partial}{\partial z}G_{\rm p}(z) = \frac{3N\gamma_1\lambda_{\rm p}^2}{8\pi}{\rm Im}\rho_{31}(z). \tag{2b}$$

Where, N represents the density of atom, $\lambda_{\rm c}$ and $\lambda_{\rm p}$ are the wavelength of the driving and probe fields, respectively. Now we discuss the effect of atomic movement bringing forth Doppler broadening on LWI system. When single atom moves along the axis of z with the velocity of ν, the frequency equation of the probe field can be written as:

$$\omega_{\rm p}(\nu) = \omega_{\rm p}(1 \pm \nu/c). \tag{3a}$$

The negative (positive) sign corresponds to co-propagating (counter-propagating) atom and field, where $\omega_{\rm p}$ represents the frequency of the probe field when the atom is immobile. Analogously, the frequency equation of the driving field can be written as:

$$\omega_{\rm c}(\nu) = \omega_{\rm c}(1 \pm \nu/c). \tag{3b}$$

Where $\omega_{\rm c}$ represents the frequency of the driving field when the atom is immobile. $\Delta_1(\nu) = \omega_{12} - \omega_{\rm c}(\nu)$ and $\Delta_2(\nu) = \omega_{13} - \omega_{\rm p}(\nu)$ are the detunings of the driving and probe fields from their relevant atomic transitions, respectively, when the atom moves. Using the Eqs.(3a) and (3b), we can get the new equations as

follows:

$$\Delta_1(\nu) = \Delta_1^0 \pm \frac{\omega_c}{\omega_p}[\Delta_2(\nu) - \Delta_2^0], \tag{4a}$$

$$\Delta_2(\nu) = \Delta_2^0 \pm \omega_p \nu / c. \tag{4b}$$

Where Δ_1^0 and Δ_2^0 are the detunings of the driving and probe fields with $\nu = 0$, namely initial detuning. Δ_1^0 and Δ_2^0 are equal to $\Delta_c(= \omega_{12} - \omega_c)$ and $\Delta_p(= \omega_{13} - \omega_p)$ respectively.

We assume that atomic velocity obeys Maxwell distribution:

$$N(\nu) = \frac{N}{\sigma\sqrt{\pi}} \exp\left(-\frac{\nu^2}{\sigma^2}\right), \tag{5}$$

where σ represents the most probable velocity, and the width of Doppler broadening induced by emission transition at the half-maximum position is:

$$\Delta\omega_{DP} = 2\sqrt{\ln 2}\frac{\sigma}{c}\omega_p. \tag{6}$$

Considering the effect of the Doppler broadening, we can write Eqs.(2a) and (2b) as:

$$\frac{\partial}{\partial z}G_c(z) = \frac{3\gamma_2\lambda_c^2}{8\pi}\int N(\nu)\mathrm{Im}\rho_{21}(z,\nu)\mathrm{d}\nu, \tag{7a}$$

$$\frac{\partial}{\partial z}G_p(z) = \frac{3\gamma_1\lambda_p^2}{8\pi}\int N(\nu)\mathrm{Im}\rho_{31}(z,\nu)\mathrm{d}\nu. \tag{7b}$$

Where $G_p$and G_c are real.

The probe gain (absorption) coefficient corresponds to the imaginary part of ρ_{31}. If $\mathrm{Im}\rho_{31} > 0$, the system exhibits gain for the probe field; if $\mathrm{Im}\rho_{31} < 0$, the probe field is attenuated. When $\rho_{11} < \rho_{33}$ and $\mathrm{Im}\rho_{31} > 0$, gain without inversion can be obtained, then LWI can be realized.

3. Numerical analysis

In the following we shall analyse the modulation role of the relative phase on the propagation effect by numerical calculation result when the probe and driving fields are co-propagating. Let us first consider the case without Doppler broadening, i.e. $\Delta\omega_{DP} = 0$.

