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Control of entanglement sudden death by Kerr medium*

Zheng Qiang(郑 强)†, Zhang Xiao-Ping(张小平), and Ren Zhong-Zhou(任中洲)

Department of Physics, Nanjing University, Nanjing 210093, *China*

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The entanglement dynamics of system, where atoms A and B interact with single mode cavity fields a and b respectively, is studied. The interaction between atom A and cavity a may be described by using the typical Jaynes–Cummings model, while that between the atom B and cavity b filled with a Kerr medium is of a two-photon process. For a certain initial atom entanglement state, there is an entanglement sudden death effect between the two atoms. The Kerr medium in the cavity b can effectively prevent the undesirable entanglement sudden death from occurring. Also, from the viewpoint of the population dynamics, we discuss why the Kerr medium can do so.

Keywords: Entanglement sudden death, Jaynes–Cummings model, Kerr
PACC: 0365, 4250

1. Introduction

Since the seminal contributions made by Einstein *et al*,[1] and lately by Bell,[2] quantum entanglement has become one of the central issues of quantum physics. Entanglement, not only plays a basic role in quantum theory, but also is a key resource in quantum information processing,[3] for example, quantum teleportation,[4,5] quantum dense coding,[6] and quantum cryptography.[7] Therefore, many methods to generate the entanglement states have been proposed.[8−11]

Entanglement dynamics is an important field. One interesting phenomenon, so called entanglement sudden death (ESD), was found by Yu and Eberly.[12] The special property of ESD is that entanglement disappears nonsmoothly in a finite time, which is beyond expectation that the entanglement may disappear smoothly. Moreover, an experiment has observed the ESD in an optical system.[13] ESD has also been observed in a system of two isolated atoms,[14−16] interacting with a Jaynes–Cummings (JC) cavity for a certain initial entangled state. In addition, the ESD also occurs in a two qubits mixed state with an Ising-type interaction,[17] a system described with the nondegenerate two photon Tavis–Cummings model,[18,19] two two-level atoms in Tavis–Cummings cavity,[20] the two qubits coupled by a spin channel with an XY-type interaction,[21] etc.

All these studies indicate that the intriguing ESD is a universal effect. However, the abrupt disappearance of entanglement may be a bad thing, for most of quantum information processes are based on entanglement. For example, in the Bennett's quantum teleportation scheme, the quantum channel between Alice and Bob is one entanglement pair. When the entanglement between Alice and Bob disappears abruptly, the quantum channel between them is also cut off, which may spoil the quantum information transferring from Alice to Bob. Therefore, from the viewpoint that the entanglement is a resource, it would be better to have some methods to control the occurrence of ESD.

In this paper, in a way similar to that in Refs.[14, 15], we consider two two-level atoms A and B interacting respectively with single mode cavity fields, a and b. The interaction between A and a is described by using the standard JC model, while that between B and b is depicted by using the two-photon JC model. The other difference between our model and that in Refs.[14, 15] is that there is an additional Kerr medium in the cavity b. So, the interaction between B and b is described by using two-photon JC model with an added Kerr medium.[22] Physically this model may be realized as if the cavity contains two

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†E-mail: qzhengnju@gmail.com

different species of Rydberg atoms: one behaves as a two-level atom and the other as the Kerr medium.[23] For simplicity, we call it Kerr medium in the following.

The rest of the paper is organized as follows. In Section 2, we review the measure of entanglement concurrence adopted in this paper. Section 3 is devoted to the entanglement dynamics of two atoms. Specifically, we discuss two types of atomic initial states. An ESD occurs only for a certain initial atom entanglement state. Adjusting the Kerr nonlinear coefficient in the cavity b, we can effectively control the occurrence of ESD. Finally the conclusions are given in Section 5.

