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Information diffusion on adaptive network*

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Based on the adaptive network, the feedback mechanism and interplay between the network topology and the diffusive process of information are studied. The results reveal that the adaptation of network topology can drive systems into the scale-free one with the assortative or disassortative degree correlations, and the hierarchical clustering. Meanwhile, the processes of the information diffusion are extremely speeded up by the adaptive changes of network topology.

Keywords: adaptive network, information diffusion, degree correlation, hierarchical clustering
PACC: 0250, 0590

1. Introduction

The past few years have witnessed a hectic activity in the complex network.[1−4] Particularly, much attention is paid to two distinct directions. On the one hand, statistical physics methods have been developed to uncover topological features of various real-world networks[5−7] and microcosmic mechanisms leading to these features.[5,6,8−10] On the other hand, many dynamical processes, such as epidemic spreading,[11−13] traffic dynamics,[14−16] synchronization of network,[17,18] and diffusive processes,[19−23] have been intensively investigated to uncover the influence of network topology on various dynamical processes. These studies not only provide deep insight into the formation and evolution of many real-world networks, but also promote the understanding of various dynamical processes taking place in them.

Usually, the evolution of networks and the dynamical processes on networks are studied separately. However, less attention has been devoted to the mechanism of interplay and feedback between them. In fact, the interplay exists universally in various real complex systems, which can significantly influence both the topological properties of the networks and the dynamical behaviours of many processes.[24] For example, in many technical distribution networks like power grids,[25] the mail network, the Internet or wireless communication networks,[26−28] the topologies can influence the dynamical states of nodes, i.e., the flow or the density of traffic. In reverse, a high load on a given link or node can cause traffic jams or electrical line failures, even an avalanche or cascading. The component failures will result in the break of links or invalidation of nodes. More examples of the interplay phenomena can be found in information networks like neural or genetic networks (see Ref.[29] and references therein). Benefited from the need to understand the nature and effects of these interplays on both the network topologies and dynamical processes, some theoretical works, based on the adaptive network, have begun to study the interplays between various dynamical processes and the evolutionary of network. A pioneer of researches in adaptive networks is to discuss a variant of the famous Bak–Sneppen model of macro-evolution which combines the population's evolutionary fitness of each node with the evolutionary dynamic of network.[30] Their results suggest that adaptive networks are capable of robust self-organization of their topology.[30] Recently, the adaptive synchronization, the traffic and epidemic dynamics have been also introduced into network evolutionary,[9,31−33] which have not only given rise to richer dynamical features, but also uncovered the profound effects of the interplay on the network topology.

As is well known, the processes of information diffusion frequently occur in both social and technological systems,[19−23] which can significantly influence network topology. For instance, in such technolog-

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ical network as World Wide Web (WWW), a great deal of information is generated and transmitted every day. If the information is of considerable value, the webs without this information tend to build the hyperlink to the web with the information. Similarly, in social systems, many innovations, important news go the rounds, where people or companies are more likely to maintain social connections with the informed ones. In reverse, these adaptive changes of connections can affect the dynamical process of information diffusion. However, many studies about the information diffusion have only focused on the static network.[19−23] Dynamical behaviours of information diffusion on the adaptive network has been rarely investigated. The effects of information diffusion on the topological properties of the network are not well understood. In this paper, we intend to explore the adaptive network by combining the dynamics of information diffusion, which is expected to reveal qualitative or quantitative effects of the adaptive nature on both the structure of network and the dynamical behaviour of the information diffusion.

2. The model

We here apply a standard model of information diffusion, which is based on cellular counters (also called voter dynamics).[34] In this model, a node in the network representing an individual can be in two discrete states: informed (I) or unaware (U). Connections between individuals along which the information can spread are represented by links. In each time step, an unaware node obtains information with a probability λ if one or more of its neighbours are in an informed state. Meanwhile, we consider that the unaware nodes rewire their U-U links to randomly selected informed nodes with a probability p, in order to obtain the information more quickly. The probability p denotes the degree of enthusiasm that individuals desire this information, and also reflects the value of information. Finally, double connections are prohibited in the rewiring process.

3. Statistical properties of the adaptive networks

Initially, the network is a random graph where the links are distributed homogeneously. At $t = 0$, one node selected randomly from network carries information ($I = 1$), and is called the Information Source (or simply the Source); all other nodes are unaware, i.e. without information ($I = 0$). Following the rule of the model mentioned above, information will flow throughout the network from the Source, and the rewiring process is terminated since all nodes are in the informed state.