In Fig.2 we plot probe gain (absorption) $\mathrm{Im}\rho_{31}$, Rabi frequencies G_c and G_p of the driving and probe fields and population difference $\rho_{11} - \rho_{33}$ as functions of propagation distance z for different values of the relative phase Φ. Where Rabi frequencies of the driving and probe fields at the entrance of the medium are $G_{cin} = 2\pi \times 25\,\mathrm{MHz}$ and $G_{pin} = 2\pi\,\mathrm{MHz}$, respectively. Other parameter values are: $\gamma_1 = 2\pi \times 3\,\mathrm{MHz}$, $\gamma_2 = 3\gamma_1$, $\lambda_c = \lambda_p = 780\,\mathrm{nm}$, $\Lambda = 0.7\gamma_1$, $p = 0.5$, $\Delta_1 = 0$, $\Delta_2 = -2\pi \times 25\,\mathrm{MHz}$ and $N = 2 \times 10^{15}/\,\mathrm{cm}^3$.

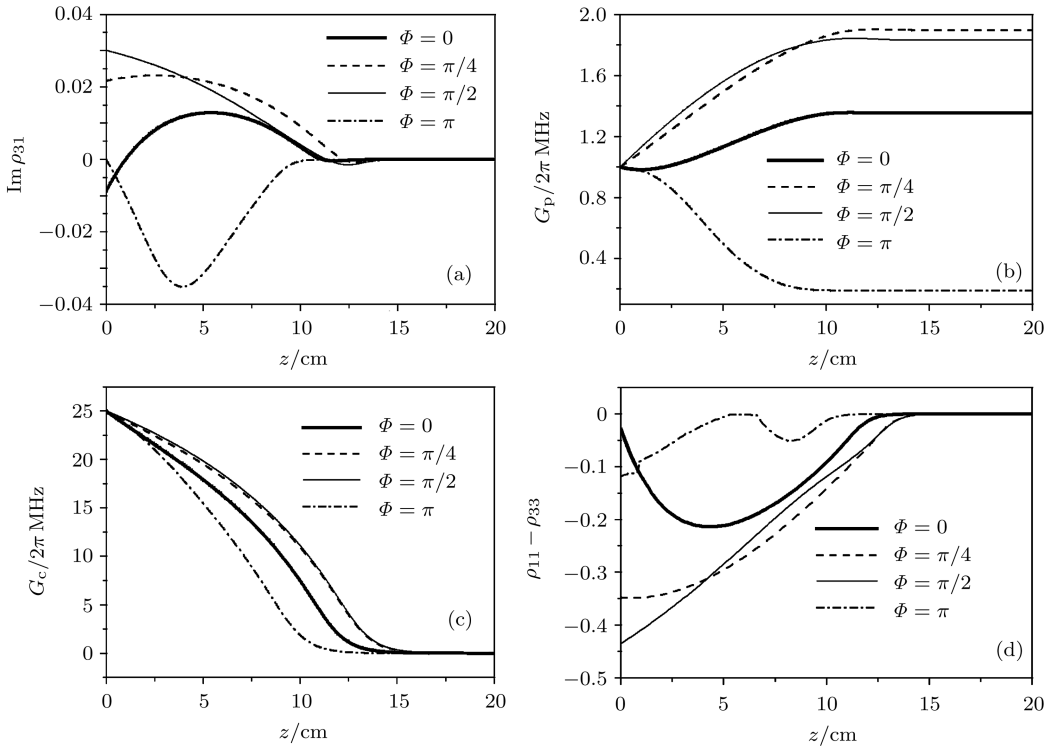


Fig.2. $\mathrm{Im}\rho_{31}$, G_p, G_c and $\rho_{11} - \rho_{33}$ as functions of propagation distance Z for different values of Φ with $\Delta\omega_{DP} = 0$, $G_{cin} = 2\pi \times 25\,\mathrm{MHz}$, $G_{pin} = 2\pi\,\mathrm{MHz}$, $\gamma_1 = 2\pi \times 3\,\mathrm{MHz}$, $\gamma_2 = 3\gamma_1$, $\lambda_c = \lambda_p = 780\,\mathrm{nm}$, $\Lambda = 0.7\gamma_1$, $p = 0.5$, $\Delta_1 = 0$, $\Delta_2 = -2\pi \times 25\,\mathrm{MHz}$ and $N = 2 \times 10^{15}/\,\mathrm{cm}^3$.

