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Mixedness of the N-qubit states with exchange symmetry*

Hu Ming-Liang(胡明亮)† and Xi Xiao-Qiang(惠小强)

Department of Applied Mathematics and Applied Physics,
Xi'an Institute of Posts and Telecommunications, Xi'an 710061, *China*

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The mixedness of the N-qubit quantum states with exchange symmetry has been studied, and the results show that the linear entropy of the single qubit reduced density matrix (RDM), which can describe the mixedness, is completely determined by the expectation values $\langle S_z\rangle$ and $\langle S_\pm\rangle$ for both the pure and the mixed states. The mixedness of the pure states can be used to describe the bipartite entanglement, as an example we have calculated the mixedness of the Dicke state and the spin squeezed Kitagawa–Ueda state. For the mixed states, we determine the mixedness properties of both the ground states and the thermal states in mean-field clusters of spin-1/2 particles interacting via the anisotropy Heisenberg XXZ interaction, and found for the ferromagnetic case ($J < 0$), the mixedness will approximate to the pairwise entanglement when the anisotropic parameter $\Delta > \Delta_c$.

Keywords: mixedness, linear entropy, symmetric quantum state
PACC: 0367, 0530, 7510J

1. Introduction

It is well known that entanglement plays an important role in the field of quantum information science (QIS). Consequently, considerable efforts have been devoted to the study of entanglement features of various types of the condensed matter systems, especially the spin chains with Heisenberg interactions.[1−15] In experiments, except for solid state and cavity quantum electrodynamics (QED) systems, recent progress show that spin chains can also be efficiently constructed with desired properties by adjusting the amplitudes and propagation directions of laser beams, or the Zeeman and the Raman fields in optical systems.[16−20] These techniques greatly facilitate the designing of quantum mechanical devices for various tasks of quantum information processing.

The mixedness of a state, which is related to quantum entanglement and central in quantum information theory, has also been a subject of several studies in recent years.[15,21−23] For instance, it has been shown that entangled states become useless for quantum teleportation on exceeding a certain degree of mixedness.[24] For a pure bipartite state, it is easy to show that the mixedness of one subsystem is equal to that of another one, and thus it can be used as a measure of the bipartite entanglement. When the state is mixed, however, this simple procedure is no longer appropriate; a so-called convex-roof construction must be used, in which the entanglement is calculated as the minimum of the entanglement of all pure state decompositions of the mixed state. In general, the calculation of this minimum is a very difficult task. Thus for mixed states, the mixedness does not in general function as an entanglement measure.

In this paper, we will study mixedness of the N-qubit quantum states for an ensemble of qubits in a completely symmetric pure or mixed state, these states are defined to be invariant under permutation of any two particles. We quantify the mixedness by the linear entropy of the single qubit reduced density matrix (RDM), which is defined as $E = 1-\mathrm{tr}(\rho_n^2)$.

2. Basic formalism

For any system consisting of spin-1/2 particles, it is straightforward to show that in the standard basis $\{|0\rangle, |1\rangle\}$, the single qubit reduced density matrix

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†E-mail: mingliang0301@xiyou.edu.cn

(RDM) is given by

$$\rho_n = \begin{pmatrix} \dfrac{1+\langle\sigma_{nz}\rangle}{2} & \langle\sigma_{n+}\rangle \\ \langle\sigma_{n-}\rangle & \dfrac{1-\langle\sigma_{nz}\rangle}{2} \end{pmatrix}, \tag{1}$$

where the first element is $\langle 0|\rho_n|0\rangle$, etc., and the angular brackets denote the expectation values of Pauli spin matrices with respect to the state ρ_n.

For any N-qubit quantum state $|\phi\rangle = a_0|00\ldots0\rangle + a_1|00\ldots1\rangle + \cdots + a_{2^N-1}|11\ldots1\rangle = \sum_{x=0}^{2^N-1} a_x|x\rangle$, one can define a unitary exchange operator $\mathcal{O}_{ij} = \sigma_{ix}\otimes\sigma_{jx}$ by its action on the tensor product basis $\mathcal{O}_{ij}|m_1\ldots m_i\ldots m_j\ldots m_N\rangle = |m_1\ldots m_j\ldots m_i\ldots m_N\rangle$, where $|m_i\rangle = |0\rangle\,(|1\rangle)$ represents the state of spin up (down). If this state admits the exchange symmetry, i.e., $\mathcal{O}_{ij}\rho\mathcal{O}_{ij}^{\dagger} = \rho$, then it is readily to show that for any Hermitian operator $\mathcal{G}$ the equality $\langle\mathcal{G}_{i\alpha}\rangle = \langle\mathcal{G}_{j\alpha}\rangle$ holds.