2. Measure of entanglement

For a bipartite state, there are some equivalent measures of the degree of entanglement, such as concurrence, entanglement entropy, negativity, and tangle. In this paper, we adopt the Wootters concurrence.[24] For two qubits, the concurrence can be calculated by

$$C(\rho) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}), \quad (1)$$

where λ_i $(i = 1, 2, 3, 4)$ are the eigenvalues in a decreasing order of the matrix

$$\boldsymbol{R} = \boldsymbol{\rho}(\boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y)\boldsymbol{\rho}^*(\boldsymbol{\sigma}_y \otimes \boldsymbol{\sigma}_y), \quad (2)$$

where $\boldsymbol{\rho}^*$ is the complex conjugation of $\boldsymbol{\rho}$ in the standard basis and $\boldsymbol{\sigma}_y$ is the Pauli matrix

$$\boldsymbol{\sigma}_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}. \quad (3)$$

The concurrence is monotone for a bipartite entanglement, $C = 0$ corresponds to a separable state, and $C = 1$ to a maximal entanglement one. With a density matrix

$$\boldsymbol{\rho} = \begin{pmatrix} a & 0 & 0 & w \\ 0 & b & z & 0 \\ 0 & z^* & c & 0 \\ w^* & 0 & 0 & d \end{pmatrix}, \quad (4)$$

the concurrence can be easily obtained as[25]

$$C(\boldsymbol{\rho}) = 2\max\{0, |z| - \sqrt{ad}, \quad |w| - \sqrt{bc}\}. \quad (5)$$

3. Entanglement dynamics of two atoms

The JC model is one of exactly solvable models in quantum optics,[26] which describes the near-resonant interaction between a single two-level atom and a single mode field. The JC model has been realized experimentally in several ways. In this paper, in a way similar to that in Refs.[14, 15], we consider two two-level atoms A and B, interacting with single mode cavity fields, a and b respectively. The Hamiltonian of the whole system is

$$H = H_1 + H_2. \quad (6)$$

The interaction between the atom A and its cavity a is described by the typical JC model, and that between the atom B and its cavity b features the two-photon process with added Kerr medium. In the dipole and rotating wave approximation, H_1 and H_2 become

$$H_1 = w_1\sigma_z^A + \nu_1 a^\dagger a + g(a^\dagger\sigma_-^A + a\sigma_+^A),$$

and

$$\begin{aligned} H_2 =& w_2\sigma_z^B + \nu_2 b^\dagger b + k(b^{\dagger 2}\sigma_-^B + b^2\sigma_+^B) \\ &+ \chi b^{\dagger 2}b^2. \end{aligned} \quad (7)$$

Here w_1 and w_2 are the atomic transition frequencies, ν_1 and ν_2 are the frequencies of cavity modes, g and k each are the strength of interaction between the cavity and atoms, χ is the dispersive part of the third-order nonlinearity of Kerr medium, $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$ is the atomic inversion operator, $\sigma_\pm$ $(\sigma_+ = |e\rangle\langle g|$, $\sigma_- = |g\rangle\langle e|)$ are the raising and lowering operators of atom, and $a^\dagger(a)$ and $b^\dagger(b)$ are the annihilation (creation) operators of the two cavity fields.

From H, we can see that there is no interaction between cavities a and b, and between the cavity $b(a)$ and atom A (B). We consider only the case where the two cavities are lossless. Obviously, there are different concurrences between different subsystems, such as $C_{\mathrm{A}a}$, C_{AB}, $C_{\mathrm{B}a}$, $C_{\mathrm{B}b}$, etc. In this paper, we focus on the concurrence of two atoms, C_{AB}.

Now we first discuss the action of Kerr Hamiltonian,

$$\chi b^{\dagger 2}b^2|n\rangle = \chi n(n-1)|n\rangle. \quad (8)$$

Expression (8) demonstrates that the Kerr Hamiltonian does not couple the different photon states, and has no effect on the photon states with $n = 0, 1$. In

practice, Kerr nonlinear coefficient is the third order nonlinear interaction and usually very small. Fortunately, giant Kerr nonlinearities have been theoretically predicted[27] and firstly experimentally measured to be 10^6 greater than those in the conventional optical materials.[28] And the Kerr nonlinearities can be enhanced by several orders of magnitude in ultracold atomic systems by using electromagnetically induced transparency.[29] Therefore, it is assumed that in this paper the Kerr coefficient can be adjusted in the range $[0, 10]$.

