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Controlling the amplitude of soliton in a growing Bose–Einstein condensate by means of Feshbach resonance*

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By using Darboux transformation, this paper studies analytically the nonlinear dynamics of a one-dimensional growing Bose–Einstein condensate (BEC). It is shown that the growing model has an important effect on the amplitude of the soliton in the condensates. In the absence of the growing model, there exhibits the stable alternate bright solitons in the condensates. In the presence of the growing model, the obtained results show that the amplitude of the bright soliton decreases (increases) for the BEC growing coefficient $\Omega < 0$ ($\Omega > 0$). Furthermore, we propose experimental protocols to manipulate the amplitude of the bright soliton by varying the scattering length via the Feshbach resonance in the future experiment.

Keywords: Feshbach resonance, Bose–Einstein condensate, bright soliton
PACC: 2340, 0530L, 1110L

1. Introduction

Since the bright and dark solitons in the Bose–Einstein condensates (BECs) have been observed experimentally, it has received extensive attention in the field of nonlinear matter waves.[1−8] An intriguing topic in this field is that the s-wave scattering length (SL) $a_{\rm s}$ (representing interatomic interactions) can be adjusted by using Feshbach resonance (FR).[9] Experimentally, the strength of $a_{\rm s}$ can be adjusted in a considerable region. When the SL is switched from a positive to a negative number, the BECs abruptly loss atoms.[10] Meanwhile, the growing modes of some atoms, such as ^{87}Rb,[11] ^{7}Li,[12] and ^{23}Na,[13] have been realized by means of FR. Theoretically, the corresponding Gross-Pitaevskii (GP) equation is

$$\begin{aligned} \mathrm{i}\hbar\Psi_t = &-\frac{\hbar^2}{2m}\Psi_{xx} + \frac{4\pi\hbar^2 a_{\rm s}}{m}|\Psi|^2\Psi \\ &+ V_{\rm trap}\Psi + \mathrm{i}\Omega\Psi, \end{aligned} \tag{1}$$

where $V_{\rm trap}$ is external potential, $a_{\rm s}$ is the time-dependent SL, m is the atomic mass, and the coefficient Ω is the gain (loss) term for $\Omega > 0$ ($\Omega < 0$). For $\Omega > 0$, the GP equation represents the mechanism of loading the thermal cloud into the BEC by optically pumping atoms from the external source. When Ω is negative, the GP equation describes a BEC which is continuously depleted due to losing atoms.

To date, the investigation on the growing BEC is one of hot topics of matter waves. For example, under the consideration of the growing model, Drummond *et al*[14] predicted that the centre of mass oscillates violently in the trap, which results in enhanced centre of mass temperature. Atre *et al*[15] revealed that solitons undergo rapid amplification, collapse, and revival of condensates in the presence of the growing model. Furthermore, the model of the growing BEC is of special importance in the field of an atom laser. Thus, we here study the nonlinear dynamics of a one-dimensional (1D) growing BEC. Meanwhile, we consider how an external time-dependent periodic potential influences the nonlinear dynamics of a growing BEC with the interatomic attractive interactions. The rest of this paper is organized as follows. In section 2, by using Darboux transformation we obtain the exact solutions of the 1D GP equation with gain or loss.

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Subsequently, we study how to control the amplitude of the bright soliton by FR in section 3. The last section is a brief summary.

2. The exact solutions of the 1D growing GP equation

Based on above ideas, we may set that the time-dependent external potential in Eq.(1) is $V_{\rm trap} = 2\cos^2(\omega_\perp T/2)$. Then, we introduce some dimensionless variables $x = \sqrt{2}X/a_\perp$, $t = \omega_\perp T$, $\psi = \sqrt{2\pi a_{\rm B}}a_\perp \Psi$, $v = V_{\rm trap}/(\hbar\omega_\perp)$, and $\Gamma = \Omega/(\hbar\omega_\perp)$. Here $\omega_\perp$ is the transverse oscillator frequency, $a_\perp = \sqrt{\hbar/(m\omega_\perp)}$ is the transverse oscillator length, and $a_{\rm B}$ is the Bohr radius. Substituting them into Eq.(1), we obtain a dimensionless nonlinear Schrödinger equation

