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Design and FPGA Implementation of a new hyperchaotic system*

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In this paper, a new four-dimensional autonomous hyperchaotic system is designed for generating complex chaotic signals. In the design, its parameters are selected according to the requirements for chaos and hyperchaos. The hyperchaotic nature is verified theoretically by using the bifurcation analysis and demonstrated experimentally by the implementation of an analogue electronic circuit. Moreover, the Field Programmable Gate Array (FPGA) technology is applied to implementing a continuous system in a digital form by using a chip of Altera Cyclone II EP2C35F484C8. The digital sequence generated from the FPGA device is observed in our experimental setup.

Keywords: Hyperchaos, bifurcation, FPGA implementation
PACC: 0545

1. Introduction

One major task for applying chaos-based information systems lies in the design of chaotic systems and the hardware implementation of reliable nonlinear circuits for generating various chaotic signals. Creating chaos, therefore, becomes a key issue and a focal topic for research.[1−15]

Hyperchaos is defined as the chaotic system with more than one positive Lyapunov exponent, that is, its dynamics is expanded in different directions simultaneously. It means that hyperchaos has an increasing randomness and a higher unpredictability that can be used to improve the security of chaotic communication systems. Therefore, the interest in hyperchaos is increasing due to its possible application in improving secure communications.[16]

Many analogue implementations of chaotic systems as electronic circuits were reported in the last decades, such as the well-known Chua circuit,[17] KCM circuit[18] and modified hyperchaotic Chua circuit[16] and so on. However, in some chaotic information systems a digital implementation may be necessary in, such as, chaos-based CDMA communication and various chaotic digital information systems.

There are several techniques to implement chaotic systems in a digital form, such as switched-capacitor techniques,[19] very large scale integration (VLSI) technique,[20] analogue complementary metal-oxide semiconductor (CMOS) technology[21] and digital signal processing (DSP) processors.[22]

The FPGA, which takes advantage of its capacity, function and reliance, has a wider application in modern digital systems. It will be preferred to design the circuit of complicated digital hardware.[23] Hence, it is significant to apply the FPGA technology to the realization of chaotic systems in a digital form in the hardware encryption of various information security systems and the chaotic secret communications.

Many digital FPGA implementations of chaotic systems have been proposed in the literature.[21−26] But most of them are devoted to the implementation of low-dimensional discrete and continuous chaotic systems. Their simple structure, which leads the secrete key space to be too small, will be no longer safe to the information masked by simplex low-dimensional chaotic system.[23] As mentioned above, a higher-dimensional continuous hyperchaotic system has the characteristics of complicated evolution processes, high randomness and large key space; it will be much more useful in various chaos-based information systems. However, there does not seem to be any re-

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port on the digital hardware implementation of continuous hyperchaos based on the FPGA. Therefore, the digital FPGA implementation of continuous hyperchaotic system is a nontrivial task with interesting impact on chaos-based engineering applications.

In this paper, we start with a new hyperchaotic system which contains one cubic nonlinear term and two cross-product terms. Its existence is verified not only by theoretical and numerical analyses but also by the implementation of an analogue electronic circuit. Then taking the proposed hyperchaotic system as an example, the FPGA technology is applied to obtaining digital hyperchaotic sequences. This method can be applied to other continuous autonomous chaotic/hyperchaotic systems and also possibly to chaotic information security systems and secrecy communication fields.

2. Theoretical design

2.1. A four-dimensional system

In pursuit of making a chaotic system hyperchaotic, we have recently found a hyperchaotic attractor from the following four-dimensional system:

$$\begin{aligned} \dot{x} &= a(y-x)+yz^2, \\ \dot{y} &= cx-y-xz+u, \\ \dot{z} &= xy-bz, \\ \dot{u} &= -kx, \end{aligned} \tag{1}$$

where a, b, c and k are constants to be determined.

In the designing of system (1), one is to analyse the structure of system (1) and determine the system parameter values such that the system can become hyperchaotic.

