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Transverse shift of the transmitted beam of a non-uniformly polarized paraxial light beam at dielectric interfaces*

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This paper reports that the central position of the reflected and transmitted beams of a nonlinear polarized light beam at the interface between two media undergoes transverse shifts. It presents a solution to the problem of transverse shift of a non-uniformly polarized paraxial light beam transmitting through interfaces between two homogeneous media by using a two-form amplitude and an extension matrix to represent the vector angular spectrum of a three-dimensional (3D) light beam. It derives general formula for the transverse shift of the transmitted beam, and discusses the shift of a well-collimated beam transmitting through an interface between two homogeneous media and a thin dielectric slab.

Keywords: transverse shift, paraxial beam, polarization, transmitted beam
PACC: 4225G, 4225J

1. Introduction

It has been already known that the totally reflected light beam undergoes shift of position in the plane of incidence (longitudinal) or normal to the plane of incidence (transverse). The longitudinal shift is well known as the Goos–Hänchen (GH) effect,[1−4] and the transverse shift includes the polarization-dependent transverse Imbert–Fedorov (IF) shift in the reflection or refraction of the beam on a sharp boundary,[4−8] and the splitting of rays of different polarizations in a smoothly inhomogeneous medium.[5,6,9−11]

As we know, there are several papers about the transverse shift in partial reflection and refraction, but their theoretical predictions and issues remain still controversial,[5,12,13] to the authors' knowledge, the shift of the transmitted beam at an interface between two homogeneous media has not been experimentally measured, and the transverse shift of a beam transmitting through a thin dielectric slab has not been discussed. Recently, Li has advanced an unified theory for the GH and IF effects by representing the vector angular spectrum of a 3D light beam in terms of a two-form angular spectrum consisting of its two orthogonal polarizations, and from this theory, the quantization characteristics of the IF effect are obtained such that the eigenstates of the IF effect are the two orthogonal circular polarizations.[4] The purpose of this paper is to discuss the transverse shift of a non-uniformly polarized paraxial light beam, such as paraxial Gaussian beam, transmitting through interfaces between two homogeneous media by using a two-form amplitude and an extension matrix to represent the vector angular spectrum of a 3D light beam. The two-form amplitude describes the polarization state of the angular spectrum, while the extension matrix describes the polarization feature of the vector angular spectrum of the beam. We derive general formula for the transverse shift of the transmitted beam of a non-uniformly polarized paraxial beam. It shows that the shift is related to the polarization state and the polarization feature of the angular spectrum, the transmission coefficients for s- and p-polarizations. We also calculate the transverse shift of the transmitted beam of a well-collimated beam transmitting through an interface between two homogeneous media and a thin dielectric slab.

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2. General theory

Consider a 3D light beam propagating parallel to the x axis in a homogeneous and isotropic medium, the vector electric field of the beam is expressed in terms of its vector angular spectrum as follows,[4]

$$E(r) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A(k_y,k_z)\exp(\mathrm{i}k\cdot r)\mathrm{d}k_y\mathrm{d}k_z, \quad (1)$$

where time dependence $\exp(-\mathrm{i}\omega t)$ is implied. $A = (A_xA_yA_z)^{\mathrm{T}}$ is the vector amplitude of the angular spectrum, $k = (k_xk_yk_z)^{\mathrm{T}}$ is the wave vector satisfying $k_x^2 + k_y^2 + k_z^2 = k^2$, $k = 2\pi/\lambda$, λ is the wavelength in the medium, superscript T means transpose. The integration limits $\pm\infty$ for both variables k_y and k_z means that the beam is supposed to satisfy paraxial approximation, and will be omitted hereafter. When this beam intersects the plane $x = 0$, the position coordinate of the centroid of the beam in the transverse direction is defined as[4]

$$\langle z\rangle = \frac{\iint \Psi^\dagger z\,\Psi\mathrm{d}y\mathrm{d}z}{\iint \Psi^\dagger\,\Psi\mathrm{d}y\mathrm{d}z} = \frac{\iint A^\dagger \mathrm{i}\dfrac{\partial A}{\partial k_z}\mathrm{d}k_y\mathrm{d}k_z}{\iint A^\dagger A\mathrm{d}k_y\mathrm{d}k_z}, \quad (2)$$

where $\Psi = E(r)\,|_{x=0}$ is the electric field distribution on the plane $x = 0$, $\partial/\partial k_z$ means partial derivative with respect to k_z when k_y is fixed, and superscript $\dagger$ stands for transpose conjugate.