$$\begin{aligned} {\rm i}\psi_t =& -\psi_{xx} + 2\frac{a_{\rm s}}{a_{\rm B}}|\psi|^2\psi \\ &+ 2\cos^2(t/2)\psi + {\rm i}\Gamma\psi, \end{aligned} \tag{2}$$

where $\Gamma = k$, and the time-dependent SL can be achieved due to the FR[9]

$$a_{\rm s}(t) = a_0\left(1 - \frac{\Delta}{B(t) - B}\right). \tag{3}$$

Here a_0 is the SL far from the resonance, $B(t)$ is the time-dependent external magnetic field, B and Δ are the resonance position and width, respectively. In real experiments, various forms of the time dependence of $B(t)$ have been explored.[16,17] We here assume that the external magnetic field is exponentially ramped down with the time. So, the corresponding variation relation between the SL and the time is $a_{\rm s} = -a_{\rm B}\exp(-2kt)$.

Subsequently, by using the Darboux transformation,[18–22] the so-called seed solution of Eq.(2) can be chosen as follows:

$$\psi_0 = \exp[{\rm i}\sqrt{2}x + kt - {\rm i}(\sin t + t)]. \tag{4}$$

In order to obtain an exact solution of Eq.(2) by using the seed solution Eq.(4), we first introduce the following UV pair function:

$$\Phi_x = U\Phi, \quad \Phi_t = V\Phi, \quad \Phi = (\phi_1, \phi_2)^{\rm T}. \tag{5}$$

Here $U = \begin{pmatrix} \lambda & p \\ q & -\lambda \end{pmatrix}$, $V = \begin{pmatrix} A & B \\ C & -A \end{pmatrix}$, with $p = \sqrt{g}\psi\exp({\rm i}\sin t + {\rm i}t)$, $A = 2{\rm i}\lambda^2 + {\rm i}g|\psi|^2$, $B = 2{\rm i}\sqrt{g}\lambda\psi\exp[{\rm i}(\sin t + t)] + {\rm i}\sqrt{g}\psi_x\exp[{\rm i}(\sin t + t)]$, and $C = -2{\rm i}\sqrt{g}\lambda\overline{\psi}\exp[-{\rm i}(\sin t + t)] + {\rm i}\sqrt{g}\overline{\psi_x}\exp[-{\rm i}(\sin t + t)]$. Here λ is a complex constant, $g = \exp(-2kt)$, and $q = -\overline{p}$. From the compatibility condition $\partial^2\Phi/\partial x\partial t = \partial^2\Phi/\partial t\partial x$, we have $U_t - V_x + [U, V] = 0$. Secondly, we perform the Darboux transformation

$$\begin{aligned} \psi =& \psi_0 + 2\left(\lambda + \overline{\lambda}\right)\frac{\overline{\sigma}}{1 + |\sigma|^2} \\ &\times \exp[kt - {\rm i}(\sin t + t)], \end{aligned} \tag{6}$$

where ψ_0 is the seed solution Eq.(4) and $\sigma = \phi_2/\phi_1$. Consequently we obtain the following exact solution

$$\begin{aligned} \psi =& \left[-1 + 2\frac{(a^2-1)\cos\varphi + {\rm i}a\sqrt{a^2-1}\sin\varphi}{a^2\cosh\theta + a\sqrt{a^2-1}\sinh\theta - \cos\varphi}\right] \\ &\times \exp[{\rm i}\sqrt{2}x + kt - {\rm i}(\sin t + t)], \end{aligned} \tag{7}$$

where $\theta = 2\sqrt{a^2-1}(x - 2\sqrt{2}t)$, $\varphi = 4a\sqrt{a^2-1}t$, and a is an arbitrary real constant. Equation (7) represents a soliton embedded in the background.[18] For the sake of convenience, we choose $a = 4.0$ in the following calculation.

3. Controlling of the amplitude of the bright soliton

Based on the experiment of ^{23}Na atoms,[10] the atomic mass $m = 3.82 \times 10^{-26}$ kg, the transverse oscillator frequency $\omega_\perp = 2\pi \times 100$ Hz, so the time and space units correspond to 1.6 ms and 1.49 μm in reality, respectively. And the FR position is at 853G and 907G (1G=10^{-4} T).[10] The SL far from the resonance has been found spectroscopically to be $52a_{\rm B}$ at zero field, and increases to the triplet SL of $85a_{\rm B}$ at high magnetic fields.[23] We may choose the FR position B=907G, the off-resonant SL $a_0 = 70a_{\rm B}$, the resonance width Δ=1G and the external magnetic field $B(t) = 907 + \Delta/\left\{1 + 1/[70\exp(2kt)]\right\}$.

