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A scheme for demonstration of four-photon de Broglie wavelength*

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We propose a scheme to effectively generate a four-photon path-entangled number state [the NOON state i.e. $\frac{1}{\sqrt{2}}(|N,0\rangle + |0,N\rangle)$] for the demonstration of four-photon de Broglie wavelength. Our scheme requires only linear optical elements, photon detectors and post-selections which are all within the reach of current technology.

Keywords: quantum information, NOON states, linear optical elements
PACC: 0365, 4250

Recently, much attention[1−13] has been focused on the path-entangled number states, e.g. the so-called NOON state

$$|\mathrm{NOON}\rangle = \frac{1}{\sqrt{2}}(|N,0\rangle + \mathrm{e}^{\mathrm{i}\phi}|0,N\rangle). \quad (1)$$

These two-mode N-photon maximally entangled states are particularly interesting because a single-photon phase shift of χ induced in one of the two components changes the relative phase of the two terms by $N\chi$. Thus, an effective de Broglie wavelength λ/N can be attributed to this quantum state. It is just like the case of a heavy massive molecule[14] consisting of N atoms, though here the photons are definitely not bound to each other. This special property of NOON state may be utilized to enhance the spatial resolution in microscopy and lithography[15] or to make the interferometry and spectroscopy achieve a phase sensitivity at the Heisenberg limit.[16−18] Thus, it is of great interest to produce NOON states. And various schemes have been proposed theoretically[1−4,7−9,12,13] and realized experimentally.[5,6,10,11,18] In this paper, we propose a new scheme to effectively generate an NOON state for the demonstration of four-photon de Broglie wavelength. Compared with former proposal,[5] our scheme is simple and robust, and can obtain a much higher four-photon count rates.

First, we consider two beams of collinear type-II parametric-down converted (PDC) photon pairs, which can be written in the following form:

$$|\psi_a\rangle = |0\rangle + \alpha|\mathrm{HV}\rangle_a + \alpha^2|\mathrm{HHVV}\rangle_a + ..., \quad (2)$$

and

$$|\psi_b\rangle = |0\rangle + \alpha|\mathrm{HV}\rangle_b + \alpha^2|\mathrm{HHVV}\rangle_b + ..., \quad (3)$$

where H and V denote the horizontal and the vertical polarizations respectively, a and b denote the path modes a and b, α is the parameter proportional to the pump pulse amplitude. It is straightforward to write the complete product state of the two PDC photon beams as

$$\begin{aligned}|\psi_{ab}\rangle &= |\psi_a\rangle \otimes |\psi_b\rangle \\ &= |0\rangle + \alpha(|\mathrm{HV}\rangle_a + |\mathrm{HV}\rangle_b) + \alpha^2(|\mathrm{HHVV}\rangle_a \\ &\quad + |\mathrm{HHVV}\rangle_b + |\mathrm{HV}\rangle_a|\mathrm{HV}\rangle_b) + \ldots. \end{aligned} \quad (4)$$

We can easily find that the two-photon component $\alpha(|\mathrm{HV}\rangle_a + |\mathrm{HV}\rangle_b)$ is just a two-photon NOON state in which two photons are either in mode a and zero in mode b or in mode b and zero in mode a. What about the four-photon component? Unfortunately, it is not in a state of the form $|4,0\rangle + |0,4\rangle$, but of the form $|4,0\rangle + |0,4\rangle + |2,2\rangle$.

If all these four photons are of the same polarization, it would be impossible to take out the unwanted components $|2,2\rangle$ from the state of $|4,0\rangle + |0,4\rangle + |2,2\rangle$.

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But if they are of opposite polarizations, just like in our case, it would be possible to distill the four-photon NOON state $|4,0\rangle+|0,4\rangle$ out by using a novel strategy.

What is the novel strategy? It is measurement. We will design a special measurement that responds only to the first and second terms, i.e. the NOON state, but not to the third term, i.e. $|2,2\rangle$, so that there will be no contribution from the $|2,2\rangle$ term and effectively we will obtain the four-photon NOON state.

A schematic of the experimental setup is depicted in Fig.1. Ultra-violet (UV) pump pulses pass through two nonlinear crystals that have been cut for collinear type-II phase matching. Each crystal has a small probability to emit a beam of infrared (IR) photon pairs with opposite polarizations—H and V. After separating the UV pulses from IR pulses by passing the beam through two dichroic mirrors (DM), the four-photon component of the two IR beams can be written as

$$\frac{1}{\sqrt{3}}(|\mathrm{HHVV}\rangle_a + \mathrm{e}^{\mathrm{i}4\Delta\phi}|\mathrm{HHVV}\rangle_b + \mathrm{e}^{\mathrm{i}2\Delta\phi}|\mathrm{HV}\rangle_a|\mathrm{HV}\rangle_b), \tag{5}$$

where $\Delta\phi$ is the difference in phase between paths a and b.