2.2. Parameter selection for hyperchaos

Obviously, system (1) satisfies the requirements for hyperchaos, that is, (1) the minimal number of the coupled first-order autonomous ordinary differential equations must be at least four; (2) the number of terms in the coupled equations giving rise to instability must be at least two, of which at least one must have a nonlinear function.[8]

On the other hand, some basic ideas of chaotification must be followed for generating chaos/hyperchaos from system (1), namely:

(1) it is dissipative.

(2) there exist some unstable equilibria.

Following the two ideas, we consider a volume V in a certain domain of the state space. For system (1), one has

$$\nabla V = \frac{\partial \dot{x}}{\partial x}+\frac{\partial \dot{y}}{\partial y}+\frac{\partial \dot{z}}{\partial z}+\frac{\partial \dot{u}}{\partial u} = -(a+b+1) \quad . \tag{2}$$

Therefore, to ensure system (1) being dissipative, it is required that

$$a+b+1>0. \tag{3}$$

On the other hand, it is easy to see that $S_0(0,\ 0,\ 0,\ 0)$ is the only equilibrium of system (1). By considering the Jacobian matrix of the equilibrium and calculating its eigenvalues, we can investigate the stability of S_0.

By linearizing system (1) at the equilibrium, we can easily obtain the Jacobian matrix

$$J = \begin{bmatrix} -a & a & 0 & 0 \\ c & -1 & 0 & 1 \\ 0 & 0 & -b & 0 \\ -k & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

and the characteristic equation

$$f(\lambda) = (\lambda+b)(\lambda^3+(1+a)\lambda^2+(a-ac)\lambda+ak) = 0, \tag{5}$$

which provides an eigenvalue $\lambda_1 = -b$ along with the following characteristic equation:

$$f(\lambda) = \lambda^3+(1+a)\lambda^2+a(1-c)\lambda+ak = 0. \tag{6}$$

To make equilibrium S_0 unstable, thereby yielding a hyperchaotic/chaotic possibility for system (1), the coefficients of Eq.(6) may be selected to satisfy the following conditions according to the Routh–Hurwitz criterion and inequality (3):

$$\begin{aligned} &a+b+1>0, \quad (1+a)>0, \\ &a(a+1)(1-c)-ak<0. \ \text{ and } \ ak>0. \end{aligned} \tag{7}$$

From inequalities (7), if we take the parameters a, b, $k>0$, conditions (7) reduce to

$$c_{\mathrm{b}} > 1-k/(a+1). \tag{8}$$

According to inequality (8), taking $a=34$, $b=9$, and $k=15$, it is required that $c_{\mathrm{b}}>0.5714$ in order to make S_0 unstable. If $c=0.5724>c_{\mathrm{b}}$, the corresponding eigenvalues of Eq.(5) for equilibrium S_0 are as follows:

$$\lambda_1 = -9, \quad \lambda_2 = -35.0105,$$

and

$$\lambda_{3,4} = 0.0053 \pm 3.8167\mathrm{i}.$$

So, the equilibrium S_0 is unstable with a saddle-focus point.

If $c < c_\mathrm{b}$, i.e. $c = 0.5624$, the corresponding eigenvalues of Eq.(5) for equilibrium S_0 are as follows:

$$\lambda_1 = -9, \quad \lambda_2 = -34.9913,$$

and

$$\lambda_{3,4} = -0.0043 \pm 3.8177\mathrm{i}.$$

So the equilibrium S_0 is stable. However, for $c = c_\mathrm{b} = 0.5714$, the corresponding eigenvalues of Eq.(5) for equilibrium S_0 are

$$\lambda_1 = -9, \quad \lambda_2 = -35,$$

and

$$\lambda_{3,4} = \pm 3.8173\mathrm{i},$$

implying that a Hopf bifurcation occurs at a value of

$$c_\mathrm{h} = 1 - k/(a+1).$$

2.3. The new hyperchaos

Under the conditions of inequalities (7) and (8), our numerical simulations show that a hyperchaotic attractor is obtained when $a = 34$, $b = 9$, $c = 20$, and $k = 15$ as shown in Fig.1, and its Lyapunov exponents are $L_1 = 3.681$, $L_2 = 0.497$, $L_3 = 0$, and $L_4 = -48.183$.

