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An applicable and resource-economical scheme for the teleportation of W state via spin-path entangled quantum channel*

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In this paper, we accomplish the teleportation of an unknown three-particle maximally entangled W state by using a spin-path entangled quantum channel which may be realized experimentally based on the advanced theory and technique in Bose–Einstein condensate (BEC) of molecule, micro-fabricated wave guide and simple quantum logic gate. Similarly, we can make an arbitrary n-particle entangled Greenberger–Horne–Zeilinger (GHZ) state ($n \geq 4$) teleported through this kind of quantum channel. It may have important applications due to its resource-economic and practical features.

Keywords: teleportation, spin-path entanglement, W state, fidelity
PACC: 4250

1. Introduction

Quantum teleportation, first introduced by Bennett *et al* in 1993,[1] has aroused the interest of the physics community for many years. It holds some promise of many useful applications in quantum communication and quantum computing. Generally speaking, it consists of three steps. The first step is to prepare an entangled pair of particles, which is shared between the sender (Alice) and the receiver (Bob). The second step is to make a joint measurement by Alice of the unknown system and one particle of the entangled pair in the Bell basis. In the last step, a classical communication from Alice to Bob allows him to reconstruct the unknown state at his end following appropriate unitary transformations. This protocol is known as entanglement swapping.[2]

Based on this protocol, several kinds of schemes for the teleportation of entangled state have been presented. The teleportation of a two-particle entangled state in various conditions has been presented in Refs.[3–6]. As any three-particle entangled state can be converted by means of stochastic local operations and classical communication (SLOCC), that is to say, by means of local operations and classical communication (LOCC) without imposing that it has to be achieved with certainty,[7,8] into one of two standard forms, namely either the Greenberger–Horne–Zeilinger (GHZ) or the W state;[9] after we have accomplished the teleportation of the W or GHZ state, we can reconvert it into the initial entangled state if we wish, so it is worthy studying. Yang and Guo[10] have proposed protocols for the teleportation of a three-particle entangled state by using three Einstein–Podolsky–Rosen (EPR) pairs. Lu has presented a scheme for the teleportation of a three-particle entangled state from Alice to Bob via a GHZ triplet state and an EPR pair as quantum channels.[11] Talking about the teleportation of three-particle W state in Ref.[12], the protocol can be achieved via a two-particle entangled state and a three-particle entangled GHZ state that are used as quantum channels, and this can also be accomplished by using three pairs of two-particle entangled states as quantum channels.[13] But, the above protocol has to use so many entangled resources as channels, which is dissatisfied with our expectation. Recently, Li and Feng[14] have proposed a protocol to realize the teleportation of a three-particle entangled GHZ state by using just two pairs of EPR states as quantum channels without giving its specific form, so we need to scheme a protocol more experimental and resource-economical to over-

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come the said deficiency. In this paper, we are to accomplish the teleportation of an unknown three-particle maximally entangled W state from Alice to Bob by using a spin-path double entangled state[15] of two fermionic atoms as quantum channels, an assistant particle that belongs to Bob, together with some simple quantum logic gates and Pauli operators such as σ_z. We also find that the teleportation of unknown n-particle ($n \geq 4$) GHZ state can be realized by using the same channel and method.

2. Teleportation of three-particle entangled W state

Before the main steps of our teleportation, we should start with the discussion about the quantum channel we will use below, and it is the core of our process. The protocol to realize the spin-path entanglement has been fully studied.[15] By using this scheme, one can create a spin-path entanglement with high fidelity if the two fermionic atoms are dissociated by sweeping the magnetic field fast across Feshbach resonance[16−19] after a diatomic molecule in a spatial superposition state has been prepared by using RF induced potentials.[20,21] As for definiteness, according to Ref.[17], it can be easily implemented by using $^6Li_2$ molecules formed from two fermionic ^{6}Li, after dissociation, the two fermions will propagate along a single guide, either path a or path b, as a maximally entangled spin state, e.g. $|\psi^+\rangle_{\rm spin}$ if the molecule 50-50 BS is installed. As the two possibilities for the two atoms to decay along the two branches are indistinguishable, the fermionic atoms must be in a state, e.g.,

$$|\phi^+\rangle_{\rm path} \otimes |\psi^+\rangle_{\rm spin}, \tag{1}$$

which is maximally entangled both in path and in spin degrees of freedom.