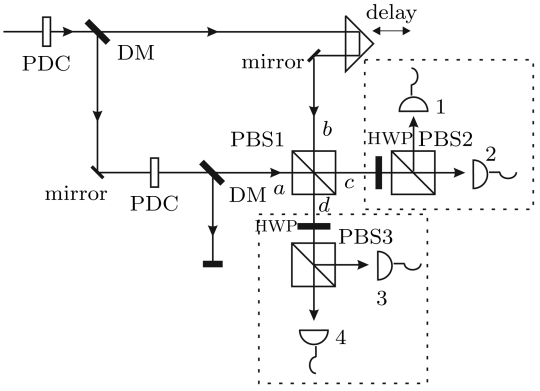


Fig.1. Schematic setup for the demonstration of four-photon de Broglie wavelength, where PDC stands for parametric down-conversion crystal; DM, dichroic mirror; PBS, polarization beamsplitter; HWP, half-waveplate with its optical axis at 67.5°. The equipments in the dash box are used to perform a measurement which responds only to $|\mathrm{HH}\rangle$ or $|\mathrm{VV}\rangle$ component but not to $|\mathrm{HV}\rangle$ component.

In order to obtain the four-photon NOON state, the $|\mathrm{HV}\rangle_a|\mathrm{HV}\rangle_b$ component should be wiped off. Thus, the two IR photon beams are then steered to a polarization beam splitter (PBS1) where the path length of each beam has been adjusted such that they arrive at the PBS almost simultaneously. Here "almost simultaneously" means that the difference in path between paths a and b is smaller than the IR photon coherence length. As the PBS transmits H and reflects V polarization, after passing through the PBS, the four-photon state can be converted into

$$\frac{1}{\sqrt{3}}(|\mathrm{HH}\rangle_c|\mathrm{VV}\rangle_d + \mathrm{e}^{\mathrm{i}4\Delta\phi}|\mathrm{VV}\rangle_c|\mathrm{HH}\rangle_d + \mathrm{e}^{\mathrm{i}2\Delta\phi}|\mathrm{HV}\rangle_c|\mathrm{HV}\rangle_d). \tag{6}$$

Before arriving at detectors, both photons in modes c and d further pass through a half-waveplate(HWP) and a PBS respectively. The optical axes of both HWPs are at 67.5°, converting $|\mathrm{H}\rangle$ and $|\mathrm{V}\rangle$ into $|+\rangle$ and $|-\rangle$ polarizations respectively, where $|+\rangle = \frac{1}{\sqrt{2}}(|\mathrm{H}\rangle + |\mathrm{V}\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}}(|\mathrm{H}\rangle - |\mathrm{V}\rangle)$. Thus, the four-photon state after the two HWPs can be written as

$$\frac{1}{\sqrt{3}}(|++\rangle_c|--\rangle_d + \mathrm{e}^{\mathrm{i}4\Delta\phi}|--\rangle_c|++\rangle_d + \mathrm{e}^{\mathrm{i}2\Delta\phi}|+-\rangle_c|+-\rangle_d). \tag{7}$$

After photons in modes c and d have passed through PBS2 and PBS3 respectively, a coincidence detection of all four outputs is then performed. Only the first two items in expression (7) make contributions to the fourfold coincidence. This can be easily understood by the fact below. Two photons $|+-\rangle$ in the same spatial mode passing through a PBS will never exit from different ports, for the state can be rewritten in the following form:

$$|+-\rangle = \frac{1}{2}(|\mathrm{H}\rangle + |\mathrm{V}\rangle)(|\mathrm{H}\rangle - |\mathrm{V}\rangle) = \frac{1}{2}(|\mathrm{HH}\rangle - |\mathrm{VV}\rangle). \tag{8}$$

As a result, we can say that, by post-selecting four photons which respectively arrive at detectors 1-4 at the same time, a four-photon NOON state

$$\frac{1}{\sqrt{2}}(|\mathrm{HHVV}\rangle_a + \mathrm{e}^{\mathrm{i}4\Delta\phi}|\mathrm{HHVV}\rangle_b) \tag{9}$$

has been effectively produced before PBS1. Thus, the fourfold coincidence rate of detectors 1–4 would be proportional to $1 + \cos 4\Delta\phi$, that is a fourfold reduction in wavelength (Fig.2(c)). A schematic of this relation is depicted in Fig.2, comparing the resulting pure four-photon interference effect (Fig.2(c)) with the well-known two-photon interference (Fig.2(b)) and the single-photon interference (Fig.2(a)) obtained with the same set-up.