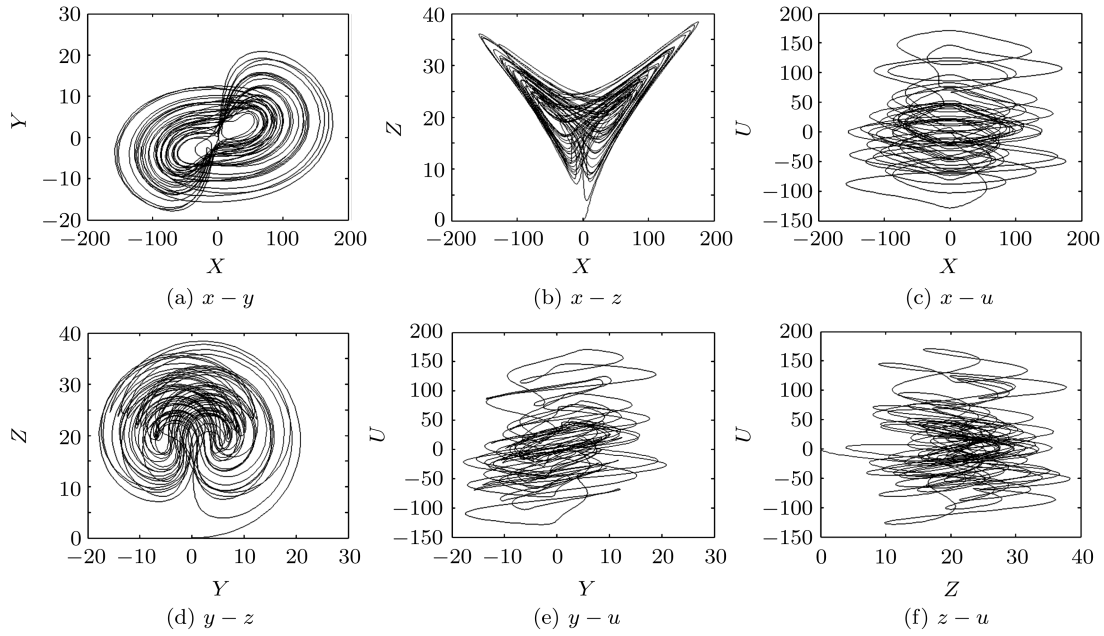


Fig.1. Phase portraits of hyperchaotic system (1) with $a = 34$, $b = 9$, $c = 20$, and $k = 15$.

2.4. Lyapunov exponents and bifurcations

In the following, we analyse the bifurcations in detail for system (1). The Lyapunov exponent spectrum and the corresponding bifurcation diagram of state variable x with respect to parameter c are shown in Fig.2, where a, b, and k are fixed while c is variable in the region $c \in [10, 30]$.

When $c \in (10, 10.16]$, the system is chaotic with only one positive Lyapunov exponent; while for $c \in [10.16, 22.67]$, the system becomes hyperchaotic with two positive Lyapunov exponents. In the region of $c \in [22.67, 23.91)$, the system becomes chaotic again. With c increasing further, the system undergoes a quasi-periodic window, a small chaotic region, and finally evolves into a periodic orbit.