Now suppose that Alice wants to teleport a three-particle maximally entangled W state

$$|\Psi_W\rangle_{123} = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)_{123} \tag{2}$$

to Bob, and the two other entangled states they share and take as quantum channels are

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{45} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{67}, \tag{3}$$

where particles 4, 6 are on the side of Alice while particles 5, 7 are on the side of Bob. From the above brief discussion, it is easy to know that here we need two EPR pairs, four micro-fabricated wave guides as quantum channels. For definitude, a schematic drawing of the set-up for our teleportation process is shown in Fig.1.

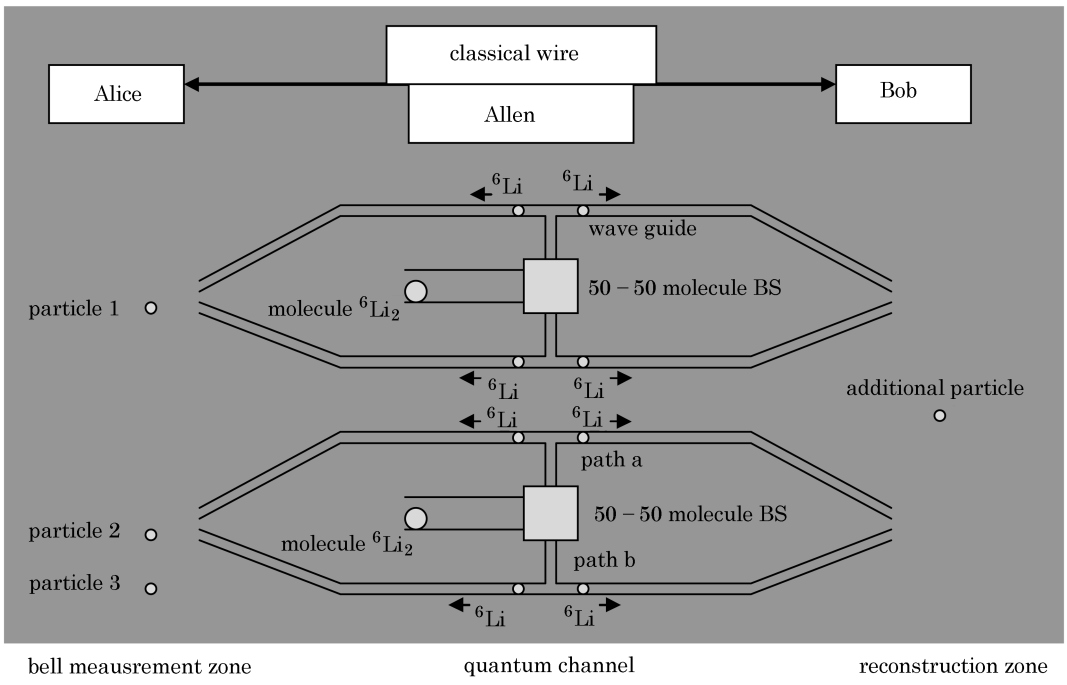


Fig.1. Schematic drawing of teleportation of an unknown three-particle maximally entangled W state.

Now we come time to our teleportation process. After the said quantum channel has been set up successfully as in Fig.1, Alice wants to make the unknown three-particle entangled W state in Eq.(2) teleported to Bob. She should first give a signal through a classical wire to a person who controls the quantum channel, such as Allen, then Allen turns on the channel. And the total initial state is

$$|\Psi\rangle_{\mathrm{I}} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{45} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{67} \otimes \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)_{123}. \quad (4)$$

Then by using the technique of entanglement swapping, Alice performs the Bell measurement on particles 1, 4 and 2, 6 separately, and the sixteen possible outcomes are given in Table 1.

Table 1. Bell measurements result of Alice and the relevant states of particles 3, 5 and 7.

bell measurement of Alice	relevant state of particles 3, 5and 7
$\|\phi^+\rangle_{26}\|\phi^\pm\rangle_{14}$	$\frac{1}{4\sqrt{3}}(\|100\rangle \pm \|001\rangle + \|010\rangle)_{357}$
$\|\phi^-\rangle_{26}\|\phi^\pm\rangle_{14}$	$\frac{1}{4\sqrt{3}}(\|100\rangle \pm \|001\rangle - \|010\rangle)_{357}$
$\|\phi^+\rangle_{26}\|\psi^\pm\rangle_{14}$	$\frac{1}{4\sqrt{3}}(\|101\rangle \pm \|000\rangle + \|011\rangle)_{357}$
$\|\phi^-\rangle_{26}\|\psi^\pm\rangle_{14}$	$\frac{1}{4\sqrt{3}}(\|101\rangle \pm \|000\rangle - \|011\rangle)_{357}$
$\|\psi^+\rangle_{26}\|\phi^\pm\rangle_{14}$	$\frac{1}{4\sqrt{3}}(\|110\rangle \pm \|011\rangle + \|000\rangle)_{357}$
$\|\psi^-\rangle_{26}\|\phi^\pm\rangle_{14}$	$\frac{1}{4\sqrt{3}}(\|110\rangle \pm \|011\rangle - \|000\rangle)_{357}$
$\|\psi^+\rangle_{26}\|\psi^\pm\rangle_{14}$	$\frac{1}{4\sqrt{3}}(\|111\rangle \pm \|010\rangle + \|001\rangle)_{357}$
$\|\psi^-\rangle_{26}\|\psi^\pm\rangle_{14}$	$\frac{1}{4\sqrt{3}}(\|111\rangle \pm \|010\rangle - \|001\rangle)_{357}$