3.1. Degree distribution

Many real-world networks are scale-free. The degree distribution $P(k)$ of these networks, which gives the probability that a node in the network possesses

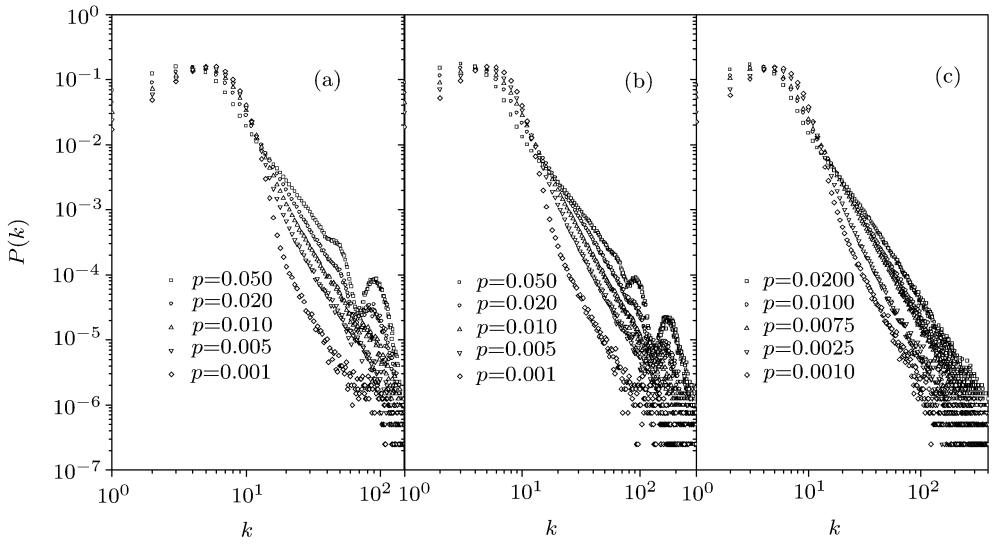


Fig.1. Degree distributions for the networks with three different spreading probabilities λ=0.02 (a), 0.01 (b), and 0.005 (c). The simulations have been run in networks with the average degree $< k > \approx 6$ containing 10000 nodes and averaging over 400 different realizations.

k links, follows the power-law form $P(k) \propto k^{-\gamma}$, with exponents varying in the range $2 < \gamma < 3$.[1−3,6] In present adaptive network, the scale-free feature is also reserved. Several examples of power-law degree distributions are illustrated in Fig.1. The scale-free regime could be well defined despite the deviation in the small degree region and the slope does not depend on the network size in the double-logarithmic plot. Moreover, the exponent γ varies with the parameter p and the spreading rate λ. For several different p and λ, the exponent γ as a function of p is shown in Fig.2. We find that γ decreases with the increase of p. For the same value of p, the larger the λ, the larger the γ. It should be mentioned that the scale-free feature can emerge only when the parameter p is close to the spreading rate λ, i.e., $p \approx \lambda$. It indicates that both the parameter p and the spreading rate λ are not independent of each other. The value of p reflects the importance of information. The larger the p is, the more important the information is, which would lead to a higher spreading probability.

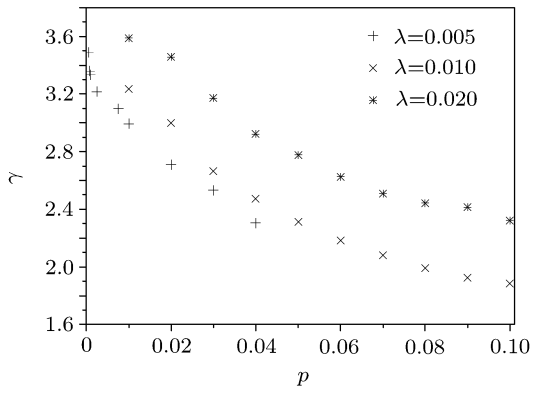


Fig.2. Exponent γ in $P(k) \propto k^{-\gamma}$ as a function of the adaptive rewiring probability p, with λ=0.02, 0.01, and 0.005.