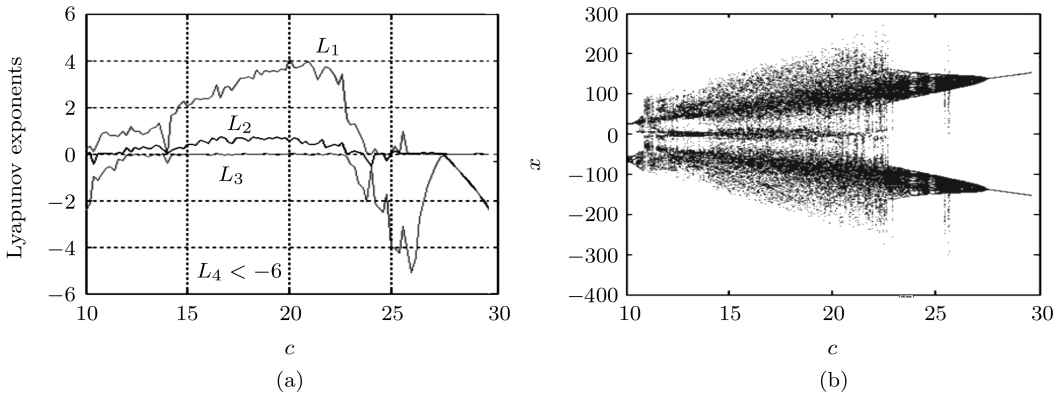


Fig.2. Lyapunov exponent spectrum and bifurcation diagram of system (1) with $a = 34$, $b = 9$, $k = 15$, and $c \in [10, 30]$.

3. Experimental demonstration

3.1. Circuit design

In this section, an electronic circuit is designed to physically verify the new hyperchaotic system (1) as shown in Fig.3, where analogue multipliers are employed to perform the required nonlinear cross-product terms in Eq.(1). The adjustable parameters a, b, c and k in system (1) can be respectively implemented by adjusting the linear resistors $R_1 - R_2$, R_8, R_{16}, and R_{22} in Fig.3.

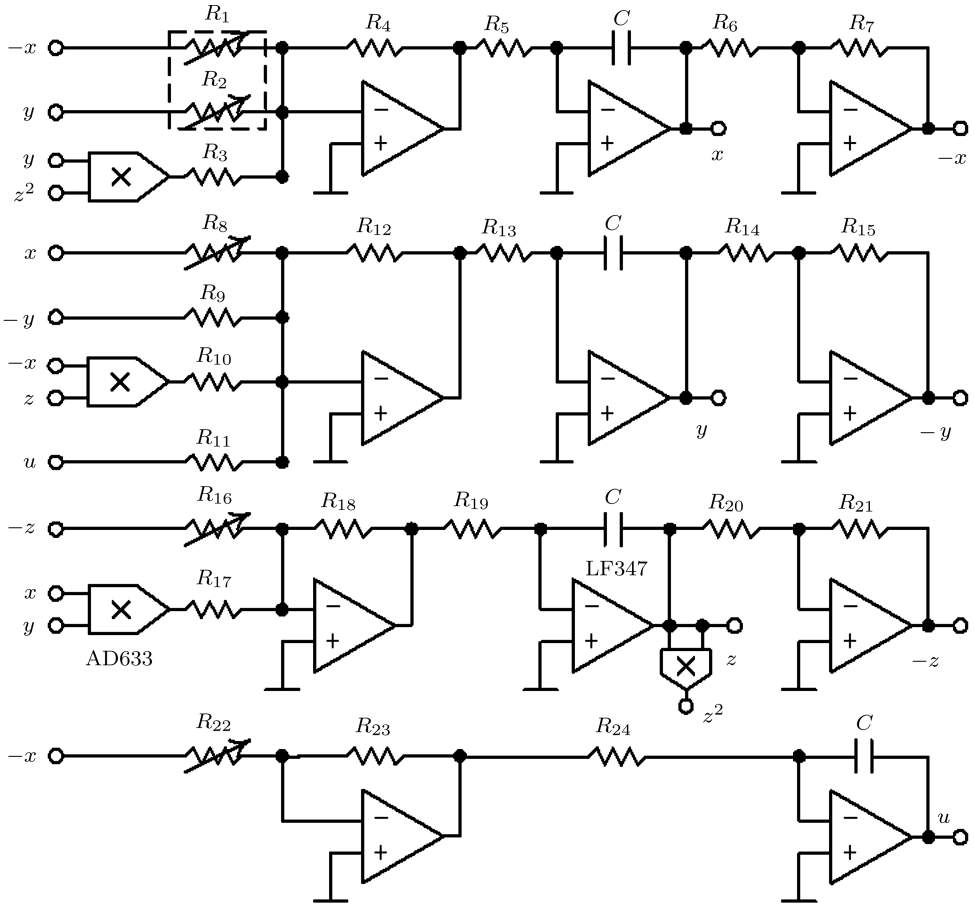
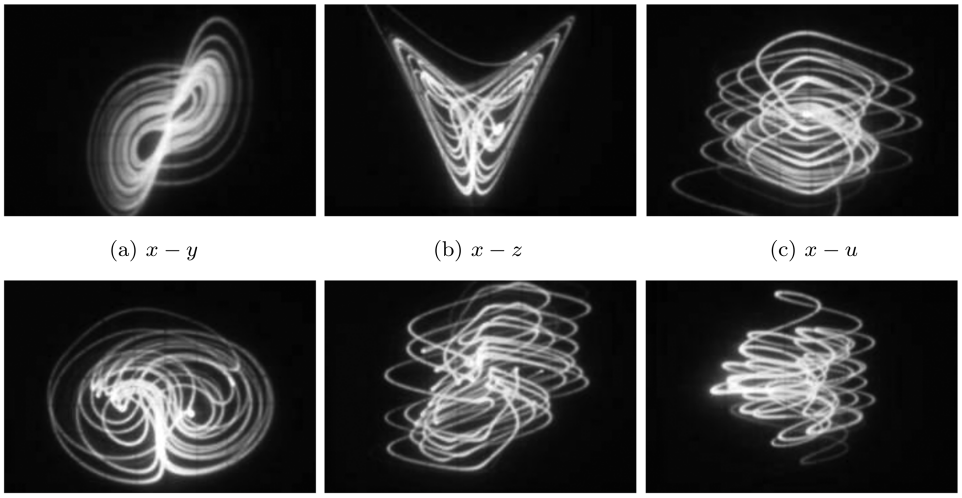


