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High efficient scheme for remote state preparation with cavity QED *

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In this paper, a scheme is proposed for remote state preparation (RSP) with cavity quantum electrodynamics (QED). In our scheme, two observers share two-atom nonmaximally entangled state as quantum channels and can realize remote preparation of state of an atom. We also propose a generalization for remote preparation of N-atom entangled state by (N+1)-atom GHZ-like state ($N \geq 2$). By this scheme, one single-atom projective measurement is enough for the RSP of a qubit or N-atom entangled state, and the probability of success for RSP is unity. Furthermore, we have considered the case where observers use W-like state as quantum channels to realize RSP of a qubit. We compare our scheme with existing ones.

Keywords: remote state preparation, cavity QED, entangled state
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1. Introduction

Entangled states play an important role in quantum information processing.[1−5] By applying entangled quantum channels, remote state preparation (RSP) was first addressed by Pati[6] and Lo[7] who studied special ensembles of states. In their scheme, two observers [a sender (Alice) and a receiver (Bob)] share a two-particle entangled state $\left|\Psi^{-}\right\rangle_{12} = (|0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2)/\sqrt{2}$ as quantum channels. Alice wants to indirectly transmit an arbitrary qubit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ to Bob, where Alice has classical knowledge of this state. To remotely prepare $|\psi\rangle$ in Bob's place, Alice takes one of $|\Psi^{-}\rangle_{12}$ and measures it on the basis $\{|\psi\rangle, |\psi_\perp\rangle\}$, where $|\psi_\perp\rangle$ stands for the orthogonal state to $|\psi\rangle$. If Alice's measurement result is $|\psi_\perp\rangle$, she sends her result to Bob through a classical channel, then Bob knows his particle is in the state $|\psi\rangle$. If the result is $|\psi\rangle$, Bob obtains the state $|\psi_\perp\rangle$. This protocol has some similarities to teleportation, so RSP is called teleportation of known state, which means that Alice knows classically what state is to be transmitted. But compared with teleportation, the protocol of RSP shows that classical communication cost is one bit per qubit, half that of teleportation. Furthermore, Alice does not possess a physical instance of the state to be transmitted in RSP, that is, RSP needs less resources than teleportation. Recently, Bennett and his co-workers[8] explored the tradeoff between entanglement and classical communication required for RSP and discussed RSP capacities of general quantum channels. Low entanglement RSP[9] and oblivious RSP[10] were also investigated. Based on the protocol of RSP introduced by Pati,[6] schemes of RSP of an EPR state and entangled state of many particles were proposed.[11−15] An experimental implementation of RSP scheme has also been reported with NMR.[16]

RSP shows that the receiver Bob possibly obtains the state $|\psi_\perp\rangle$ with certain probability, where the state $|\psi_\perp\rangle$ is orthogonal to the state $|\psi\rangle$. For some special ensembles of states, e.g., equatorial states, Bob can transform $|\psi_\perp\rangle$ into $|\psi\rangle$ by applying rotation operations. However, in general, nonequatorial states, the corrective transformation $|\psi_\perp\rangle \rightarrow |\psi\rangle$ is antiunitary, and Bob cannot obtain the $|\psi\rangle$, that is, RSP fails. So we may think the probability of success for RSP is smaller than teleportation whose probability of success is unity. In this paper, we will give a scheme of RSP where Alice can remotely transmit an arbitrary qubit with a probability of unity. Our scheme is based on the strong coupling system of atoms and

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cavity field.

The paper is organized as follows. In Section 2, we discuss the RSP of an atom state and the RSP of N-atom entangled state ($N \geq 2$). In Section 3, we use W-like state as quantum channels, and investigate the RSP of an atom state. In Section 4, we compare our scheme with existing schemes and briefly discuss the experimental feasibility of our scheme.

