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Time-domain analytic solutions of two-wire transmission line excited by a plane-wave field*

Ni Gu-Yan(倪谷炎)[a)†], Yan Li(颜 力)[b)], and Yuan Nai-Chang(袁乃昌)[c)]

[a)] *College of Science, National University of Defense Technology, Changsha* 410073, *China*
[b)] *College of Aerospace and Material Engineering, National University of Defense Technology, Changsha* 410073, *China*
[c)] *College of Electronic Science and Engineering, National University of Defense Technology, Changsha* 410073, *China*

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This paper reports that an analytic method is used to calculate the load responses of the two-wire transmission line excited by a plane-wave directly in the time domain. By the frequency-domain Baum–Liu–Tesche (BLT) equation, the time-domain analytic solutions are obtained and expressed in an infinite geometric series. Moreover, it is shown that there exist only finite nonzero terms in the infinite geometric series if the time variate is at a finite interval. In other word, the time-domain analytic solutions are expanded in a finite geometric series indeed if the time variate is at a finite interval. The computed results are subsequently compared with transient responses obtained by using the frequency-domain BLT equation via a fast Fourier transform, and the agreement is excellent.

Keywords: transmission line, analytic solution, BLT equation, coupling formulation.
PACC: 1220, 4110

1. Introduction

Transmission-line models, which are approximations to a rigorous electromagnetic analysis, are frequently used in performing engineering studies for electrical systems.[1−6] In order to analyse large multiconductor cable harnesses in aircraft, the Baum–Liu–Tesche (BLT) equation was obtained,[1,7,8] which is a frequency domain matrix equation describing the behaviour of voltages and currents at all of the junctions (interconnections) of the multiconductor cables in the network.

Recently, an extension of the BLT equation concept was developed to include the effects of electromagnetic (EM) field propagation in space (along trajectories that can be thought of as being an imaginary field propagation 'tube'). In Ref.[9], the possibility of nonplane wave propagation was modelled, and the coupling of these EM fields to a transmission line was taken into account. Moreover, the re-radiation of EM fields from the transmission-line current was included in the model. With these effects included, an extended BLT equation was formulated, with the observables being a combination of both transmissionline voltages and E-field responses. A numerical study of this extended BLT equation was reported in Refs.[9] and [10]. The original BLT equation was formulated in the frequency domain and relied on Fourier transform methods to obtain transient responses.[7−10]

In some cases, however, a direct time-domain solution may be preferred. Baum[11] has described the possibility of casting the BLT equation into an equivalent time-domain form in which the inverse of the "resonance" matrix is expanded in a geometric series, and the solution is expressed in terms of convolution operations. Based on the time-domain BLT equation, Tesche[4,5] developed the transient BLT solution to the special case of a single transmission-line section to calculate the currents and voltages at the terminating loads of the line directly in the time domain by evaluating several convolution integrals.

In the early days, based on the transmission line approximation, three different coupling formulations, i.e. Taylor, Agrawal and Rachidi coupling formulations, have been proposed in the literature for evaluating voltages induced by an external EM field on a transmission line.[12−14]

Especially, for the line-plane transmission line,

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†Corresponding author. E-mail: guyan-ni@163.com

Nucci and Xie used a numerical method showing that the total induced voltages predicted by these three coupling formulations are identical.[15,16] On the other hand, for two-wire transmission line excited by a plane-wave, applying the frequency domain BLT equation, Ni *et al* obtained the frequency-domain analytic solutions of the load voltages and currents of the line with Taylor and Agrawal coupling formulations, and showed that two analytic solutions are identical indeed.[17] In this paper, we will discuss the time domain analytic solutions of the load total voltages and currents of the two-wire transmission line excited by a plane-wave field, and compare them with the corresponding transient responses obtained from the frequency domain solutions via a fast Fourier transform (FFT).