For simplicity, the initial condition of two atoms is assumed to be in the following form:

$$|\varphi_{\rm atom}\rangle = \cos(\alpha)|eg\rangle + \sin(\alpha)|ge\rangle, \tag{9}$$

where the first index represents the state of atom A and the second one refers to the state of B. When $\alpha = \pi/4$, the $|\varphi_{\rm atom}\rangle$ reduces to the Bell state. And the initial states of the two cavities are in vacuum state

$$|\varphi_{\rm c}\rangle = |00\rangle, \tag{10}$$

where the first index represents the state of cavity a and the second one refers to the state of b. Therefore, the initial condition of the whole system is obtained as

$$|\varphi_0\rangle = |\psi_{\rm c}\rangle|\psi_{\rm atom}\rangle. \tag{11}$$

With the initial state of the whole system $|\varphi_0\rangle$, the state of system at any time t can be written as

$$\begin{aligned}|\varphi(t)\rangle =& x_1|eg00\rangle + x_2|gg10\rangle + x_3|ge00\rangle \\ &+ x_4|gg02\rangle.\end{aligned} \tag{12}$$

Substituting the state $|\varphi(t)\rangle$ into the Schrödinger equation ($\hbar$=1), under the resonance conditions, one can obtain the equations of amplitudes x_i ($i = 1, 2, 3, 4$) as

$$\begin{aligned}{\rm i}\dot{x_1} &= gx_2, \\ {\rm i}\dot{x_2} &= gx_1, \\ {\rm i}\dot{x_3} &= \sqrt{2}kx_4,\end{aligned}$$

and

$${\rm i}\dot{x_4} = \sqrt{2}kx_3 + 2\chi x_4. \tag{13}$$

The reduced density matrix of atoms A and B is

$$\rho_{\rm AB} = {\rm Tr_c}[|\varphi(t)\rangle\langle|\varphi(t)|], \tag{14}$$

where $\rm Tr_c$ denotes tracing over the photon states.

In the basis $\{\, |ee\rangle, |eg\rangle, |ge\rangle, |gg\rangle \,\}$, $\rho_{\rm AB}$ is rewritten as

$$\rho_{\rm AB} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & |x_1|^2 & x_3x_1^* & 0 \\ 0 & x_1x_3^* & |x_3|^2 & 0 \\ 0 & 0 & 0 & |x_2|^2 + |x_4|^2 \end{pmatrix}. \tag{15}$$

Figures 1 and 2 show the temporal evolutions of concurrence C of reduced atomic density matrix $\rho_{\rm AB}$. The parameters in Fig.1 are $\alpha = \pi/4$, $g = 1$, and $k = 0.2$. Figures 1(a), 1(b), 1(c), and 1(d) correspond to $\chi = 0$, 0.1, 2, and 5, respectively. There is no ESD between two atoms A and B for an initial state $|\varphi_0\rangle$. The similar conclusions have been obtained in Refs.[14, 15] and Ref.[18].

Figure 1 shows another property of entanglement dynamics of two atoms. When there is no Kerr medium, i.e. $\chi = 0$ (Figs.1(a) and 2(a)), the temporal evolution of the concurrence fluctuates without any distinct period. With the increase of Kerr nonlinearity coefficient, the fluctuation of the concurrence declines and it becomes quasi-periodic. Therefore, the Kerr medium can stabilize the entanglement between the two atoms, which may be useful in quantum information process based on the entanglement. Figure 2 has a different parameter setting and displays the similar property to that in Fig.1.