2. RSP of an atom state and RSP of N-atom entangled state

We first consider the RSP of an atom state (a qubit). Assuming that the state $|\psi\rangle$ of an atom to be remotely prepared can be expressed as

$$|\psi\rangle = \alpha\,|g\rangle + \beta\,|e\rangle\,, \tag{1}$$

where $|\alpha|^2+|\beta|^2 = 1$, we choose α to be a real number and β a complex number. In Bloch sphere representation, α and β can be expressed as $\alpha = \cos(\theta/2)$ and $\beta = \sin(\theta/2)\exp(\mathrm{i}\varphi)$. According to the protocol of RSP,[6] Alice knows the classical description about the state $|\psi\rangle$. To transmit the state $|\psi\rangle$, Alice and Bob need to share an entangled state as quantum channels. Here we can assume that Alice is also a provider of entangled resources, that is, quantum channels will be prepared by Alice. Because Alice knows the two parameters α and β, she can produce an entangled state $|\Psi\rangle_{12}$ of two atoms

$$|\Psi\rangle_{12} = \beta\,|e\rangle_1\,|e\rangle_2 + \alpha\,|g\rangle_1\,|g\rangle_2\,, \tag{2}$$

which is regarded as quantum channels. Now Alice should introduce the preparation of the state $|\Psi\rangle_{12}$. Here two two-level atoms are chosen, whose states can be expressed as $|g\rangle$ (the ground state), and $|e\rangle$ (the excited state). When the atoms are one by one injected into the cavity field, resonance of atom-field occurs between $|g\rangle$ and $|e\rangle$. So the interaction[17,18] between atom and field can be regarded as the Jaynes–Cummings model (JCM). In the interaction picture, the atom–field interaction is described by the JCM Hamiltonian

$$H_{\mathrm{I}} = \varepsilon(a^+S_- + a^-S_+), \tag{3}$$

where a^+ and a^- are the creation and annihilation operators for the cavity mode, respectively, ε is the atom-field coupling strength, and $S_- = |g\rangle\langle e|$ and $S_+ = |e\rangle\langle g|$ are the lowering and raising operators describing the transition between $|g\rangle$ and $|e\rangle$. Supposing that atom 1 initially prepared in the ground state $|g\rangle_1$, Alice sends this atom through the cavity field, initially in the number state $|1\rangle_{\mathrm{c}}$. The atom-field wave functions after an interaction time t_{int} read[19]

$$\begin{aligned}&|g\rangle_1\,|1\rangle_{\mathrm{c}} \to |\Psi\rangle_{1,\mathrm{c}}\\ =&\cos(\Omega_0 t_{\mathrm{int}}/2)\,|g\rangle_1\,|1\rangle_{\mathrm{c}}\\ &-\sin(\Omega_0 t_{\mathrm{int}}/2)\,|e\rangle_1\,|0\rangle_{\mathrm{c}}\,,\end{aligned} \tag{4}$$

where Ω_0 is the Rabi frequency.

Next, Alice sends atom 2 in initial state $|g\rangle_2$ to cross the cavity field. Atom 2 is tuned to interact with the cavity field and undergoes a π pulse. So the whole system evolves to

$$\begin{aligned}&|\Psi\rangle_{1,\mathrm{c}}\,|g\rangle_2 \to |\Psi\rangle_{12,\mathrm{c}}\\ =&[\cos(\Omega_0 t_{\mathrm{int}}/2)\,|g\rangle_1\,|e\rangle_2\\ &+\sin(\Omega_0 t_{\mathrm{int}}/2)\,|e\rangle_1\,|g\rangle_2]\,|0\rangle_{\mathrm{c}}\,.\end{aligned} \tag{5}$$

From Eq.(5) we find that the cavity field gets out of the vacuum state after the interaction of atom-field, so we can neglect the effect of dissipation of cavity field on the state of the two atoms.