For convenience of later use, we define the collective spin operator $S_\alpha = \sum_{n=1}^{N}\sigma_{n\alpha}/2$ ($\alpha = x$, y, z), then by using the standard angular momentum coupling theory, we can rewrite the state $|\phi\rangle$ in the form $|\phi\rangle = \sum_{S=0,1/2}^{N/2}\sum_{m=-S}^{S} c_{S,m}|S,m\rangle$ in the orthonormal basis, where S and m are respectively the total and the z component of the total spin, with S equals to one of the values beginning from $S=0$ ($S=1/2$) to $N/2$ for even (odd) number N, and $m = -S, -S+1, \ldots, S-1, S$.

For quantum state with exchange symmetry, it is readily seen that

$$\langle S_\alpha\rangle = \frac{1}{2}N\langle\sigma_{n\alpha}\rangle, \quad \langle S_\pm\rangle = N\langle\sigma_{n\pm}\rangle. \tag{2}$$

From the unit trace of the density matrices, it follows immediately that for qubits the equality $1-\mathrm{tr}(\rho_n^2) = 2\det(\rho_n)$ holds, thus from the definition of the linear entropy one can obtain

$$\begin{aligned} E &= \frac{1}{2}(1-\langle\sigma_{nz}\rangle^2) - 2\langle\sigma_{n+}\rangle\langle\sigma_{n-}\rangle \\ &= \frac{1}{2} - \frac{2}{N^2}\left(\langle S_z\rangle^2 + \langle S_+\rangle\langle S_-\rangle\right). \end{aligned} \tag{3}$$

From the obvious fact that $S_z|S,m\rangle = m|S,m\rangle$, and $S_\pm|S,m\rangle = \sqrt{(S\mp m)(S\pm m+1)}|S,m\pm1\rangle$, one can immediately obtain

$$\begin{aligned} \langle S_z\rangle &= \sum_{S=0,1/2}^{N/2}\sum_{m=-S}^{S} c_{S,m}^2 m, \\ \langle S_\pm\rangle &= \sum_{S=0,1/2}^{N/2}\sum_{m=-S}^{S} c_{S,m}c_{S,m\pm1} \\ &\quad\times\sqrt{(S\mp m)(S\pm m+1)}. \end{aligned} \tag{4}$$

If the N-qubit state is mixed $\rho = \sum_i |\phi_i\rangle p_i\langle\phi_i| = \sum_i p_i\rho_i$ ($p_i>0$, $\sum_i p_i = 1$), in a similar way to obtain Eq.(4), one will find

$$\langle S_z\rangle = \sum_i p_i\langle S_{iz}\rangle, \quad \langle S_\pm\rangle = \sum_i p_i\langle S_{i\pm}\rangle, \tag{5}$$

where $\langle S_{iz}\rangle$ and $\langle S_{i\pm}\rangle$ denote the corresponding expectation values with respect to the state ρ_i of the ith subsystem.

For the special cases $\rho = |S,m\rangle\langle S,m|$ and $\rho = \sum_{Sm}|S,m\rangle p_{Sm}\langle S,m|$ ($p_{Sm}>0$, $\sum_{Sm}p_{Sm}=1$), one will obtain $\langle S_z\rangle = m$, $\langle S_\pm\rangle = 0$ and $\langle S_z\rangle = \sum_{Sm}p_{Sm}m$, $\langle S_\pm\rangle = 0$ respectively.

From Eqs.(3), (4) and (5), one can conclude that for any N-qubit quantum state with exchange symmetry, the mixedness of the single-qubit RDM is completely determined by the expectation values $\langle S_z\rangle$ and $\langle S_\pm\rangle$.

3. Mixedness of pure states

For pure states, the mixedness of a subsystem is equal to that of another one, and thus can be used as a measure of the bipartite entanglement.

Firstly, the usual symmetric Dicke state $|\mathrm{Dicke}\rangle = |N/2,m\rangle$ is considered. From Eqs.(3) and (4) one can explicitly obtain $E = 1/2 - 2m^2/N^2$. Clearly, the linear entropy for the Dicke state is symmetric with respect to $m=0$. E decreases with the increase of $|m|$, and disappears when $|m|$ arrives at its maximum value $N/2$. Comparing $E = 1/2 - 2m^2/N^2$ with Eq.(17) in Ref.[25], one will obtain a direct connection between mixedness (measured by the linear entropy) and pairwise entanglement (measured by the concurrence) given by $C = \left(NE - \sqrt{E(N^2E - 2N + 2)}\right)/(N-1)$.