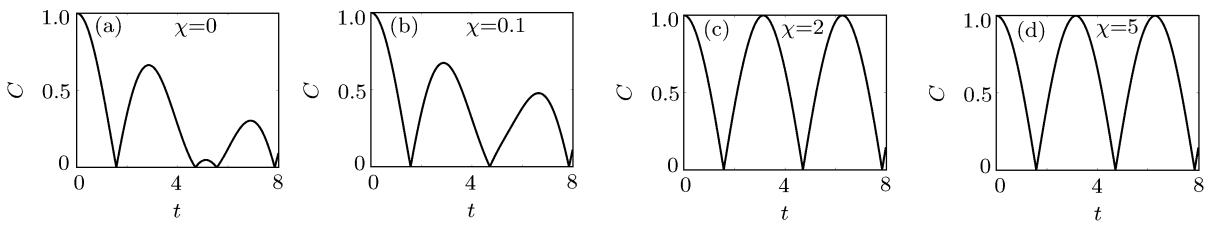


Fig.1. Evolutions of the concurrence of $\rho_{\rm AB}$ for an initial state of atoms $|\varphi_{\rm atom}\rangle = \cos(\alpha)|eg\rangle + \sin(\alpha)|ge\rangle$ with $\alpha = \pi/4$, $g = 1$ and $k = 0.2$, showing no entanglement sudden death.

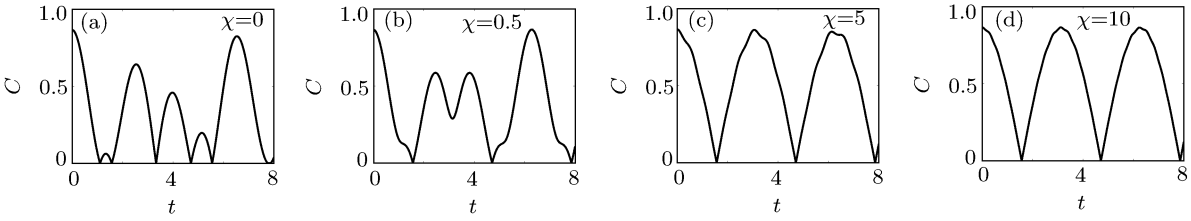


Fig.2. Evolutions of the concurrence of $\rho_{\rm AB}$ for an initial state of atoms $|\varphi_{\rm atom}\rangle = \cos(\alpha)|eg\rangle + \sin(\alpha)|ge\rangle$ with $\alpha = \pi/6$, $g = 1$ and $k = 1$, showing no entanglement sudden death.

Alternatively, we consider other type of initial state

$$|\phi_0\rangle = |\phi_c\rangle|\phi_{\rm atom}\rangle, \tag{16}$$

with

$$|\phi_{\rm atom}\rangle = \cos(\alpha)|ee\rangle + \sin(\alpha)|gg\rangle,$$
$$|\phi_{\rm c}\rangle = |00\rangle. \tag{17}$$

If the initial state of the whole system is $|\phi_0\rangle$, the state of system at any time t can be written as

$$|\phi(t)\rangle = y_1|ee00\rangle + y_2|gg00\rangle + y_3|ge10\rangle + y_4|eg02\rangle + y_5|gg12\rangle. \tag{18}$$

Substituting the state $|\phi(t)\rangle$ into the Schrödinger equation ($\hbar$=1), one can obtain the equations of amplitudes under the resonance conditions

$$\begin{aligned} {\rm i}\dot{y_1} &= gy_3 + \sqrt{2}ky_4, \\ {\rm i}\dot{y_2} &= 0, \\ {\rm i}\dot{y_3} &= gy_1 + \sqrt{2}ky_5, \\ {\rm i}\dot{y_4} &= \sqrt{2}gy_1 + 2\chi y_4 + gy_5, \end{aligned}$$

and

$${\rm i}\dot{y_5} = \sqrt{2}ky_3 + 2\chi y_5 + gy_4. \tag{19}$$

The reduced density matrix of atoms A and B is

$$\rho_{\rm AB} = {\rm Tr_c}[|\phi(t)\rangle\langle|\phi(t)|]. \tag{20}$$

In the basis { $|ee\rangle$, $|eg\rangle$, $|ge\rangle$, $|gg\rangle$ }, $\rho_{\rm AB}$ is rewritten as

$$\rho_{\rm AB} = \begin{pmatrix} |y_1|^2 & 0 & 0 & y_2y_1^* \\ 0 & |y_4|^2 & 0 & 0 \\ 0 & 0 & |y_3|^2 & 0 \\ y_1y_2^* & 0 & 0 & |y_2|^2 + |y_5|^2 \end{pmatrix}. \tag{21}$$