Now Alice lets atom 2 cross a microwave field S_{R} of frequency ν_{R} close to resonance with the $|g\rangle \to |e\rangle$ transition. In the interaction picture, the state of atom 2 $|\psi(t)\rangle_2$ satisfies the following equation:[20]

$$\frac{\partial}{\partial t}\,|\psi(t)\rangle_2 = -\frac{\mathrm{i}}{\hbar}V(t)\,|\psi(t)\rangle_2\,. \tag{6}$$

A formal solution of Eq.(6) is

$$|\psi(t)\rangle_2 = U_I(t)\,|\psi(0)\rangle_2\,, \tag{7}$$

where

$$\begin{aligned}U_{\mathrm{I}}(t) =&\cos\left(\frac{\Omega_{\mathrm{R}}t}{2}\right)(|a\rangle\langle a| + |b\rangle\langle b|)\\ &+\mathrm{i}\sin\left(\frac{\Omega_{\mathrm{R}}t}{2}\right)(\mathrm{e}^{-\mathrm{i}\phi}\,|a\rangle\langle b|\\ &+\mathrm{e}^{\mathrm{i}\phi}\,|b\rangle\langle a|).\end{aligned} \tag{8}$$

In Eq.(8), Ω_{R} is Rabi frequency and ϕ is the phase of dipole matrix element.

From Eqs.(7) and (8), if we choose $\Omega_{\mathrm{R}}t = \pi$ and $|\psi(0)\rangle_2 = |g\rangle_2$ or $|e\rangle_2$, we can obtain

$$\begin{aligned}&|g\rangle_2 \to \mathrm{i}\mathrm{e}^{-\mathrm{i}\phi}\,|e\rangle_2\,,\\ &|e\rangle_2 \to \mathrm{i}\mathrm{e}^{\mathrm{i}\phi}\,|g\rangle_2\,.\end{aligned} \tag{9}$$

Then the state of atoms 1 and 2 can be written as

$$\begin{aligned}|\Psi\rangle_{12} =&\sin(\Omega_0 t_{\mathrm{int}}/2)\mathrm{e}^{-\mathrm{i}2\phi}\,|e\rangle_1\,|e\rangle_2\\ &+\cos(\Omega_0 t_{\mathrm{int}}/2)\,|g\rangle_1\,|g\rangle_2\,,\end{aligned} \tag{10}$$

where the common phase factor $\exp(\mathrm{i}(\phi+\pi/2))$ is discarded. We may adjust the atom–cavity interaction time to the required value corresponding to $\cos(\Omega_0 t_{\mathrm{int}}/2)=\alpha=\cos(\theta/2)$ and $\sin(\Omega_0 t_{\mathrm{int}}/2)=\sin(\theta/2)$, and the atom-microwave interaction time to the required value corresponding to $\exp(-\mathrm{i}2\phi)=\exp(\mathrm{i}\varphi)$, then Eq.(10) can be rewritten as

$$|\Psi\rangle_{12}=\beta|e\rangle_1|e\rangle_2+\alpha|g\rangle_1|g\rangle_2. \tag{11}$$

So Alice prepares a two-atom entangled state. She and Bob share $|\Psi\rangle_{12}$ as quantum channels. We assume atom 1 is for Alice and atom 2 for Bob. In order to realize RSP of $|\psi\rangle$, Alice lets atom 1 cross a classical field and undergo the transition

$$\begin{aligned}|e\rangle_1 &\xrightarrow{\pi/2 \text{ classical pulse}} \frac{1}{\sqrt{2}}(|g\rangle_1+|e\rangle_1),\\ |g\rangle_1 &\xrightarrow{\pi/2 \text{ classical pulse}} \frac{1}{\sqrt{2}}(|g\rangle_1-|e\rangle_1).\end{aligned} \tag{12}$$

According to Eqs.(12), we rewrite Eq.(11) as

$$\begin{aligned}|\Psi\rangle_{12}=&\frac{1}{\sqrt{2}}|e\rangle_1(\beta|e\rangle_2-\alpha|g\rangle_2)\\ &+\frac{1}{\sqrt{2}}|g\rangle_1(\alpha|g\rangle_2+\beta|e\rangle_2).\end{aligned} \tag{13}$$

From Eq.(13), if Alice has result $|g\rangle_1$ and sends this to Bob through classical channel, Bob knows that atom 2 is in the state $|\psi\rangle$. If the result is $|e\rangle_1$ plus classical information from Alice, Bob only needs to apply an appropriate rotation operation on his atom, and atom 2 finally reaches the input state $|\psi\rangle$. So RSP is successfully realized with unity probability.