Denote $A = A_{\mathrm{s}} + A_{\mathrm{p}} = A_{\mathrm{s}}\tilde{s} + A_{\mathrm{p}}\tilde{p}$,[14] where A_{s} and A_{p} are the complex amplitudes of the s- and p-polarization states, $\tilde{s}$ and $\tilde{p}$ are their respective unit vectors, $\tilde{s}$ and $\tilde{p}$ and the wave vector k are mutually orthogonal. The vector angular spectrum A can be expressed as

$$A = \begin{pmatrix} s_x & p_x \\ s_y & p_y \\ s_z & p_z \end{pmatrix}\tilde{A} \equiv P\tilde{A}, \quad (3)$$

where $\tilde{A} = \begin{pmatrix} A_s \\ A_p \end{pmatrix}$ is the two-form amplitude of the angular spectrum and P is the extension matrix describing the extension of the two-form amplitude $\tilde{A}$ to the 3D Cartesian coordinate.[14] We consider a paraxial beam with principal wave vector $(k\cos\theta_0, k\sin\theta_0, 0)^{\mathrm{T}}$ rotated by an angle around the x axis, θ_0 is the angle between the propagation axis and the x axis. It is found that the extension matrix is symmetric with respect to the x axis that the polarization distribution of the angular spectrum is symmetric with respect to x axis, and the beam is non-uniformly polarized since the s- and p-polarizations of different plane-wave elements are generally in different directions.[4] The extension matrix elements can be solved as

$$\begin{aligned} s_y &= -\frac{k_z}{(k_y^2+k_z^2)^{1/2}}, \\ s_z &= \frac{k_y}{(k_y^2+k_z^2)^{1/2}}, \\ p_x &= -\frac{(k_y^2+k_z^2)^{1/2}}{k}, \\ p_y &= \frac{k_xk_y}{k(k_y^2+k_z^2)^{1/2}}, \\ p_z &= \frac{k_xk_z}{k(k_y^2+k_z^2)^{1/2}}. \end{aligned} \quad (4)$$

From Eqs.(3) and (4), we obtain the following expressions:[4]

$$A^\dagger A = \tilde{A}^\dagger\tilde{A}, \quad (5)$$

$$A^\dagger\frac{\partial A}{\partial k_z} = \tilde{A}^\dagger\frac{\partial\tilde{A}}{\partial k_z} + \frac{k_xk_y}{k(k_y^2+k_z^2)}(A_s^*A_p - A_sA_p^*). \quad (6)$$

Substituting Eqs.(5) and (6) into Eq.(2), we can obtain the transverse shift of the centroid of a polarized paraxial light beam reflecting and transmitting in the transverse direction on the plane $x = 0$.

3. The transverse shift of the transmitted beam at an interface between two media

Consider a paraxial light beam propagating from a homogeneous medium with refractive index n_1 to another medium with refractive index n_2. Without loss of generality, we consider an arbitrarily polarized incident beam of the following two-form amplitude:

$$\tilde{A}_i = \begin{pmatrix} l_{i1} \\ l_{i2} \end{pmatrix} A = \tilde{L}_iA, \quad (7)$$

where $\tilde{L}_i = \begin{pmatrix} l_{i1} \\ l_{i2} \end{pmatrix}$ describes the polarization state of the angular spectrum of the beam and is assumed to satisfy the normalization condition $\tilde{L}_i^\dagger\tilde{L}_i = 1$. The angular distribution function A is assumed to be a positively-definite sharply-peaked symmetric function

around the principal axis (k_{y0}, k_{z0}) and satisfies the normalization condition

$$\iint A^2(k_y, k_z)\mathrm{d}k_y\mathrm{d}k_z = 1. \tag{8}$$

For simplicity, we consider the following Gaussian distribution function:[4]

$$A = \left(\frac{w_y w_z}{\pi}\right)^{1/2} \exp\left[-\frac{w_y^2}{2}(k_y - k_{y0})^2\right] \times \exp\left(-\frac{w_z^2}{2}k_z^2\right), \tag{9}$$

where $w_y = w_0/\cos\theta_0$, $w_z = w_0$ and w_0 is half the width of the beam at waist. Angular distribution function (9) indicates that the axis of the beam, represented by its principal axis $(k_{y0}, k_{z0}) = (k\sin\theta_0, 0)$, is perpendicular to the z axis. $\Delta\theta = 1/k\omega_0$ is half the divergence angle of the beam, and $\Delta\theta \ll 1$ means the beam satisfying paraxial approximation.