Figures 3 and 4 are the temporal evolutions of concurrence $C(\rho_{\rm AB})$. In Fig.3, the parameters are $\alpha = \pi/4$, $g = 1$, and $k = 0.2$. Figures 3(a), 3(b), 3(c), and 3(d) correspond to χ=0, 0.1, 2, and 5, respectively. There is an ESD effect between two atoms in this initial state when there is no Kerr medium [Fig.3(a)]. Gradually increasing the Kerr nonlinearity coefficient, the ESD disappears. That means the Kerr medium can efficiently repress the ESD. Therefore, one can control the occurrence of ESD by using the Kerr medium. This may be useful for the quantum information process based on the entanglement. When the entanglement is needed, we turn on the Kerr effect; when it is not needed, we can turn off it. The parameter setting in Fig.4 is different from that in Fig.3, but two effects, ESD and Kerr depressing ESD, are similar.

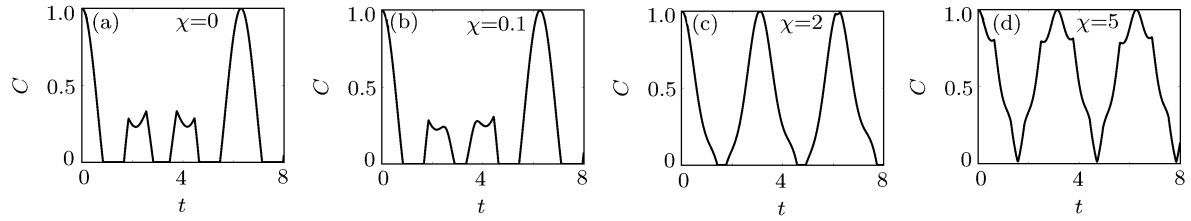


Fig.3. Evolutions of the concurrence of $\rho_{\rm AB}$ for an initial state of atoms $|\varphi_{\rm atom}\rangle = \cos(\alpha)|ee\rangle + \sin(\alpha)|gg\rangle$ with $\alpha = \pi/4$, $g = 1$ and $k = 0.2$, showing no Kerr medium only in (a) and no ESD only in (d).

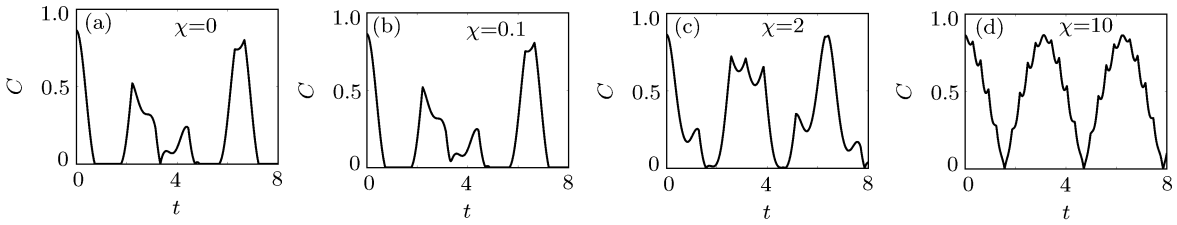


Fig.4. Evolutions of the concurrence of $\rho_{\rm AB}$ for an initial state of atoms $|\varphi_{\rm atom}\rangle = \cos(\alpha)|ee\rangle + \sin(\alpha)|gg\rangle$ with $\alpha = \pi/6$, g=1 and k=1, showing no Kerr medium only in (a) and no ESD only in (d).