According to our protocol of RSP, we can remotely prepare N-atom entangled state $|\psi\rangle_{12\ldots N}$

$$\begin{aligned}|\psi\rangle_{12\ldots N}=&\alpha|g\rangle_1|g\rangle_2\ldots|g\rangle_N\\ &+\beta|e\rangle_1|e\rangle_2\ldots|e\rangle_N,\end{aligned} \tag{14}$$

where $N\geq 2$.

To do this, Alice and Bob use $(N+1)$-atom GHZ-like state as quantum channels. Applying cavity QED technique, Alice first prepares the GHZ-like state $|\Psi\rangle_{12\ldots N(N+1)}$,

$$\begin{aligned}|\Psi\rangle_{12\ldots N(N+1)}=&\beta|e\rangle_1|e\rangle_2\ldots|e\rangle_N|e\rangle_{N+1}\\ &-\alpha|g\rangle_1|g\rangle_2\ldots|g\rangle_N|g\rangle_{N+1}\\ \Rightarrow&|\Psi\rangle_{12\ldots N(N+1)}\\ =&\frac{1}{\sqrt{2}}|e\rangle_1(\beta|e\rangle_2|e\rangle_3\ldots|e\rangle_N|e\rangle_{N+1}\\ &-\alpha|g\rangle_2|g\rangle_3\ldots|g\rangle_N|g\rangle_{N+1})\\ &+\frac{1}{\sqrt{2}}|g\rangle_1(\alpha|g\rangle_2|g\rangle_3\ldots|g\rangle_N|g\rangle_{N+1}\\ &+\beta|e\rangle_2|e\rangle_3\ldots|e\rangle_N|e\rangle_{N+1}),\end{aligned} \tag{15}$$

where Alice possesses atom 1 and Bob possesses atoms 2, 3, …, N, $N+1$. Next, Alice needs to detect atom 1 in state $|e\rangle_1$, or $|g\rangle_1$, and then sends her result of measurement to Bob through classical channel. With the help of Alice, Bob can recover $|\psi\rangle_{12\ldots N}$ by applying the appropriate quantum rotation operations or do nothing on his atoms. By choosing suitable quantum channels, the process of RSP of the state $|\psi\rangle_{12\ldots N}$ is deterministic.

3. RSP of a qubit by W-like state

W-like[21] state can be expressed as

$$|\Psi\rangle=a|001\rangle+b|010\rangle+c|100\rangle, \tag{16}$$

where $|a|^2+|b|^2+|c|^2=1$, and its properties and application have been reported in Ref.(21). By cavity QED, the scheme of preparation of W-like state was proposed.[22] Here we assume W-like state can be expressed as

$$\begin{aligned}|W\rangle_{12,\mathrm{c}}=&\frac{\alpha}{\sqrt{2}}|g\rangle_1|g\rangle_2|1\rangle_{\mathrm{c}}+\frac{\beta}{\sqrt{2}}|g\rangle_1|i\rangle_2|0\rangle_{\mathrm{c}}\\ &+\frac{1}{\sqrt{2}}|e\rangle_1|g\rangle_2|0\rangle_{\mathrm{c}},\end{aligned} \tag{17}$$

where $|\alpha|^2+|\beta|^2=1$, and W-like state includes atoms 1, 2 and cavity field. The two possible states of atom 1 are $|g\rangle$ and $|e\rangle$, while the states of atom 2 are $|g\rangle$ and $|i\rangle$, and cavity field has two possible states $|0\rangle$ and $|1\rangle$.