Fig.3. Circuit diagram for realizing the new hyperchaotic system (1).

3.2. Experimental results

The observations from the experimental setup are shown in Figs.4(a)–4(f), where resistors R_1, R_4, R_5, R_6, R_{12}, R_{13}, R_{14}, R_{15}, R_{18}, R_{19}, R_{20}, R_{21}, R_{23}, R_{24} each are 10 kΩ; R_2 and R_7 each are 45 kΩ; R_3 = 290 Ω; R_8 is 4.5 kΩ; R_9 and R_{11} each are 65 kΩ; R_{10} = 4.05 kΩ; R_{16} = 1 kΩ; R_{17} = 40 kΩ; R_{22} = 120 kΩ. Capacitors each are 10 nF. OP-Amps are LF347. Multiplies are AD633. By comparing the results of Fig.1 and those of Fig.4, we can find a good agreement between the numerical simulations and the experimental measurements for the new hyperchaotic system.



(a) $x-y$ (b) $x-z$ (c) $x-u$

(d) $y-z$ (e) $y-u$ (f) $z-u$

Fig.4. Phase portraits of the hyperchaotic system in the experimental setup. The voltages of supplies for all active devices are ±14 V . The sensitivities of both vertical and horizontal axes are 3V / div.

4. Digital implementation based on FPGA

In this section, a digital circuit that implements the proposed hyperchaotic system in a digital form is designed. The circuit is translated into the VHDL (very high speed integrated circuit hardware description language) code, and some performances of the circuit will be demonstrated by computer simulations. Then it can be downloaded into target hardware and the ALTERA FPGA chip of EP2C35F484C8. The digital sequences generated from FPGA can be observed in simulated and practical experiments.