In the absence of the growing model, i.e., $k = 0$, the corresponding SL is $a_{\rm s} = -a_{\rm B}$ [see Eq.(3)]. The density distribution of the condensates at different time is shown in Fig.1. When $t = 0$ (see the solid line), a sharp peak can be seen. Obviously, it can be regarded as a bright soliton. Comparing with the case at $t = 0.05$ (see the dashed line), we find that the amplitude of the bright soliton varies small and its width changes large. However, the bright soliton at $t = 0.1$ (see the dotted line) has the same amplitude and width as that at $t = 0$. Similar phenomena can also be found among the other same time intervals. This behaviour implies the stable alternate bright solitons appearing in the condensates. Such a dynamic

stability in bright soliton is attributed mainly to the constant attractive interactions.

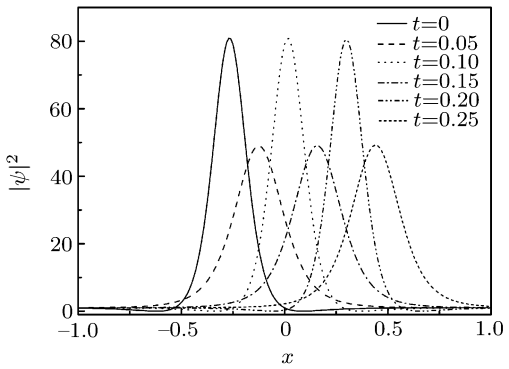


Fig.1. In the absence of the growing model, the density distribution of the condensates with different time.

In the presence of the growing model, we first choose $k = -2$ as a typical example of the parameter k being negative number. The variation of the external magnetic field in this case is shown in Fig.2. We see that the external magnetic field varies from 907.98G to 907.13G, which approaches the FR position, after the time passing 1.5, i.e. ≈2.4 ms in reality. The corresponding SL is plotted in inserted picture of Fig.2. We find that the absolute value of SL increases with increasing time. It means that the interatomic interactions become stronger. In this case, the density distribution of the condensates with different time is depicted in Fig.3. One can find that a bright soliton appears in the condensates at $t = 0$ (the solid line). When the time is going on, the amplitude of the bright soliton decreases exponentially. It illustrates that the condensates loss atoms when the external magnetic field approaches the FR position. Similar results can be found when the parameter k is other negative number. The validity of GP equation relies on the condition that the system is dilute and weakly interacting: $n \mid a_{\rm s} \mid^3 \ll 1$, where n is the average density of the condensate. In the real experiment of ^{23}Na atoms,[10] the atomic densities $n \approx 10^{15}\,{\rm cm}^{-3}$. And in Fig.2, when T=2.4 ms, the absolute value of SL $|a_{\rm s}| = 400a_{\rm B}$, so $n|a_{\rm s}|^3 < 10^{-2} \ll 1$. This means that the stability of soliton and the validity of 1D approximation are maintained. Thereby, we conclude that the experimental protocol is: firstly, a condensate containing a bright soliton approaches an external magnetic field strength 907.98G; then, let the magnetic field varies from 907.98G to 907.13G, which approaches the FR position 907G; after the time passing 2.4 ms, one may observe that the amplitude of the bright soliton decreases exponentially.

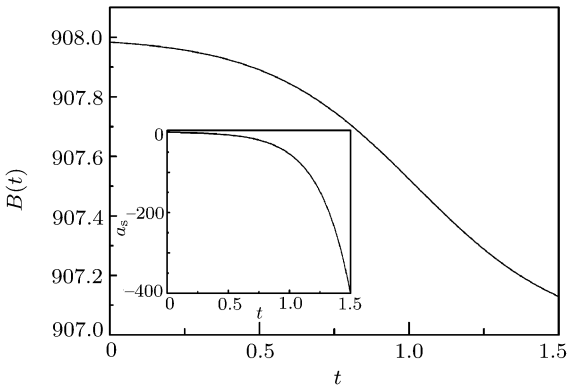


Fig.2. The external magnetic field approaches the Feshbach resonance position. The units of the magnetic field and SL are G and $a_{\rm B}$, respectively.