2. The frequency-domain BLT equation for two-wire transmission line

The BLT equations provide a compact expression for the load current and voltage, and can be used for studying the behaviour of the terminal responses for many different types of excitation fields.[1]

Without losing generality, let us consider the case of a lossy two-wire line (see Fig.1). The two-wire line is illuminated by an incident EM field. Denote $\mathcal{L}$ the length of the line, Z_1 and Z_2 the load impedances at $x=0$ and $x=\mathcal{L}$ ends, respectively, and Z_c the characteristic impedance of the line. Let $\rho_i = \dfrac{Z_i - Z_c}{Z_i + Z_c}$, for $i = 1, 2$. Then, at the end of the line, the induced voltages and currents can be expressed in matrix form as

$$\begin{pmatrix} I(0) \\ I(\mathcal{L}) \end{pmatrix} = \frac{1}{Z_c} \begin{pmatrix} 1-\rho_1 & 0 \\ 0 & 1-\rho_2 \end{pmatrix} \times \begin{pmatrix} -\rho_1 & \mathrm{e}^{\gamma\mathcal{L}} \\ \mathrm{e}^{\gamma\mathcal{L}} & -\rho_2 \end{pmatrix}^{-1} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}, \tag{1}$$

$$\begin{pmatrix} V(0) \\ V(\mathcal{L}) \end{pmatrix} = \begin{pmatrix} 1+\rho_1 & 0 \\ 0 & 1+\rho_2 \end{pmatrix} \times \begin{pmatrix} -\rho_1 & \mathrm{e}^{\gamma\mathcal{L}} \\ \mathrm{e}^{\gamma\mathcal{L}} & -\rho_2 \end{pmatrix}^{-1} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}, \tag{2}$$

where $(S_1, S_2)^{\mathrm{T}}$ denotes the source vector, which is different for different coupling formulations.

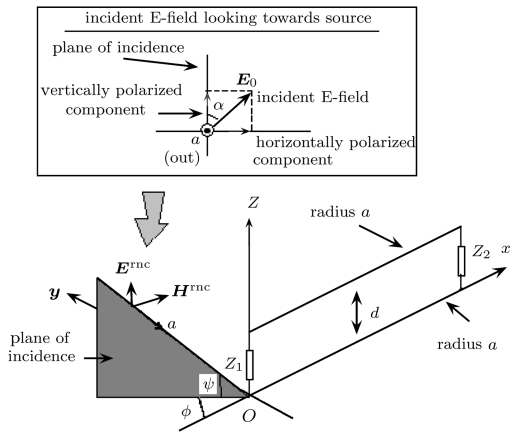


Fig.1. Isolated two-wire line excited by an incident plane wave.

The boundary conditions are

$$V(0) = -Z_1 I(0), \quad V(\mathcal{L}) = Z_2 I(\mathcal{L}).$$

Especially, for a plane wave excitation, as shown in Fig.1, this field is described by angles of incidence ψ and ϕ, as well as a polarization angle α, which defines the E-field vector direction relative to the vertical plane of incidence. Denote the magnitude of the incident E-field by E_0. Ni *et al* obtained the source vector as follows:[17]

$$\begin{aligned} S_1 = {} & \frac{[\mathrm{e}^{(\gamma - jk\cos\psi\cos\phi)\mathcal{L}} - 1]E_0}{2(\gamma - jk\cos\psi\cos\phi)} \\ & \times [jkd(\cos\alpha\cos\phi + \sin\alpha\sin\psi\sin\phi) \\ & - YZ_c\mathrm{d}\cos\alpha\cos\psi], \end{aligned} \tag{3}$$

$$\begin{aligned} S_2 = {} & \frac{[\mathrm{e}^{(-\gamma\mathcal{L} - jk\mathcal{L}\cos\psi\cos\phi)} - 1]\mathrm{e}^{\gamma\mathcal{L}}E_0}{2(\gamma + jk\cos\psi\cos\phi)} \\ & \times [jkd(\cos\alpha\cos\phi + \sin\alpha\sin\psi\sin\phi) \\ & + YZ_c\mathrm{d}\cos\alpha\cos\psi]. \end{aligned} \tag{4}$$

By Eqs.(1)–(4), it is easy to deduce analytic solution of the load response frequency-domain approximation for plane-wave excitation.