Next we see mixedness properties of another symmetric quantum state, i.e., the spin squeezed Kitagawa–Ueda state $|KU\rangle = \exp(-\mathrm{i}\chi t S_x^2)|1\rangle^{\otimes N}$.[26] From Refs.[25, 26] one can easily obtain $\langle S_z\rangle = -(N/2)\times\cos^{N-1}(\chi t)$ and $\langle S_\pm\rangle = 0$. Substituting these results into Eq.(3) yields $E = 1/2 - \cos^{2N-2}(\chi t)/2$. Obviously, the linear entropy here changes periodically with the increase of χt with a period of π, and increases as N increases since $\partial E/\partial N = \sin^{2N-2}(\chi t)\geq 0$. When $N\to\infty$, E attains its maximum value 0.5.

4. Mixedness of mixed states

To see the mixedness of the symmetric mixed states, we consider a cluster of spin-1/2 particles of arbitrary sizes, with the Hamiltonian given by

$$\hat{H} = J \sum_{j>i=1}^{N} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \Delta \sigma_{iz}\sigma_{jz}) + B \sum_{i=1}^{N} \sigma_{iz}, \quad (6)$$

where $\boldsymbol{\sigma}_i = (\sigma_{ix}, \sigma_{iy}, \sigma_{iz})$ denotes the vector of Pauli matrices, J is the coupling constant with $J < 0$ ($J > 0$) corresponding to the ferromagnetic (antiferromagnetic) case, and the parameter Δ controls the degree of the anisotropy. Here every spin is coupled to every other spin with equal strength. Such a study is fed by two motivations. On the one hand, mean field systems are amenable to exact analytical treatment as we will see, one can determine the mixedness by the sole knowledge of the partition function. On the other hand, these studies provide the first approximation for the analysis of other models.

The above Hamiltonian admits the exchange symmetry, which can be rewritten in terms of the total spin operators in the form

$$\hat{H} = 2J\left(\boldsymbol{S}^2 + \Delta S_z^2\right) + 2BS_z, \quad (7)$$

where we have ignored an additive constant. Equation (7) is exactly solvable, with the corresponding eigenvalues and eigenstates given by

$$\begin{aligned} &\varepsilon(S,m) = 2JS(S+1) + 2J\Delta m^2 + 2Bm, \\ &|\psi(S,m)\rangle = |S,m\rangle, \end{aligned} \quad (8)$$

where $S = N/2, N/2-1, \ldots, 0(1/2)$ for even (odd) number N, and $m = -S, -S+1, \ldots, S-1, S$.

For this model, the partition function and the magnetization per site are analytically obtained as

$$Z = \sum_{S=0,1/2}^{N/2} g(N,S) \times \sum_{m=-S}^{S} \mathrm{e}^{-\beta[2JS(S+1)+2J\Delta m^2+2Bm]},$$

$$\langle S_z \rangle = \frac{1}{Z} \sum_{S=0,1/2}^{N/2} g(N,S) \times \sum_{m=-S}^{S} m\mathrm{e}^{-\beta[2JS(S+1)+2J\Delta m^2+2Bm]}. \quad (9)$$

Here $g(N,S)$ is the multiplicity of the eigenvector $|S,m\rangle$, which is given by

$$g(N,S) = \begin{pmatrix} N \\ N/2-S \end{pmatrix} \frac{2S+1}{N/2+S+1}, \quad (10)$$

where $\begin{pmatrix} N \\ N/2-S \end{pmatrix}$ is the binomial coefficient, i.e., $\begin{pmatrix} N \\ k \end{pmatrix} = \dfrac{N!}{k!(N-k)!}$.

For an arbitrary quantum number S, the sum of m begins from $m = -S$ to $m = S$, therefore it is direct to check from Eqs.(3) and (9) that the mixedness is symmetric with respect to the zero-magnetic field $B = 0$ at any temperature, irrespective of the anisotropic parameter Δ, and at the critical point $B = 0$, Eq.(9) gives $\langle S_z \rangle = 0$. Combination of this with Eq.(3) yields $E \equiv 0.5$. So we only discuss the linear entropy of the model in the regions of $B > 0$ in the following section.