3.2. Degree correlation

Assortative or disassortative mixing patterns of degree (also referred to as the degree-degree correlations) are possessed by many real complex networks.[35] For instance, many social networks exhibit that the nodes with high connectivity (hubs) connect more preferably to the hubs, which are called the assortative mixing networks. On the opposite side, many technological and biological networks show disassortative mixing, i.e., the hubs are preferably connected to the nodes with small degree. In this stage, most of network models that have been proposed to simulate the structures of various real networks are somewhat incomplete due to the absence of these mixing patterns.To explore dynamical mechanism or microcosmic origin of the feature of the degree correlations still continue.

A simple and coarse-grained manner to quantify the degree correlations is the average nearest neighbour function $\overline{k}_{nn}(k)$.[36] It can be calculated from the conditional probability $P(j|k) = P(j,k)\langle k\rangle/[kP(k)]$ as

$$\overline{k}_{nn}(k) = \sum_j jP(j|k). \tag{1}$$

A network with an (de-)increasing $\overline{k}_{nn}(k)$ is called (dis-)assortative correlation. In Fig.3, we report $\overline{k}_{nn}(k)$ as a function of k for the present networks for a fixed spreading probability λ=0.01. An adjustable correlation pattern can be obtained by adjusting the parameter p. We also calculate $\overline{k}_{nn}(k)$ for different spreading probability λ, same result is found, which indicates that the degree correlations likely originated from the adaptation of the systems.

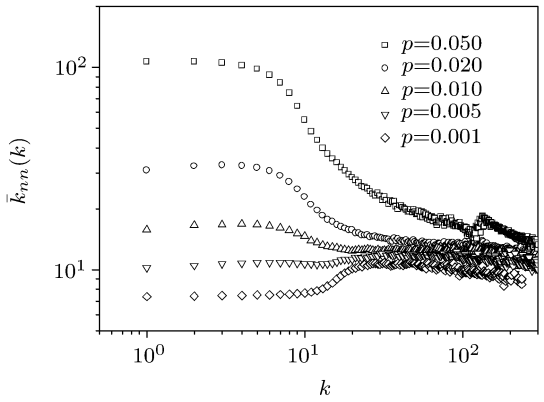


Fig.3. Average degree of the neighbours of a node with connectivity k in the adaptive network, with spreading probability λ=0.01.

3.3. Hierarchical clustering

Hierarchical clustering, existing in many real networks, such as Actor network, WWW and Internet, indicates that these networks are fundamentally modular.[37] In society, the modules represent groups of friends or co-workers; in the WWW, they denote communities with shared interests. This clearly identifiable modular organization is at the origin of the high clustering coefficient observed in many real networks. It can be characterized in a quantitative manner using a scaling law $C(k) - k^{-\beta}$,[38] which describes the dependence of the clustering coefficient on the

node degree. We argue that this scaling law quantifies the coexistence of a hierarchy of nodes with different degrees of clustering and applies to the present adaptive networks as well. In Fig.4, different relations between $C(k)$ and k are shown for λ=0.01. Particularly, when $p \geq 0.1$, it asymptotically follows the $1/k$ law, indicating the presence of the hierarchical architecture. In contrast, when $p \simeq 0.005$, the clustering coefficient is independent of k, i.e., the scaling law does not apply.

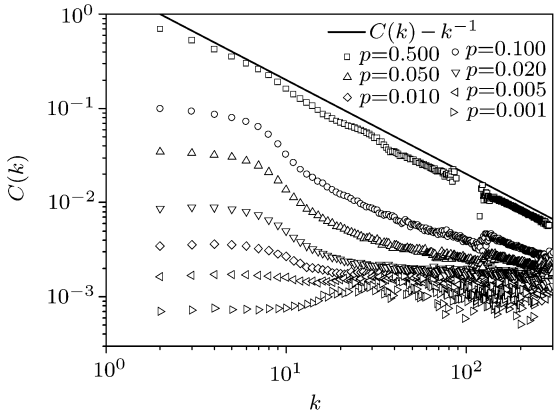


Fig.4. The $C(k)$ curves for different p values. The networks' parameters used are the same as that in Fig.1.