3. Time-domain analytic solutions of the load voltages

In this section, we will discuss transient response for the plane wave excitation and derive time-domain analytic solutions of the load induced voltages and currents for the two-wire line.

Substituting $\gamma = j\omega/v$ into the BLT equation (2), we easily get

$$V(0) = \frac{(1+\rho_1)\mathrm{e}^{-2j\omega\mathcal{L}/v}}{1-\rho_1\rho_2\mathrm{e}^{-2j\omega\mathcal{L}/v}}(\rho_2 S_1 + \mathrm{e}^{j\omega\mathcal{L}/v}S_2), \quad (5)$$

$$V(\mathcal{L}) = \frac{(1+\rho_2)\mathrm{e}^{-2j\omega\mathcal{L}/v}}{1-\rho_1\rho_2\mathrm{e}^{-2j\omega\mathcal{L}/v}}(\rho_1 S_2 + \mathrm{e}^{j\omega\mathcal{L}/v}S_1). \quad (6)$$

Let

$$A_1 = \frac{d(\cos\alpha\cos\phi + \sin\alpha\sin\psi\sin\phi - \cos\alpha\cos\psi)}{2-2\cos\psi\cos\phi},$$

$$A_2 = \frac{d(\cos\alpha\cos\phi + \sin\alpha\sin\psi\sin\phi + \cos\alpha\cos\psi)}{2+2\cos\psi\cos\phi}.$$

Substituting $\gamma = j\omega/v$ and $k = \omega\sqrt{\mu\epsilon}$ into Eqs.(3) and (4), we have

$$\begin{aligned} S_1 &= -A_1E_0(1-\mathrm{e}^{j\omega(1/v-\sqrt{\mu\epsilon}\cos\psi\cos\phi)\mathcal{L}}),\\ S_2 &= -A_2E_0(1-\mathrm{e}^{-j\omega(1/v+\sqrt{\mu\epsilon}\cos\psi\cos\phi)\mathcal{L}})\mathrm{e}^{j\omega\mathcal{L}/v}. \end{aligned}$$

Hence, by Eqs.(5) and (6), we obtain the following equations for the frequency ω:

$$\begin{aligned} V(0) &= \frac{(1+\rho_1)E_0(\omega)}{1-\rho_1\rho_2\mathrm{e}^{-2j\omega\mathcal{L}/v}}\Big[-\rho_2A_1(\mathrm{e}^{-2j\omega\mathcal{L}/v} - \mathrm{e}^{-j\omega(1/v+\sqrt{\mu\epsilon}\cos\psi\cos\phi)\mathcal{L}}) - A_2(\mathrm{e}^{-j\omega\mathcal{L}/v}\\ &\quad -\mathrm{e}^{-j\omega(2/v+\sqrt{\mu\epsilon}\cos\psi\cos\phi)\mathcal{L}})\mathrm{e}^{j\omega\mathcal{L}/v}\Big], \quad (7)\\ V(\mathcal{L}) &= \frac{(1+\rho_2)E_0(\omega)}{1-\rho_1\rho_2\mathrm{e}^{-2j\omega\mathcal{L}/v}}\Big[-\rho_1A_2(\mathrm{e}^{-2j\omega\mathcal{L}/v} - \mathrm{e}^{-j\omega(3/v+\sqrt{\mu\epsilon}\cos\psi\cos\phi)\mathcal{L}})\mathrm{e}^{j\omega\mathcal{L}/v}\\ &\quad -A_1(\mathrm{e}^{-j\omega\mathcal{L}/v} - \mathrm{e}^{-j\omega\sqrt{\mu\epsilon}\cos\psi\cos\phi\mathcal{L}})\Big]. \quad (8) \end{aligned}$$