Suppose Alice, Bob and Charlie share this W-like state. Atom 1 is for Charlie, atom 2 is for Bob, and Alice has the cavity field. The state that Alice wants to transmit to Bob is $|\psi\rangle=\alpha|g\rangle+\beta|i\rangle$. In order to realize RSP of $|\psi\rangle$, Charlie firstly needs to detect his atom on the basis $\{|g\rangle_1, |e\rangle_1\}$, and then sends his result to Alice and Bob through classical channels. From Eq.(17), if Charlie's result is $|e\rangle_1$, Bob cannot recover the $|\psi\rangle$ from his atom. If Charlie's result is $|g\rangle_1$, then the state of atom 2 and cavity field evolves to

$$|\Psi\rangle_{2,\mathrm{c}}=\alpha|g\rangle_2|1\rangle_{\mathrm{c}}+\beta|i\rangle_2|0\rangle_{\mathrm{c}}. \tag{18}$$

Next, Alice performs measurement on cavity field on the basis $\left\{\frac{1}{\sqrt{2}}(|0\rangle_{\mathrm{c}}+|1\rangle_{\mathrm{c}}), \frac{1}{\sqrt{2}}(|0\rangle_{\mathrm{c}}-|1\rangle_{\mathrm{c}})\right\}$. If Alice has $\frac{1}{\sqrt{2}}(|0\rangle_{\mathrm{c}}+|1\rangle_{\mathrm{c}})$ as her result, and sends it to Bob, then Bob knows that atom 2 is in the state $|\psi\rangle$. If the measurement result is $\frac{1}{\sqrt{2}}(|0\rangle_{\mathrm{c}}-|1\rangle_{\mathrm{c}})$ plus classical information from Alice and Charlie, Bob can recover the $|\psi\rangle$ from atom 2 by performing appropriate unitary operation on his atom. The probability of success for RSP by using W-like state is 1/2.

4. Conclusion

In summary, we have proposed a scheme for RSP. Our scheme is based on present cavity QED technique,[23,24] and our aim is to search for an appropriate physical system for RSP. Two different cases of RSP are discussed: In the first case, Alice and Bob share two-atom nonmaximally entangled state to transmit a qubit, and share $(N+1)$-atom GHZ-like state to remotely prepare N-atom entangled state. Our studies show that the probability of success for RSP is unity. Present reports[11−14] show RSP is probabilistic, because Bob possible obtains the orthogonal state $|\psi_\perp\rangle$ of the input state $|\psi\rangle$. For the general state, $|\psi_\perp\rangle$ is not transformed into $|\psi\rangle$ by unitary operation. But in our scheme, by selecting suitable quantum channels, the process of RSP is deterministic. Furthermore, in our scheme, single-atom detections and single qubit gates are enough for RSP of a qubit, or N-atom entangled state. So the implementation of our scheme is easier than existing schemes.[14] In the second case, we consider the RSP of a qubit by using W-like state. Studies show the probability of success is 1/2, which is bigger than 1/3 reported by Liu[15] who used W state as quantum channels to realize RSP of a qubit. Compared with previous schemes, our process of RSP is more efficient and easier to perform. Finally, we have briefly discussed the feasibility of our scheme. In cavity QED, the heart of this system is a microwave photon trap, composed of high-Q superconducting cavity, which stores a few photon field in a small volume of space for duration as long as milliseconds. For Rydberg atoms injected one by one into the cavity, the current experiment shows that these atoms combine a huge dipole coupling to a single photon with lifetime $(3\times10^{-2}\,\mathrm{s})$ that is far larger than the cavity crossing time $(2\times10^{-2}\,\mathrm{s})$, that is, coupling to the environment is weak enough so that coherent atom-field interaction overwhelms dissipative processes.[19] Therefore, based on present available cavity QED techniques, our scheme may be experimentally feasible.

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