Finally, we discuss why the Kerr medium can depress the ESD from the view point of population dynamics. The two panels in Fig. 5 correspond to Kerr nonlinear coefficient $\chi = 0$ and $\chi = 10$. In each panel, from top to bottom, the sub-figure shows the temporal evolutions of the concurrence C, and the population $P_i = |y_i|^2$ ($i = 1,\ 3,\ 4,\ 5$), separately. Note $P_1 + P_3 + P_4 + P_5$=Const as P_2 is invariant. On the other hand, there are entanglement invariants[15] in the two closed JC models. Therefore, it is believed that the reason why the ESD of $\rho_{\rm AB}$ disappears may be due to the transferring of the entanglement into other subsystems. In our second atomic initial state case, the total entanglement of system originates from the initial entanglement state of atom $|\phi_{\rm atom}\rangle = \cos(\alpha)|ee\rangle + \sin(\alpha)|gg\rangle$. Note that the entanglement between atoms A and B is mostly determined by P_1. When $P_1 \simeq 0$, there exists the ESD. This is just the case of the second curve of $\chi = 0$ in Fig.5. The reason why $P_1 \simeq 0$ is population transferring to P_3, P_4, and P_5 due to the interactions between the atoms and cavities. As mentioned in Section 1, the Kerr medium has the effect only on the photon state $|n\rangle$ ($n \geq 2$). When $\chi = 10$, then $P_4 \simeq P_5 \simeq 0$ and $P_1 \neq 0$, i.e. the Kerr medium inhibits the amplitudes of state $|gg12\rangle$ and $|eg02\rangle$. Therefore, there is no ESD.

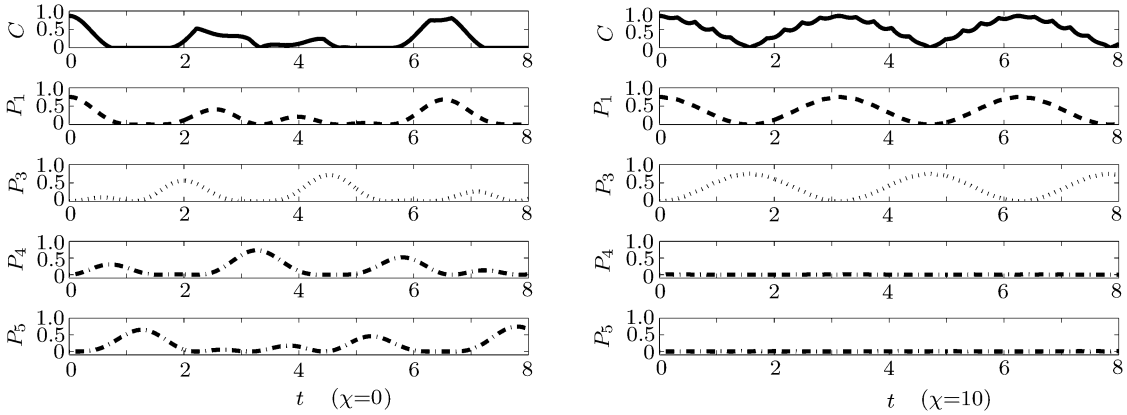


Fig.5. The temporal evolutions of the concurrence C and the population distributions $P_i = |y_i|^2$ ($i = 1,\ 3,\ 4,\ 5$) for an initial state of atoms $|\varphi_{\rm atom}\rangle = \cos(\alpha)|ee\rangle + \sin(\alpha)|gg\rangle$ with $\alpha = \pi/6$, $g = 1$ and $k = 1$.

4. Conclusion

By using the entanglement degree concurrence, we investigate the entanglement dynamics of two two-level atoms A and B that interact with single mode cavity field, a and b, respectively. The interaction between atom A and cavity a may be described by using the typical JC model, while that between atom B and cavity b features a two photon process with added Kerr medium. We study two types of atomic initial states, and find there occurs the entanglement sudden death effect only for a certain initial atom state. By adjusting the Kerr nonlinear coefficient in the cavity b, one can effectively control the occurrence of entanglement sudden death when it is not desirable. This may be useful for quantum information processing based on the entanglement. From the view point of population dynamics, we also discuss the reason why the Kerr medium can repress entanglement sudden death.

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