Since $|\rho_1\rho_2\mathrm{e}^{-2j\omega\mathcal{L}/v}| < 1$, we can rewrite the fraction as

$$\frac{1}{1-\rho_1\rho_2\mathrm{e}^{-2j\omega\mathcal{L}/v}} = \sum_{n=0}^{\infty}(\rho_1\rho_2\mathrm{e}^{-2j\omega\mathcal{L}/v})^n.$$

Substituting the above formula into Eqs.(7) and (8), we have

$$\begin{aligned} V(0) &= (1+\rho_1)E_0(\omega)\times\sum_{n=0}^{\infty}(\rho_1\rho_2)^n\Big[-\rho_2A_1(\mathrm{e}^{-2(n+1)j\omega\mathcal{L}/v} - \mathrm{e}^{-j\omega((2n+1)/v+\sqrt{\mu\epsilon}\cos\psi\cos\phi)\mathcal{L}})\\ &\quad -A_2(\mathrm{e}^{-j2n\omega\mathcal{L}/v} - \mathrm{e}^{-j\omega((2n+1)/v+\sqrt{\mu\epsilon}\cos\psi\cos\phi)\mathcal{L}})\Big], \quad (9)\\ V(\mathcal{L}) &= (1+\rho_2)E_0(\omega)\times\sum_{n=0}^{\infty}(\rho_1\rho_2)^n\Big[-\rho_1A_2(\mathrm{e}^{-(2n+1)j\omega\mathcal{L}/v} - \mathrm{e}^{-j\omega((2n+2)/v+\sqrt{\mu\epsilon}\cos\psi\cos\phi)\mathcal{L}})\\ &\quad -A_1(\mathrm{e}^{-j(2n+1)\omega\mathcal{L}/v} - \mathrm{e}^{-j\omega(2n/v+\sqrt{\mu\epsilon}\cos\psi\cos\phi)\mathcal{L}})\Big]. \quad (10) \end{aligned}$$

Suppose that a function $F(\omega)$ is the Fourier transform of a function $f(t)$ and τ is a constant. Then, the inverse Fourier transform of $\mathrm{e}^{-j\omega\tau}F(\omega)$ is $f(t-\tau)$. So we can deduce the time-domain solutions of the load response by Eqs.(9) and (10). Using $\sqrt{\mu\epsilon} = 1/v$, we get the time-domain analytic solution of the load induced voltages for the two-wire line as following:

$$\begin{aligned} v(0,t) &= (1+\rho_1)\times\sum_{n=0}^{\infty}(\rho_1\rho_2)^n\Big[-\rho_2A_1(E_0(t-2(n+1)\mathcal{L}/v) - E_0(t-(2n+1+\cos\psi\cos\phi)\mathcal{L}/v))\\ &\quad -A_2(E_0(t-2n\mathcal{L}/v) - E_0(t-(2n+1+\cos\psi\cos\phi)\mathcal{L}/v))\Big], \quad (11)\\ v(\mathcal{L},t) &= (1+\rho_2)\times\sum_{n=0}^{\infty}(\rho_1\rho_2)^n\Big[-\rho_1A_2(E_0(t-(2n+1)\mathcal{L}/v) - E_0(t-(2n+2+\cos\psi\cos\phi)\mathcal{L}/v))\\ &\quad -A_1(E_0(t-(2n+1)\mathcal{L}/v) - E_0(t-(2n+\cos\psi\cos\phi)\mathcal{L}/v))\Big]. \quad (12) \end{aligned}$$

By the terminate condition $i(0,t) = u(0,t)/Z_1$ and $i(\mathcal{L},t) = u(\mathcal{L},t)/Z_2$, it is easy to obtain time-domain analytic solution of the load induced currents.