From Fig.2 we can see that the relative phase Φ has remarkable effect on variation rules of $\mathrm{Im}\rho_{31}$, G_c, G_p and $\rho_{11}-\rho_{33}$. Figure 2(a) displays that with the increase of propagation distance, Rabi frequency (strength) of the driving field decreases monotonically until it becomes zero, but the decreasing rate is different for different value of Φ, so length of the distance in which the driving field interacting with atoms is different, and this length also determines the length of the distance in which probe gain occurs. From the viewpoint of energy conservation, increase of the probe field originates from decrease of the driving field.

Figures 2(b) and 2(c) show that: when $\Phi=\pi$, in the range about from $z=0\,\mathrm{cm}$ to $z\approx 9.5\,\mathrm{cm}$, with the increase of propagation distance, the probe field is always absorbed and so the Rabi frequency (strength) of the probe field decreases monotonically. When $\Phi=0,\pi/4,\pi/2$, we can get probe gain, but the size and variation rule of the gain and thereby Rabi frequency of the probe field are obviously different for different value of Φ. From Fig.2(d) we know that when the gain is gotten, population difference $\rho_{11}-\rho_{33}$ is always negative, i.e., inversion between the levels 1 and 3 doesn't occur, so the gain is inversionless, i.e. LWI gain. It is clear that the LWI gain and the steady value (LWI output) of the probe field corresponding to $\Phi=\pi/4$ or $\Phi=\pi/2$ are considerably larger than those corresponding to $\Phi=0$, i.e., those only SGC is considered. LWI output and the average value of LWI gain for $\Phi=\pi/4$ are larger than those for $\Phi=\pi/2$.

Now we study the case with Doppler broadening ($\Delta\omega_\mathrm{DP}=2\pi\times 80\,\mathrm{MHz}$) as shown in Fig.3. Comparing Fig.3 with Fig.2, we see that with the increase of propagation distance, variation rules of $\mathrm{Im}\rho_{31}$, G_c, G_p and $\rho_{11}-\rho_{33}$ in the case with Doppler broadening are similar to those without Doppler broadening except that LWI output and the average value of LWI gain for $\Phi=\pi/2$ are larger than those for $\Phi=\pi/4$. The most obvious difference between both cases with and without Doppler broadening is that the maxima $(\mathrm{Im}\rho_{31})_\mathrm{max}$ and $(G_\mathrm{p})_\mathrm{max}$ of LWI gain and Rabi frequency (LWI output) of the probe field when Doppler broadening presents are considerably smaller than those when Doppler broadening is absent. When Doppler broadening is absent, $(\mathrm{Im}\rho_{31})_\mathrm{max}\approx 0.03$, $(G_\mathrm{p})_\mathrm{max}\approx 2\pi\times 3\,\mathrm{MHz}$; when Doppler broadening presents, $(\mathrm{Im}\rho_{31})_\mathrm{max}\approx 0.016$, $(G_\mathrm{p})_\mathrm{max}\approx 2\pi\times 1.6\,\mathrm{MHz}$. This shows that Doppler broadening will obviously attenuate propagation-dependent LWI gain and output.