4.1. Ground-state mixedness

In this section we investigate the mixedness at zero absolute temperatures. We assume that the $T = 0$ ground state is an equal superposition of the relevant states, which can be shown properly by taking the zero-temperature limit of the thermal state $\rho(T) = Z^{-1}\exp(-\beta\hat{H})$.

We first consider the ferromagnetic case ($J < 0$). If $\Delta \geq 0$, from Eq.(8) it is easy to show that the ground-state energy is $\varepsilon_0 = \varepsilon(N/2, -N/2)$. Thus from Eq.(5) one obtain $\langle S_z \rangle = -N/2$, which gives the result $E \equiv 0$ in the full regions of the magnetic field $B > 0$. If $\Delta < 0$, the ground-state energy is obtained from Eq.(8) as

$$\varepsilon_0 = \begin{cases} \varepsilon(S,-S)(S = 0, 1/2), & \text{if } \ 0 < B < (2S+1)J\Delta, \\ \varepsilon(S,-S)(1/2 < S < N/2), & \text{if } \ (2S-1)J\Delta < B < (2S+1)J\Delta, \\ \varepsilon(S,-S)(S = N/2), & \text{if } \ B > (2S-1)J\Delta, \end{cases} \quad (11)$$

with the corresponding ground state vector $|S,-S\rangle$. Thus one will obtain the magnetization as $\langle S_z\rangle=-S$, which gives the linear entropy $E=1/2-2S^2/N^2$. Clearly, with the increase of the applied magnetic field B, the linear entropy decreases by K steps, where $K=(N+2)/2$ for even number N, and $K=(N+1)/2$ for odd number N. When $B>(2S-1)J\Delta$, spins are all aligned in the direction of applied magnetic field and hence $E=0$.

Now we discuss the antiferromagnetic case ($J>0$). If $\Delta>-1$, the ground-state energy is

$$\varepsilon_0=\begin{cases}\varepsilon(S,-S)(S=0,1/2), & \text{if } 0<B<(2S+2)J+(2S+1)J\Delta,\\ \varepsilon(S,-S)(1/2<S<N/2), & \text{if } 2SJ+(2S-1)J\Delta<B<(2S+2)J+(2S+1)J\Delta,\\ \varepsilon(S,-S)(S=N/2), & \text{if } B>2SJ+(2S-1)J\Delta,\end{cases} \tag{12}$$

with the corresponding ground state vector given by $|S,-S\rangle$. Hence one still has $\langle S_z\rangle=-S$ and $E=1/2-2S^2/N^2$. Obviously, with increasing value of the applied magnetic field B, the linear entropy still decreases by K steps, where K being the same as that of $J<0$ and $\Delta<0$. If $\Delta\leq-1$, the expressions of the linear entropy for even and odd number of sites N are different.

For even number N, if $-(N+2)/N<\Delta\leq-1$, the ground-state energy is

$$\varepsilon_0=\begin{cases}\varepsilon(0,0), & \text{if } B<J+JN(1+\Delta)/2,\\ \varepsilon(N/2,-N/2), & \text{if } B>J+JN(1+\Delta)/2,\end{cases} \tag{13}$$

which gives the linear entropy to be $E=0.5$ and $E=0$, respectively. If $\Delta\leq-(N+2)/N$, the ground-state energy is $\varepsilon_0=\varepsilon(N/2,-N/2)$ in the whole regions of $B>0$, which yields $E=0$.

For odd number N, if $-(N+3)/(N+1)<\Delta\leq-1$, the ground-state energy is

$$\varepsilon_0=\begin{cases}\varepsilon(1/2,-1/2), & \text{if } B<J(N+1)(1+\Delta)/2+J,\\ \varepsilon(N/2,-N/2), & \text{if } B>J(N+1)(1+\Delta)/2+J,\end{cases} \tag{14}$$

which still gives $E=0.5$ and $E=0$, respectively. If $\Delta\leq-(N+3)/(N+1)$, the ground-state energy is $\varepsilon_0=\varepsilon(N/2,-N/2)$ in the whole region of $B>0$, which yields $E=0$.

4.2. Thermal-state mixedness

Let us now turn to the more realistic case of nonzero temperatures. The state of the system here is given by temperature-dependent density matrix $\rho(T)=Z^{-1}\exp(-\beta\hat{H})$, and the linear entropy of the thermal state is completely determined by the magnetization expressed in Eq.(9), from which it is easy to check that $\langle\sigma_{nz}\rangle=2\langle S_z\rangle/N\leq0$. Moreover, for any Hermitian operator Λ satisfying $\Lambda^2=1$ we have $|\langle\Lambda\rangle|\leq1$, thus we obtain $-1\leq\langle\sigma_{nz}\rangle\leq0$. Combining this with Eqs.(3) and (9), we immediately arrive at the following two Propositions.