4. Application to the time-domain solutions

In order to show how to apply the time-domain analytic solution, and check on its validity, a numerical example has been considered by using the geometry of Fig.1. The following parameters, used in Ref.[1], have been chosen to define a baseline geometry for this comparison:

line length $\mathcal{L} = 30\,\mathrm{m}$,

wire separation $d = 0.2\,\mathrm{m}$,

wire radius $a = 0.0015\mathrm{m}$,

characteristicimpedance $Z_c = 586\,\Omega$,

load resistance $Z_1 = Z_2 = 293\,\Omega$,

incident angles $\phi = 0,\ \psi = \pi/3$,

polarization angle $\alpha = 0$,

incident E − field : double exponential waveform

$$E(t) = \begin{cases} 1.05 \times (\mathrm{e}^{-4\times10^6 t} - \mathrm{e}^{-4.76\times10^8 t}), & t \geq 0, \\ 0, & t < 0. \end{cases}$$

Then $A_1 = A_2 = 0.1$, $\rho_1 = \rho_2 = -1/3$, $\mathcal{L}/v = 10^{-7}(\mathrm{sec})$. By Eq.(12), the induced voltage $v(l,t)$ at $x = \mathcal{L}$ is as follows:

$$\begin{aligned} v(\mathcal{L},t) = {} & \frac{2}{3}\sum_{n=0}^{\infty} 9^{-n}\left\{\frac{1}{30}\left[E(t-(2n+1)\times10^{-7}) - E\left(t-\left(2n+\frac{5}{2}\right)\times10^{-7}\right)\right]\right. \\ & \left.-\frac{1}{10}\left[E(t-(2n+1)\times10^{-7}) - E\left(t-\left(2n+\frac{1}{2}\right)\times10^{-7}\right)\right]\right\} \\ = {} & \frac{2}{3}\left\{\frac{1}{30}\left[E(t-10^{-7}) - E\left(t-\frac{5}{2}\times10^{-7}\right)\right] - \frac{1}{10}\left[E(t-10^{-7}) - E\left(t-\frac{1}{2}\times10^{-7}\right)\right]\right\} \\ & +\frac{2}{27}\left\{\frac{1}{30}\left[E(t-3\times10^{-7}) - E\left(t-\frac{9}{2}\times10^{-7}\right)\right]\right. \\ & \left.-\frac{1}{10}\left[E(t-3\times10^{-7}) - E\left(t-\frac{5}{2}\times10^{-7}\right)\right]\right\} + \ldots \end{aligned} \tag{13}$$

Notice these constants t_i in these function $E(t-t_i)$ terms in the right side of Eq.(13) and sort them by an increased order after zero as follows:

$$0,\ \frac{1}{2}\times10^{-7},\ 10^{-7},\ \frac{5}{2}\times10^{-7},\ 3\times10^{-7},\ \ldots$$

Two connected constants have got an interval. According to the definition of the function $E(t)$, we know that $E(t) = 0$ if $t < 0$. Hence, by Eq.(13), we may write the nonzero terms of the expression of $v(\mathcal{L},t)$ in the following different intervals as

$$v(\mathcal{L},t) = \begin{cases} 0,\ t \in \left(0, \frac{1}{2}\times10^{-7}\right], \\ \frac{2}{3}\left[\frac{1}{10}E\left(t-\frac{1}{2}\times10^{-7}\right)\right],\ t \in \left(\frac{1}{2}\times10^{-7}, 10^{-7}\right], \\ \frac{2}{3}\left[\frac{1}{30}\times E(t-10^{-7}) - \frac{1}{10}\left(E(t-10^{-7}) - E\left(t-\frac{1}{2}\times10^{-7}\right)\right)\right],\ t \in \left(10^{-7}, \frac{5}{2}\times10^{-7}\right], \\ \ldots \end{cases}$$

This means that there exist only finite nonzero terms in Eq.(13) if t is given in a finite interval. For example, we only need $n = 2$ if we calculate $v(\mathcal{L}, t)$ for $t \in [0, 5 \times 10^{-7}]$. Applying the terminate condition $i(\mathcal{L}, t) = u(\mathcal{L}, t)/Z_2$, we calculate the induced current at $x = \mathcal{L}$ in Fig.2, which is the same as that in Ref.[1]; however, this book obtained the transient response by the frequency-domain BLT equation and Fourier transform techniques transforming a wideband spectrum into the time domain.