When the beam is reflected and transmitted at the interface between two homogeneous media, the angular spectrum of the transmitted beam can be expressed as

$$\tilde{A}_t = T\tilde{A}_i = \tilde{L}_t A, \tag{10}$$

where $T = \begin{pmatrix} T_\mathrm{s} & 0 \\ 0 & T_\mathrm{p} \end{pmatrix}$ is the transmission coefficient matrix, $\tilde{L}_t = T\tilde{L}_i = \begin{pmatrix} l_{t1} \\ l_{t2} \end{pmatrix}$ describes the polarization state of the transmitted beam, T_s and T_p are the transmission coefficients for the s- and p-polarizations, respectively. Considering Eqs.(4), (7) and (9) together, we conclude that θ_0 is the angle between the symmetry axis of the extension matrix and the beam axis. Different values of θ_0 correspond to different beams with different vector properties and intensity distributions.[14]

Applying Eqs.(2), (5), (6), (7) and (10), we obtain the z coordinate of the centroid of the incident and the transmitted beams as follows:

$$\langle z\rangle_i = \iint \frac{2i\mathrm{Im}m}{1+|m|^2}\frac{k_x k_y}{k(k_y^2+k_z^2)}A^2\mathrm{d}k_y\mathrm{d}k_z, \tag{11}$$

$$\langle z\rangle_t = \frac{\iint 2\mathrm{i}\rho\mathrm{Im}m\dfrac{k_{tx}k_{ty}}{k_t(k_{ty}^2+k_{tz}^2)}A^2\mathrm{d}k_y\mathrm{d}k_z}{\iint(1+\rho^2|m|^2)A^2\mathrm{d}k_y\mathrm{d}k_z}, \tag{12}$$

where $m = l_{i1}/l_{i2}$ is the parameter to characterize polarization of each wave, and $2\mathrm{Im}m/(1+|m|^2)$ is the polarization ellipticity, $\mathrm{Im}m = 0$ is for linear polarizations, $m = i$ for right circular polarizations and $m = -i$ for left circular polarizations, respectively. $\rho = T_s/T_p$, and

$$T_s = \frac{2\cos\theta}{\cos\theta + \sqrt{n_{21}^2 - \sin^2\theta}}, \tag{13}$$

$$T_p = \frac{2n_{21}\cos\theta}{n_{21}^2\cos\theta + \sqrt{n_{21}^2 - \sin^2\theta}}, \tag{14}$$

where θ is the incident angle, and $n_{21} = n_2/n_1$ is the relative refractive index.

The translation invariance along tangent direction brings the conservation of the respective wave vector, k_{jy} =const., k_{jz} = const., $j = i, t$, here i and t stand for the incident and transmitted wave separately.[7] The Snell's law brings $k_t\sin\theta_t = k\sin\theta$, and takes the geometrical optics problem into account, $k_x = k\cos\theta$, $k_{tx} = k_t\cos\theta_t$, where θ_t is the refraction angle. From Eqs.(11) and (12), we obtain the transverse shift of the centroid of the transmitted beam as

$$\begin{aligned} z_c &= \langle z\rangle_t - \langle z\rangle_i \\ &= \frac{\iint 2\mathrm{i}\rho_1\mathrm{Im}m\dfrac{\cos\theta_t}{k\sin\theta}A^2\mathrm{d}k_y\mathrm{d}k_z}{\iint(1+\rho_1^2|m|^2)A^2\mathrm{d}k_y\mathrm{d}k_z} \\ &\quad - \iint\frac{2i\mathrm{Im}m}{1+|m|^2}\frac{\cos\theta}{k\sin\theta}A^2\mathrm{d}k_y\mathrm{d}k_z. \end{aligned} \tag{15}$$

This is the general formula for the transverse shift of the transmitted beam. It is easily found that the transverse shift is related to the polarization state and the polarization feature of the angular spectrum, the transmission coefficients for s- and p-polarizations, the angle of incidence and the angular distribution function. For simplicity, we just discuss a well-collimated beam as an example. In this case, the integrands in Eq.(15) can be regarded as constants and thus can be taken out of the integral with k_y, k_z, θ evaluated at k_{y0}, k_{z0} and θ_0,[4] hence we get

$$z_c = \frac{\mathrm{i}\lambda\mathrm{Im}m}{\pi\tan\theta_0}\left(\frac{\rho\cos\theta_{t0}}{(1+\rho^2|m|^2)\cos\theta_0} - \frac{1}{1+|m|^2}\right), \tag{16}$$

where $\cos\theta_{t0} = \sqrt{1-\sin^2\theta_0/n_{21}^2}$. From Eq.(16), we find that:

(1) The transmitted beam will never undergo transverse shift in the follow conditions: (i) the incident beam is linearly polarized ($\mathrm{Im}m = 0$); (ii) an

arbitrary polarization beam propagates in a homogeneous and isotropic medium ($n_{21} = 1$). These are consistent with the geometrical optics.

(2) The transmitted beam undergoes transverse shift for nonlinear polarization beams (Im$m \neq 0$) in the case of $n_{21} \neq 1$, the maximum shift is for circular polarizations (Im$m = \pm i$), the transverse shift changes with the incident angle and the refractive indices of the two media. Figure 1 shows the transverse shift of the transmitted beam of a left circular polarization changing with the incident angle from $0 \sim \pi/2$ in the case of (a) $n_1 = 1$, $n_2 = 2$ and (b) $n_1 = 1$, $n_2 = 1.506$. While Fig.2 shows the transverse shift of the transmitted beam of a left circular polarization changing with the incident angle $0 \sim \theta_c$ in the case of (a) $n_1 = 2$, $n_2 = 1$ and (b) $n_1 = 1.506$, $n_2 = 1$, here θ_c is the critical angle for total reflection. The sign of the shift is reversed for the right circular polarization, this is to say, the shift is same in magnitude but opposite in direction for left and right circular polarizations, respectively. Furthermore, when $n_1 > n_2$, the shift of a circular polarization changes with the increasing difference of the refractive index between the two media, and increases with the increase of the incident angle, its maximum approaches the order of the wavelength λ that could be experimentally measured.

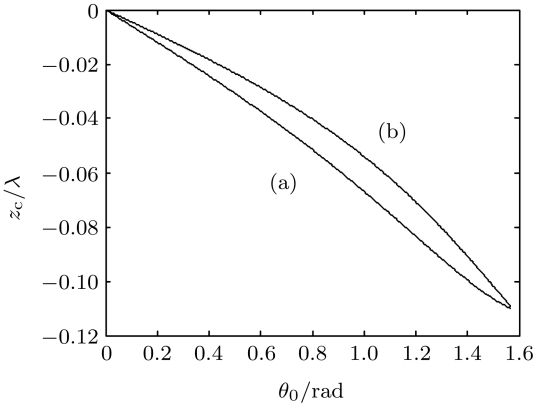


Fig.1. The transverse shift of the transmitted beam of a left circular polarization ($m = -i$) changing with the incident angle and the refractive indices (a) $n_1 = 1$, $n_2 = 2$ and (b) $n_1 = 1$, $n_2 = 1.506$.

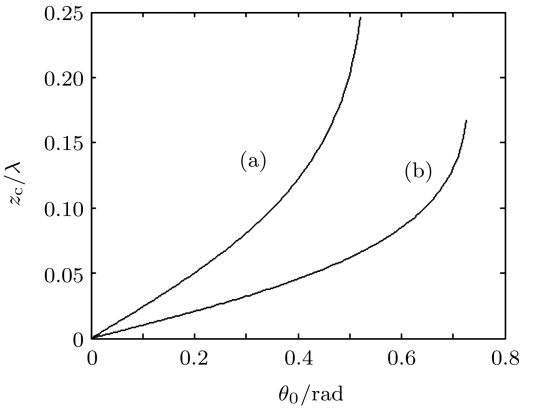


Fig.2. The transverse shift of the transmitted beam of a left circular polarization ($m = -i$) changing with the incident angle and the refractive indices (a) $n_1 = 2$, $n_2 = 1$ and (b) $n_1 = 1.506$, $n_2 = 1$.