First of all, we discretize and quantize system (1) to implement the continuous system in a digital form and obtain a binary digital sequence. Giving the equation

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = f\left(x_1, x_2,\ ...,\ x_N\right) = \lim_{\tau\to 0}\frac{x_{i(n+1)} - x_{i(n)}}{\tau} \quad (9)$$

yields

$$x_{i(n+1)} = x_{i(n)} + \tau f\left(x_{1(n)}, x_{2(n)},\ ...,\ x_{N(n)}\right), \quad (10)$$

where i = 1, 2, 3, and 4. From expression (10), system (1) can be discretized as follows:

$$\begin{aligned} x_{n+1} &= x_n + \tau\left(a(y_n - x_n) + y_n z_n^2\right), \\ y_{n+1} &= y_n + \tau\left(cx_n - x_n z_n - y_n + u_n\right), \\ z_{n+1} &= z_n + \tau\left(x_n y_n - bz_n\right), \\ u_{n+1} &= u_n - \tau k x_n. \end{aligned} \quad (11)$$

To obtain the digital sequence, i.e. the quantitative values $b_1 b_2 b_3$ ... b_m of Eq.(11), the $x_{i(n)}$ can be expressed as[26]

$$\begin{aligned} & x_{i(n+1)} \\ &= b_1 \times 2^u + b_2 \times 2^{u-1} + \cdots + b_u \times 2 \\ &\quad + b_{u+1} + b_{u+2} \times 2^{-1} + \cdots + b_m \times 2^{-v}, \end{aligned} \quad (12)$$

where $u + v + 1 = m$. The result, $\{b_{1(n)},\ b_{2(n)},\ \ldots,\ b_{m(n)}\}$, is just the digital sequence achieved from Eq.(11) at instant $n + 1$.

DSP-Builder is a DSP development tool from Altera Company, which can combine Quartus II software tool with the MathWorks Matlab/Simulink tools together. We can realize the continuous hyperchaotic

system model of the FPGA circuit in Matlab/Simulink and generate HDL (hardware description language) automatically by using the DSP-Builder.

The model of Eq.(11) is built first by using the DSP Builder blocks in the environment of Matlab/Simulink as shown in Fig.5. In the design of digital sequence, adder, delay, multiplier, amplifier, IF statement block, IO bus block, n-to-1 multiplexer block and constant block are selected from the DSP Builder component library.

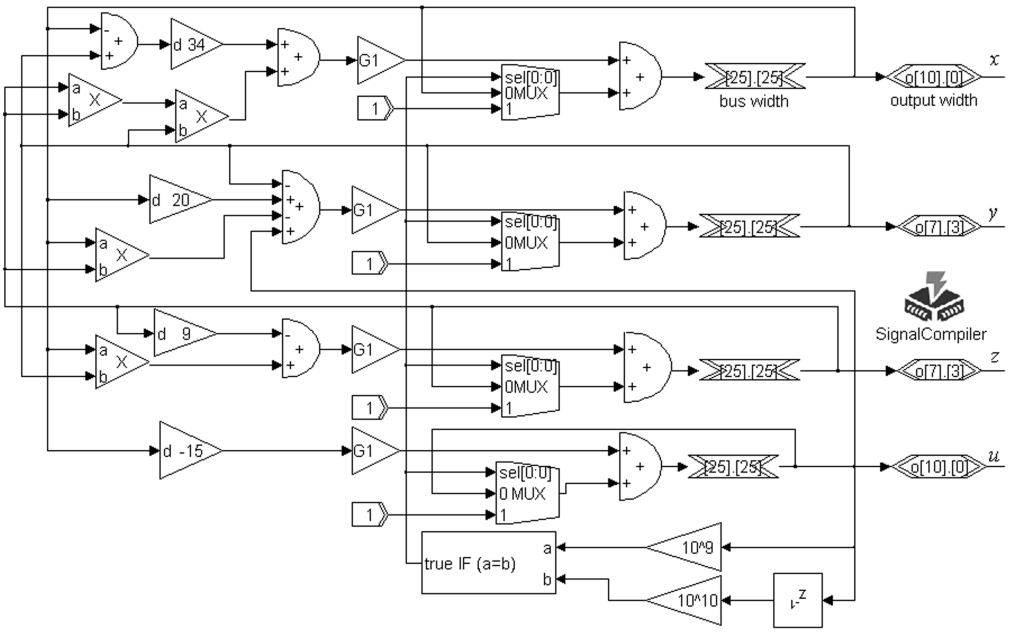


Fig.5. Digital circuit realized in the Simulink by DSP-Builder blocks.