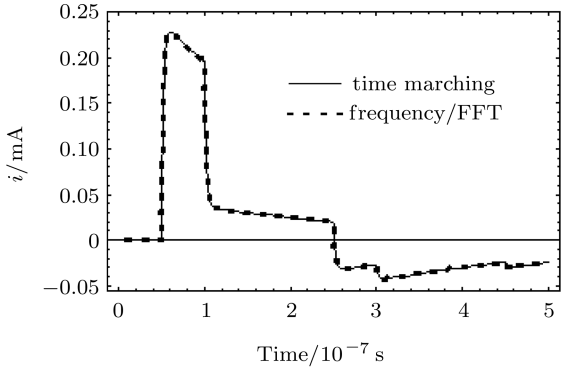


Fig.2. The induced transient current at $x = \mathcal{L}$ for the two-wire line.

In general, if we give a positive real number T_0, and denote by N the largest non-negative integer number such that

$$(2N + \cos\psi\cos\phi)\mathcal{L}/v < T_0,$$

then, for each integer number n and each real number t such that $n > N$ and $t \in [0, T_0]$, respectively, we have

$$t - (2n + \cos\psi\cos\phi)\mathcal{L}/v \leq 0.$$

This implies that

$$E(t - (2n + \cos\psi\cos\phi)\mathcal{L}/v) = 0.$$

Hence, for $t \in [0, T_0]$, by Eq.(12) and the terminate condition, we can write $v(\mathcal{L}, t)$ and $i(\mathcal{L}, t)$ as follows:

$$\begin{aligned} v(\mathcal{L},t) = {} & (1+\rho_2) \times \sum_{n=0}^{N} (\rho_1\rho_2)^n [-\rho_1 A_2 (E_0(t-(2n+1)\mathcal{L}/v) - E_0(t-(2n+2+\cos\psi\cos\phi)\mathcal{L}/v)) \\ & -A_1(E_0(t-(2n+1)\mathcal{L}/v) - E_0(t-(2n+\cos\psi\cos\phi)\mathcal{L}/v))], \end{aligned} \tag{14}$$

$$\begin{aligned} i(\mathcal{L},t) = {} & \frac{(1-\rho_2)}{Z_c} \times \sum_{n=0}^{N} (\rho_1\rho_2)^n [-\rho_1 A_2 (E_0(t-(2n+1)\mathcal{L}/v) - E_0(t-(2n+2+\cos\psi\cos\phi)\mathcal{L}/v)) \\ & -A_1(E_0(t-(2n+1)\mathcal{L}/v) - E_0(t-(2n+\cos\psi\cos\phi)\mathcal{L}/v))]. \end{aligned} \tag{15}$$

Obviously, $v(\mathcal{L}, t)$ and $i(\mathcal{L}, t)$ have only finite nonzero terms for $t \in [0, T_0]$. However, we can evaluate several convolution integrals if the currents and voltages are calculated directly in the time domain by the time-domain BLT equation. So it is more convenient to calculate or analyse the terminate transient response by Eqs.(14) and (15) than by the frequency-domain or the time-domain BLT equation.

5. Conclusion

In this paper, an analytic method is used to calculate the load responses of the two-wire transmission line directly in the time domain. For a plane-wave excitation, by the frequency-domain analytic solutions, we obtain the load time-domain analytic solutions of the line, and show that there exist only finite nonzero terms in these solutions if the time variate is at a finite interval. The example illuminates that the time-domain analytic solution is more convenient to calculate and analyse the transient response than the frequency-domain solutions. Now, we wonder whether the method can be extended to the multiconductor transmission line case. This will be discussed in our further work.

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