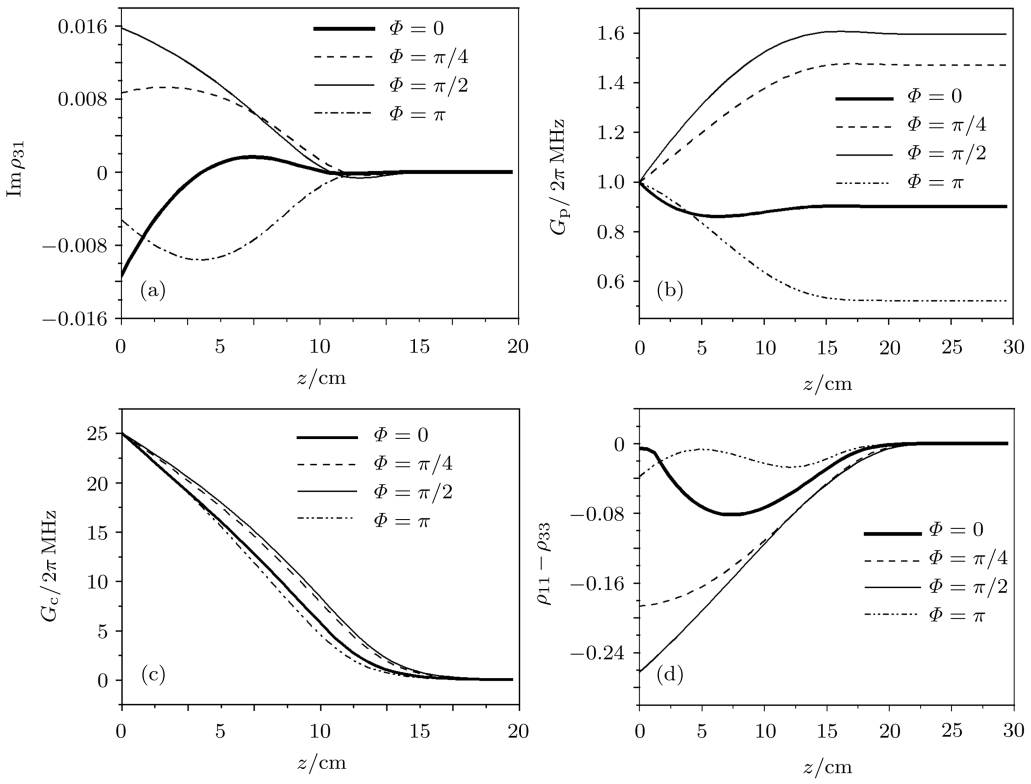


Fig.3. $\mathrm{Im}\rho_{31}$, G_p, G_c and $\rho_{11}-\rho_{33}$ as functions of propagation distance Z for different values of Φ with same parameters values as those in Fig.3 but $\Delta\omega_\mathrm{DP}=2\pi\times 80\,\mathrm{MHz}$.

Finally, it should be pointed out that it doesn't violate energy conservation that both the probe and driving fields are simultaneously attenuated when $\Phi = \pi$. This is due to that both are simultaneously scattered by the atomic medium and there isn't the energy transformation from the driving field to the probe field.

4. Conclusions

In conclusion, using the results of numerical calculation we analysed the effect of the relative phase (Φ) between the probe and driving fields on the propagation-dependent LWI gain, population difference, Rabi frequencies (strengths) of the probe and driving fields in a closed lambda-type system for both cases with and without Doppler broadening when the probe and driving fields are co-propagating. It is shown that Doppler broadening due to atomic motion and propagation effect associated with driving field depletion along the active medium decreases obviously the gain and output of the LWI. The relative phase (Φ) has a remarkable modulation role to the propagation effect on LWI. When $\Phi = \pi$, the LWI gain doesn't present and the probe field is attenuated; when $\Phi = 0$, i.e., only SGC is considered, LWI gain can be obtained, but the LWI gain is small; when $\Phi = \pi/4$ and $\Phi = \pi/2$, larger LWI gain and output can be gotten; specially, for the case with Doppler broadening when $\Phi = \pi/2$ or for the case without Doppler broadening $\Phi = \pi/4$, we can obtain the largest LWI gain and output. It is necessary to choose suitable value of the relative phase for getting the largest LWI gain and output which are much larger than those when only SGC is considered.

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