Proposition 1 For the XXZ model with all spins interacting with each other with equal strength, the mixedness of the single-qubit RDM always decreases with increasing value of the applied magnetic field B at any temperature, irrespective of the anisotropy Δ.

Proof From Eq.(9) one has

$$\begin{aligned}&\frac{\partial\langle S_z\rangle}{\partial B}\\ &=\frac{\beta N^2}{2}\left[\left(\frac{1}{Z}\sum_{S=0,1/2}^{N/2}g(N,S)\sum_{m=-S}^{S}\left(\frac{2m}{N}\right)\mathrm{e}^{-\beta\varepsilon}\right)^2\right.\\ &\quad\left.-\frac{1}{Z}\sum_{S=0,1/2}^{N/2}g(N,S)\sum_{m=-S}^{S}\left(\frac{2m}{N}\right)^2\mathrm{e}^{-\beta\varepsilon}\right]\\ &=\frac{\beta N^2}{2}\left(\langle\sigma_{nz}\rangle^2-\langle\sigma_{nz}^2\rangle\right)\leq0,\end{aligned} \tag{15}$$

where we have used the obvious fact $\langle\Lambda^2\rangle-\langle\Lambda\rangle^2\geq0$. This equation shows that $\langle S_z\rangle$ decreases as the applied magnetic field B increases, thus $\langle S_z\rangle^2$ increases as B increases, and the linear entropy E decreases as B increases. For sufficient strong magnetic field B, E disappears.

This proposition shows that for both the antiferromagnetic and the ferromagnetic cases, the mixedness of the single-qubit RDM always decreases with increasing value of the magnetic field B at any tem-

perature, irrespective of the anisotropic parameter Δ.

Proposition 2 For the XXZ model with all spins interacting with each other with equal strength, the mixedness of the single-qubit RDM always increases as Δ increases if $J > 0$, and decreases as Δ increases if $J < 0$ at any temperature, irrespective of the magnetic field B.

Proof From Eq.(9) one has

$$\begin{aligned}\frac{\partial\langle S_z\rangle}{\partial\Delta} &= \frac{\beta JN^3}{4}\left[\left(\frac{1}{Z}\sum_{S=0,1/2}^{N/2} g(N,S)\sum_{m=-S}^{S}\left(\frac{2m}{N}\right)\mathrm{e}^{-\beta\varepsilon}\right)\left(\frac{1}{Z}\sum_{S=0,1/2}^{N/2} g(N,S)\sum_{m=-S}^{S}\left(\frac{2m}{N}\right)^2\mathrm{e}^{-\beta\varepsilon}\right)\right.\\ &\quad\left.-\frac{1}{Z}\sum_{S=0,1/2}^{N/2} g(N,S)\sum_{m=-S}^{S}\left(\frac{2m}{N}\right)^3\mathrm{e}^{-\beta\varepsilon}\right]\\ &= \frac{\beta JN^3}{4}\left(\langle\sigma_{nz}\rangle\langle\sigma_{nz}^2\rangle-\langle\sigma_{nz}^3\rangle\right)\begin{cases}\geq 0, & \text{if } J>0,\\ \leq 0, & \text{if } J<0,\end{cases}\end{aligned} \tag{16}$$

where we have used the inequality $\langle\Lambda\rangle\langle\Lambda^2\rangle-\langle\Lambda^3\rangle\geq 0$. This equation shows that if $J > 0$ ($J < 0$), $\langle S_z\rangle$ increases (decreases) as the anisotropic parameter Δ increases, thus $\langle S_z\rangle^2$ decreases (increases) as Δ increases, and the linear entropy E increases (decreases) as Δ increases. When $\Delta\to\infty$, E reaches its asymptotic value $E_{\mathrm{asym}} = 0.5$ if $J > 0$ and disappears if $J < 0$.

This proposition shows that for the antiferromagnetic case, one can increase the mixedness by increasing the anisotropic parameter Δ at any temperature, while for the ferromagnetic case, one can increase the mixedness by decreasing the anisotropic parameter Δ at any temperature.