The Hardware in Loop (HIL), which is supplied by DSP-Builder, can do well in hardware simulation. In this step, Signal Compiler block and Cyclone II EP2C35F484C8 are used as a translation tool and a device respectively. The simulated results for the established HIL model of the designed digital circuit have shown that the continuous system (1) can be carried out by FPGA in hardware.

Now, we analyse and synthesize the Quartus Project File (QPF) file (the project file) that is generated by Signal Compiler block, and then download the Hardware Description Language HDL file into the EP2C35F484C8 chip and begin to run. The waveform of each port can be observed by Signal Tap II in Quartus II, as depicted in Fig.6 where the period of clock is 6.4 μs.

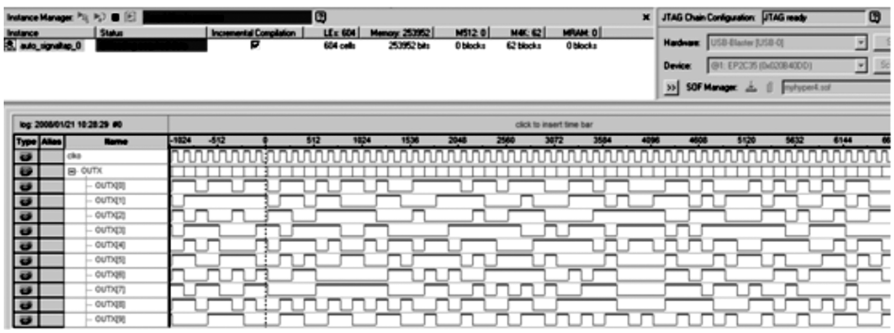


Fig.6. The result of wave stimulation.

We have made an effort to implement the hyperchaotic system (1) on the FPGA, EP2C35F484C8. The FPGA evaluation board used in our experiment is shown in Fig.7, and the digital hyperchaotic sequence is observed as shown in Fig.8. The corresponding analogue hyperchaotic sequences have also been obtained by using a D/A converter as shown in Fig.8.

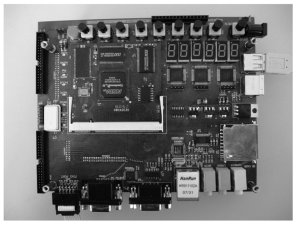
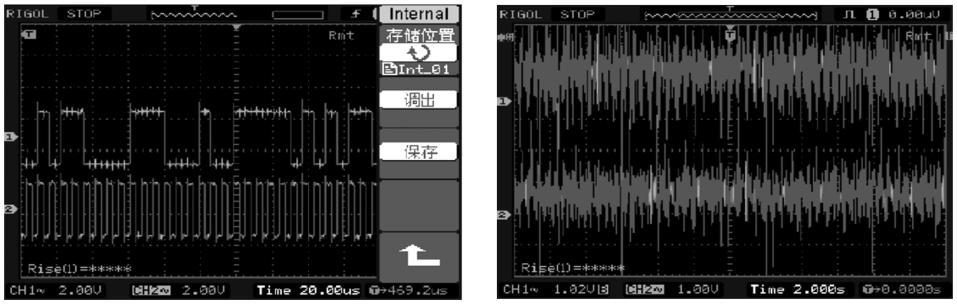


Fig.7. FPGA evaluation board used to perform digital implementation.



(a) (b)

Fig.8. Experimental results for hyperchaotic system (1) using FPGA.

5. Conclusions

A novel hyperchaotic system has been introduced in this paper. The hyperchaotic nature is verified by numerical analysis and the experimental demonstration of an analogue electronic circuit. Furthermore, a method of digitally implementing a continuous hyperchaotic system is proposed by using the FPGA technology, and the digital sequence generated from the FPGA device is obtained in our experiment. This FPGA-based method can be extended to the implementation of other continuous hyperchaotic or chaotic systems and may be used in various chaos-based information security systems and secrecy communications.

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