4. The transverse shift of a paraxial beam transmitting through a thin dielectric slab

Consider a paraxial light beam transmitting through a thin dielectric slab (the thickness is much bigger than the wavelength of the beam) with refractive index n_2 placed in a medium with refractive index n_1. The beam is incident on the upper interface and transmits out from the lower interface when it undergoes two times refractions, the angular spectrum of the transmitted beam can be expressed as

$$\tilde{A}_t = T_1 T_2 \tilde{A}_i, \tag{17}$$

where $T_1 = \begin{pmatrix} T_{s1} & 0 \\ 0 & T_{p1} \end{pmatrix}$ is the transmission matrix at the upper interface, T_{s1} and T_{p1} are the same as Eqs.(13) and (14). $T_2 = \begin{pmatrix} T_{s2} & 0 \\ 0 & T_{p2} \end{pmatrix}$ is the transmission matrix at the lower interface, and

$$T_{s2} = \frac{2\sqrt{n_{21}^2 - \sin^2\theta}}{\cos\theta + \sqrt{n_{21}^2 - \sin^2\theta}}, \tag{18}$$

$$T_{p2} = \frac{2n_{21}\sqrt{n_{21}^2 - \sin^2\theta}}{n_{21}^2\cos\theta + \sqrt{n_{21}^2 - \sin^2\theta}}. \tag{19}$$

We obtain the shift of the transmitted beam as

$$z_c = \frac{\iint \frac{2i\rho_1\rho_2 \mathrm{Im}m}{k\tan\theta} A^2 \mathrm{d}k_y \mathrm{d}k_z}{\iint (1 + \rho_1^2\rho_2^2 |m|^2) A^2 \mathrm{d}k_y \mathrm{d}k_z} - \iint \frac{2i\mathrm{Im}m}{(1 + |m|^2)k\tan\theta} A^2 \mathrm{d}k_y \mathrm{d}k_z, \tag{20}$$

where $\rho_1 = T_{s1}/T_{p1}$, $\rho_2 = T_{s2}/T_{p2}$. For a well-collimated beam, Eq.(20) can be reduced as

$$z_c = \frac{i\lambda \mathrm{Im}m}{\pi\tan\theta_0} \left(\frac{\rho_1\rho_2}{(1 + \rho_1^2\rho_2^2 |m|^2)} - \frac{1}{1 + |m|^2} \right). \tag{21}$$

From Eq.(21), we find that a linearly polarized beam transmitting through a thin dielectric slab never undergoes transverse shift. But, for a nonlinear polarization beam, when $n_{21} \neq 1$, the transverse shift of the transmitted beam is also related to the polarization state and the polarization feature of the angular spectrum, the transmission coefficients for s- and p-polarizations, and the incident angle. Figure 3 shows the transverse shift of a left circular polarization beam transmitting through a thin dielectric slab with $n_2 = 1.506$, and $n_1 = 1$ is considered. It is found

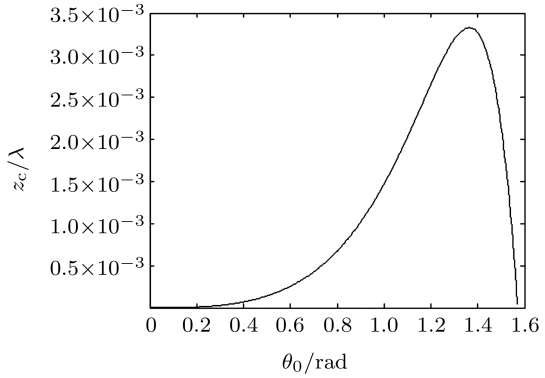


Fig.3. The transverse shift of a left circular polarization ($m = -i$) transmitting through a thin medium slab with refractive index $n_2 = 1.506$.

that the shift is different from the shift of a beam transmitting through an interface between two media, and is much smaller than the wavelength which can not be experimentally measured.

5. Conclusions

We solve the problem of the transverse shift of a non-uniformly polarized paraxial beam transmitting through an interface between two homogeneous media by using a two-form amplitude and an extension matrix to represent the vector angular spectrum of a 3D light beam. We derive general formula for the transverse shift of the transmitted beam at the interface between two media, it shows that the shift is dependent on the polarization state of the beam and is quantized with eigenstates of the opposite circular polarizations. This is in fact the translational inertial spin effect happening on the interface of two homogeneous media. For a well-collimated beam, the results are consistent with the theoretical results in Refs.[4] and [12], and the maximum shift approaches the order of the wavelength of the beam that could be experimentally measured. However, the transverse shift of a polarized paraxial beam transmitting through a thin dielectric slab is too small in comparison with the wavelength of the beam to be experimentally measured. The transverse shift of an arbitrary polarized paraxial beam transmitting through multilayer media and the method enhancing the transverse shift of the transmitted beam need further investigations.

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