Bell states are as follows:

$$|\phi^\pm\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2},$$
$$|\psi^\pm\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}. \quad (5)$$

After these two Bell measurements, the entanglement between the three-particle entangled W state has been converted into the entanglement between particles 3, 5 and 7. In order to teleport the unknown W state to Bob, Alice needs to carry out an Hadamard operation on particle 3, which will separate particle 3 from the entangled state. For instance, if the outcome of the two Bell measurements is

$$|\Psi\rangle_{357} = \frac{1}{4\sqrt{3}}(|110\rangle \pm |011\rangle + |000\rangle)_{357}, \quad (6)$$

the Hadamard operation will transform expression (6) into

$$\frac{1}{4\sqrt{6}}[(|10\rangle - |11\rangle + |00\rangle)_{57} \otimes |0\rangle_3 + (|00\rangle - |11\rangle - |10\rangle)_{57} \otimes |1\rangle_3]. \quad (7)$$

Alice measures particle 3. If the result is$|0\rangle_3$, particles 5 and 7 must be in the state

$$\frac{1}{4\sqrt{6}}(|10\rangle - |11\rangle + |00\rangle)_{57}, \quad (8)$$

otherwise, the state of particles 5 and 7 is

$$\frac{1}{4\sqrt{6}}(|00\rangle - |11\rangle - |10\rangle)_{57}. \quad (9)$$

Alice tells Bob all her results of the above measurement through the classical wire in Fig.1. According to the information, it is obvious that Bob needs an additional particle, particle 8, to reconstruct the original W state, here we need it in the $|1\rangle_8$ state, then Bob performs some relevant unitary operation S on particles 5 and 7 and the additional particle to obtain the teleported W state. For example, Bob can perform operations $U = (\sigma_z)_7(U_{\mathrm{CNOT}})_{75}(U_{\mathrm{CNOT}})_{58}$ and $U = (U_{\mathrm{CNOT}})_{75}(\sigma_z)_5(U_{\mathrm{CNOT}})_{58}$ to convert expressions (8) and (9) into a desired state with a probability of 1/32 respectively. The rested 31 results can be converted into a desired state alike, here we do not need to write out all of them, and the total probability is unity.

3. Influence for white noise on quantum channel

The above scheme has taken on a teleportation picture for us to use two pairs of maximally entangled EPR states as quantum channels. But as is known, our channel is influenced unavoidably by some noises from its surrounding environment in the real world, it is necessary to study the influence of ambient noise on the quantum channels.

A quantum state can be usually taken as a mixed entangled state in experiment. White noise is a very important kind of quantum noise especially in this paper, and we are to study the relationship between fidelity of the teleportation and white noise of the quantum channel.

According to Ref.[22], the two pairs of EPR quantum channels can be written as

$$\rho = \rho_1 \otimes \rho_2, \tag{10}$$

$$\rho_1 = p_1 \left|\phi^+\right\rangle_{45} \left\langle\phi^+\right| + \frac{1-p_1}{4} I_{45},$$

and

$$\rho_2 = p_2 \left|\phi^+\right\rangle_{67} \left\langle\phi^+\right| + \frac{1-p_2}{4} I_{67}, \tag{11}$$

where $|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$, $0 < p_1 \leq 1$, $0 < p_2 \leq 1$, and I is the identical operator of two-particle system. As for the teleported maximally entangled W state, the entanglement among the three particles is a constant,[23,24] here we do not need to consider it.