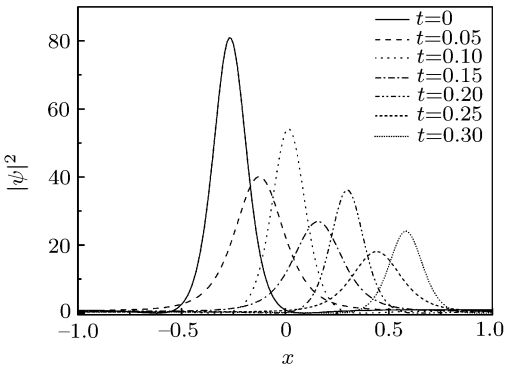


Fig.3. The density distribution of the condensates with different time for the growing coefficient $k = -2$.

Then, we consider $k = 2$ as an example of the parameter k being positive number. Similarly, the variation relation between the external magnetic field and time is plotted in Fig.4. We see that the external magnetic field deviates from the FR position when the time is going on. And when the time $t > 0.3$, i.e., $T > 0.48$ ms in reality, the external magnetic field approaches to a constant 908.000G, so we can only change the external magnetic field from 907.984G to 908.000G. The corresponding SL is depicted in inserted picture of Fig.4. One sees that the absolute value of SL decreases with increasing time. In such a condition, the density distribution of the condensates with different time is displayed in Fig.5. When the time is going on, the amplitude of the bright soliton increases exponentially. This indicates that BEC continuously replenished by pumping from the external reservoir, which should be affected by the change of attractive interatomic interactions.[12] When the external magnetic field varies from 907.984G to 908.00G,

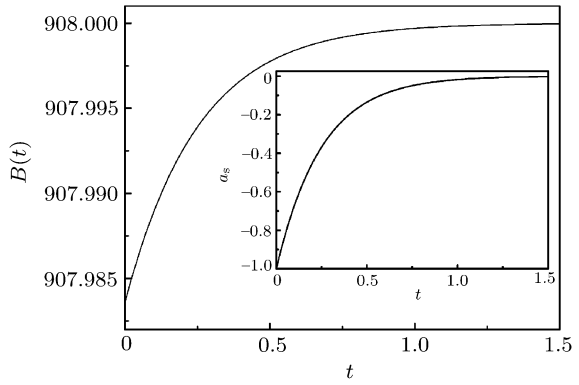


Fig.4. The external magnetic field deviates from the Feshbach resonance position. These units used are the same as Fig.2.

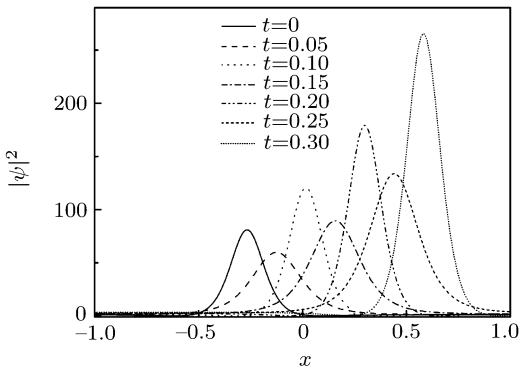


Fig.5. The density distribution of the condensates with the different time for the growing coefficient $k = 2$.

the absolute value of SL $|a_s|_{\max} = a_B$, which is also valid for ^{23}Na condensate. Thus, the stability of soliton and the validity of 1D approximation are maintained from 907.984G to 908.00G. The phenomena in Fig.5 can be observed under the current experimental condition.

4. Conclusion

In summary, by using Darboux transformation we obtain the exact solutions of the 1D GP equation with gain or loss. In the absence of the growing model, the stable alternate bright soliton can be found, whose main contribution comes from the constant attractive interactions. For the growing coefficient $\Omega < 0$, such as $k = -2$, the amplitude of the bright soliton decreases exponentially. This means that the BEC losses atoms when the external magnetic field approaches the FR position. While the growing coefficient $\Omega > 0$, (i.e., $k = 2$), the amplitude of the bright soliton increases exponentially. This indicates that BEC continuously replenished by pumping from the external reservoir when the external magnetic field deviates from the FR position. Furthermore, these phenomena can be realized by varying the scattering length via the FR in the future experiment.

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