As a final discussion, it is worth while to study the relations between the mixedness of the single-qubit RDM measured by the linear entropy and the pairwise entanglement measured by the concurrence. Following the procedure of the previous section, the concurrence of the thermal state $\rho(T) = Z^{-1}\exp(-\beta\hat{H})$ can be obtained as

$$C = \frac{1}{2}\max\left(0, |G_{xy}| - \sqrt{(1+G_{zz})^2 - 16\langle S_z\rangle^2/N^2}\right), \tag{17}$$

where the correlation functions are given by

$$\begin{aligned}G_{zz} &= \frac{1}{Z}\sum_{S=0,1/2}^{N/2} g(N,S)\sum_{m=-S}^{S}\left(\frac{4m^2-N}{N(N-1)}\right)\mathrm{e}^{-\beta[2JS(S+1)+2J\Delta m^2+2Bm]},\\ G_{xy} &= \frac{1}{Z}\sum_{S=0,1/2}^{N/2} g(N,S)\sum_{m=-S}^{S}\left(\frac{4S(S+1)-2N-4m^2}{N(N-1)}\right)\mathrm{e}^{-\beta[2JS(S+1)+2J\Delta m^2+2Bm]}.\end{aligned} \tag{18}$$

Then, from Eqs.(3) and (17), one can obtain

$$C = \frac{1}{2}\max\left(0, |G_{xy}| - \sqrt{(1+G_{zz})^2 + 8E - 4}\right). \tag{19}$$

Equation (19) implies that there does not exist direct connections between the linear entropy of the single-qubit RDM and the concurrence for the general thermal state $\rho(T)$ since C is determined by two addi-

tional correlation functions G_{zz} and G_{xy} besides $\langle S_z \rangle$. However, for the ferromagnetic case at fixed T, we notice that when the anisotropic parameter Δ exceeds a certain critical value $\Delta_{\rm c}$, the curves for the linear entropy and the concurrence almost coincide (see Fig.1), i.e., the values of E and C are approximately the same, which gives $C \approx 0.5 - 2\langle S_z \rangle^2/N^2$ when $\Delta > \Delta_{\rm c}$. Figure 1 also indicates that the critical value $\Delta_{\rm c}$ decreases with the increase of the magnetic field B.

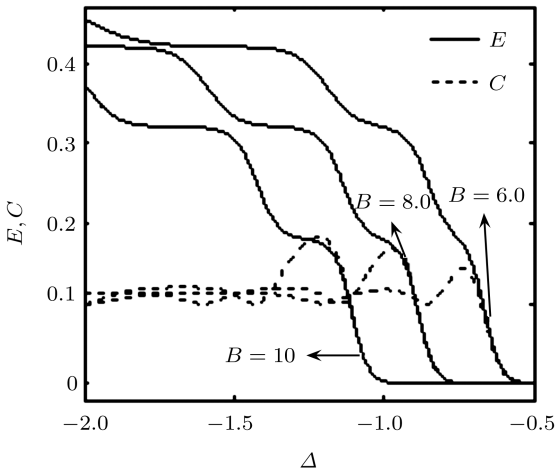


Fig.1. Linear entropy E and concurrence C as functions of the anisotropic parameter Δ. The parameter values for the plot are $J = -1$, $N = 10$, $T = 0.5$.

5. Conclusions

We have studied mixedness features of the N-qubit quantum states with exchange symmetry. The calculation of the linear entropy shows that the mixedness of the single-qubit RDM is solely determined by the expectation values $\langle S_z \rangle$ and $\langle S_\pm \rangle$ for both the pure and the mixed states. For the pure states, the mixedness can be used to measure the bipartite entanglement, as an example we have calculated the mixedness of the Dicke state and the Kitagawa–Ueda state: for the Dicke state, the linear entropy E always decreases with the increase of $|m|$, and for the Kitagawa–Ueda state, E changes periodically with the increase of χt with a period π. For the mixed states, we investigate mixedness of a mean-field cluster of spin-1/2 particles interacting via the anisotropic XXZ interaction, and the analytical expressions of the linear entropy for ground states and thermal states are obtained. The results show that for an arbitrary Δ, the linear entropy always decreases with the increase of B at fixed temperatures, while for arbitrary $B > 0$, the linear entropy always increases (decreases) as Δ increases if $J > 0$ ($J < 0$). For the ferromagnetic case ($J < 0$), the curves for the linear entropy and the concurrence almost coincide when the anisotropic parameter Δ exceeds a certain critical value $\Delta_{\rm c}$.

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