In stead of the maximally entangled EPR channel, now we should accomplish the scheme via the two mixed entangled states as quantum channels. The computation is very complicated, here we do not need to write out all of them. After receiving all classical information from Alice, by some relevant transformation, Bob can receive the state

$$\begin{aligned}
\rho_3 =& \frac{p_1(1+p_2)}{6} |001\rangle_{578} \langle 001| + \frac{(1+p_1)p_2}{6} |001\rangle_{578} \langle 010| \\
&+ \frac{3+p_1+p_2-p_1p_2}{12} |001\rangle_{578} \langle 100| + \frac{p_1p_2}{3} |010\rangle_{578} \langle 001| \\
&+ \frac{3+p_1-p_2+p_1p_2}{12} |010\rangle_{578} \langle 010| + \frac{(1+p_1)p_2}{6} |010\rangle \\
&+ \frac{3-p_1+p_2+p_1p_2}{12} |100\rangle_{578} \langle 001| + \frac{p_1p_2}{3} |100\rangle_{578} \langle 010| \\
&+ \frac{(1+p_2)p_1}{6} |100\rangle_{578} \langle 100| \\
&+ \frac{(1-p_2)p_1}{6} |010\rangle_{578} \langle 111| + \frac{(1-p_1)p_2}{6} |100\rangle_{578} \langle 111| \\
&+ \frac{3-p_1-p_2-p_1p_2}{12} |111\rangle_{578} \langle 111| + \frac{p_2(1-p_1)}{8} |111\rangle_{578} \langle 001| \\
&+ \frac{P_1(1-P_2)}{6} |111\rangle_{578} \langle 010| .
\end{aligned} \tag{12}$$

The fidelity of this state is

$$\begin{aligned}
F =& {}_{123}\langle_W \Psi| \rho_3 |\Psi_W\rangle_{123} \\
=& \frac{9+5p_1+5p_2+17p_1p_2}{36}.
\end{aligned} \tag{13}$$

In order to obtain more information, we have plotted a three-dimensional diagram of the computed F in Fig.2. It is convenient to find out that the value of F is enhanced from 1/4 to 1 with the increase of parameters P_1 and P_2. The minimum value of F is 1/4 if P_1=0 and P_2=0, and it comes to 1 if P_1=1 and P_2=1, that is to say, the state Bob received will be in a mixed state and the fidelity is dependent on P_1 and P_2. The teleported state is identical to the original W state when the two channels are perfect.

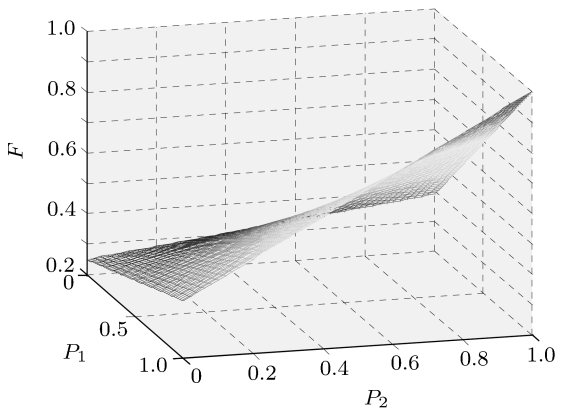


Fig.2. The variation of the fidelity in expression (12) with parameters P_1 and P_2.

4. Conclusion

In this paper we have proposed to teleport an unknown three-particle maximally entangled W state by using two specific spin–path entangled quantum channels. The existing protocols to teleport an unknown three-particle entangled state,[10−13] require at lest three pairs of two-particle entangled states or one (GHZ) triplet state and one EPR pair or one two-particle entangled state together with one there-particle entangled GHZ state or three pairs of two-particle entangled states as quantum channels. Compared with the above protocols, ours just needs two pairs of EPR states as quantum channels, which can be realized experimentally, that is to say, we have achieved a scheme that is economic in quantum-resource and applicable for practical teleportation scenario. In fact, through the investigation, we also find that the teleportation of an arbitrary three-particle entangled GHZ state can be realized easily by using a simple unitary transformation operator that converts particles on the side of Bob into the desired state after receiving all measurements from Alice. This work has been done theoretically by Li and Feng in Ref.[14]. Interestingly, we can achieve an n-particle ($n \geq 4$) entangled GHZ state teleportation through $n-1$ quantum-channel devices shown in Fig.1. As is well known, the noise of quantum channel cannot be avoided in the real world. We have also considered the relationship between fidelity of the teleported W state and the white noise of the two quantum channels. From expression (13), we find that the fidelity of the state Bob received is 1 if the two channels are perfect, and its minimum value is 1/4 when the two channels are both in the maximally mixed state. The fidelity is dependent on p_1 and p_2 and enhanced with the increase of the two parameters. This protocol may have important applications in quantum information processing, for it has practical and resource-economical features.

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