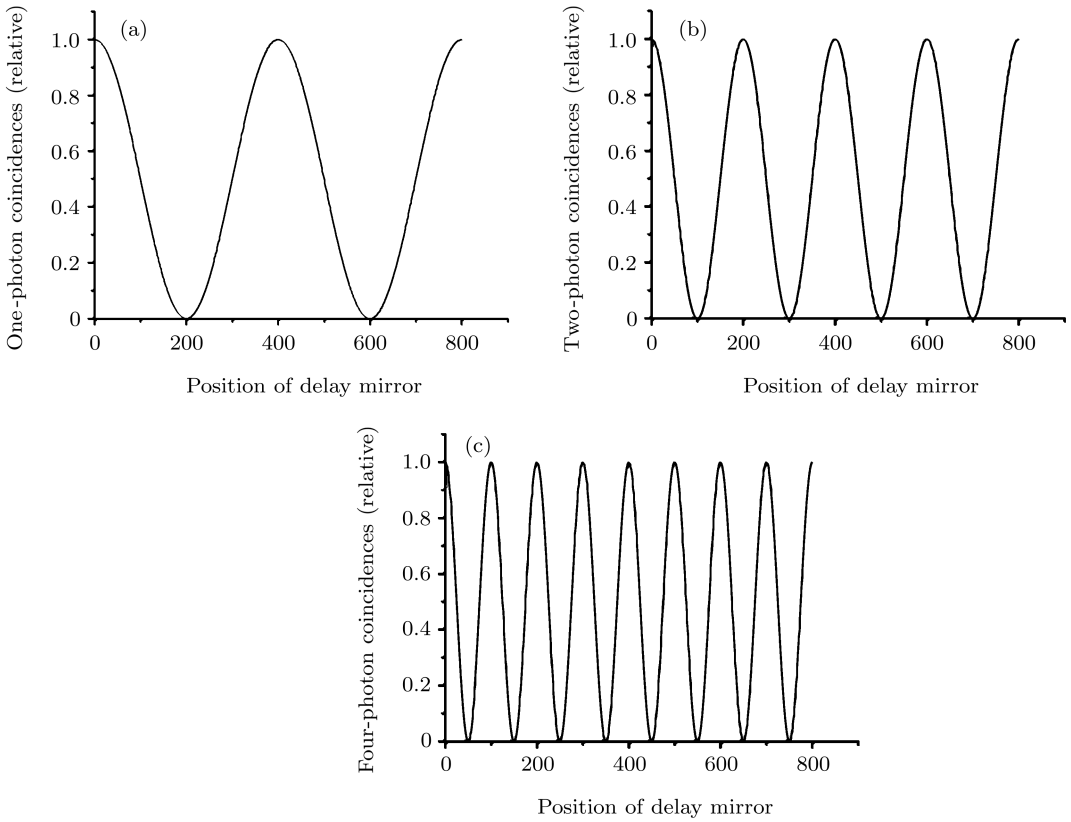


Fig.2. Pure one-, two- and four- photon interference, where the wavelength of IR photons is assumed to be 800 nm. The count rates in all these three figures are relative values. Subfigure a is for the single photon rate corresponding to the detection in mode 2. Subfigure b for the two-photon coincidence rate corresponding to the detection in modes 2 and 4. Subfigure c for The four-photon coincidence rate corresponding to the detection in modes 1–4.

Of course, in a real experiment, the visibility cannot be as that perfect as that shown in Fig.2. The imperfections of real states should be mainly caused by two reasons. First, the partial distinguishability of independent photons will cause some incoherent mixtures. Second, high-order emission of PDC pairs, especially the six-photon emissions, will give rise to undesired components.

Although our scheme has some similarities to Walther's scheme,[5] there are some significant differences between the two schemes. First, the starting state are different: in our scheme, we use two collinear type-II PDC processes to produce a state in the form of expression (5) whereas Walther's scheme starts with two entangled photon pairs that are produced by two non-collinear type-II PDC processes. All of the down-converted photons are collected in our scheme whereas in Walther's scheme only a small portion is collected. As a result, when two pump lasers have the same power, the intensity of four-photon emission in our scheme would be much higher than that in Walther's scheme, which makes our scheme more applicable to real experiments. Second, the four photons in Walther's scheme are locally separated whereas in our scheme, they are all together in the same spatial mode which shows a more visual 'four-photon' image.

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