4. Temporal behaviours of the information diffusion

Two quantities are usually used to characterize the temporal behaviours of the information diffusion.[39,40] One is the average density of informed nodes. If $I(t)$ and N denote the number of informed nodes and the total number of nodes in the network respectively, then the density of informed nodes is given by $i(t) = I(t)/N$. The average density of informed nodes is computed by the average of $i(t)$ over many independent realizations. Another is the spreading velocity, which is defined as

$$V_{\rm in}(t) = \frac{{\rm d}i(t)}{{\rm d}t} \approx \frac{I(t+1) - I(t)}{N}. \qquad (2)$$

To demonstrate the effects of the adaptive rewiring on the diffusive process, we perform extensive numerical simulations for different values of p. Figure 5(a) shows that the smaller the p is, the more slowly the information diffuses. Obviously, all the nodes will be

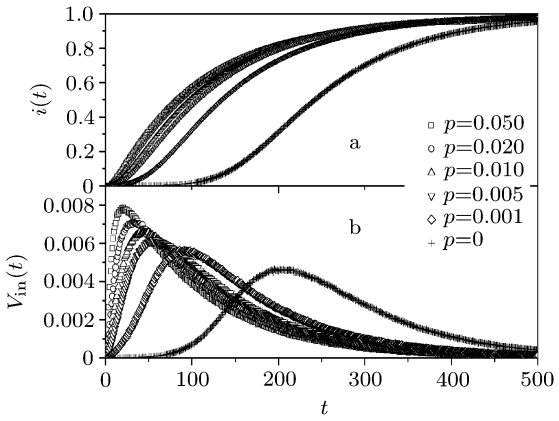


Fig.5. a Average density of informed nodes versus time in an adaptive network of 10000 nodes with average $k < k > \approx 6$. b Corresponding spreading velocity at each moment t in the adaptive networks with the same parameters.

in informed state in the limit of time, i.e., $\lim_{t\to\infty} i(t) = 1$. As is shown in Fig.5(b), the corresponding diffusive velocities have a peak at a certain time step t, where $i(t)$ increases at the fastest rate. This time step is refereed to as the moment of the outbreak or the peak time, which shows a transition from the small incidence of the information to a macroscopic outbreak. Similar to the epidemic diffusion on both un-weighted and weighted scale-free network, the diffusive velocity goes up to a peak quickly.[39,40] These results have not a fundamental deviation from the traditional results of the information diffusion on the scale-free networks.

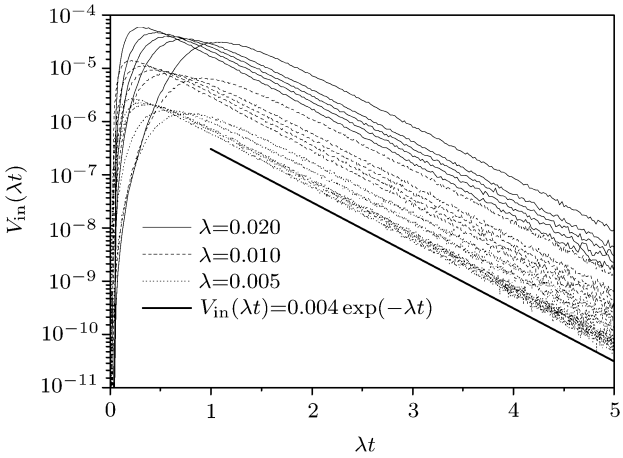


Fig.6. The scaling of the diffusive velocity after the "peak" time for the adaptive networks with different rewiring probabilities at different spreading rates. The networks' parameters used are the same as those in Fig.1.

However, what is interesting, velocity decays exponentially after the peak time (Fig.6)

$$V_{\rm in}(t) \sim \exp[-\lambda t]. \quad (3)$$

The decaying form seems to be independent of networked structures. In addition, by numerical calculating the other models, such as random graph and Barabási and Albert scale-free network,[6] we obtain the same result. This suggests that it might be a general property of the diffusion dynamics. However, the moment of the outbreak or the peak time is different. Comparing the result with that on random graph, we see that it is smaller on the adaptive network, indicating that the process of the information diffusion is speeded up by the adaptive rewiring of networks.

5. Summary and discussion

We have investigated the feedback mechanism and the interplay combining the dynamic of information diffusion with the evolutionary dynamic of network. In addition to the scale-free feature, two important properties of real networks, the degree correlations and hierarchical clustering, are affected by the adaptive rewiring mechanism, which are rarely encompassed by a single network model in the previous studies. Moreover, the temporal behaviours of information diffusion are also strongly affected by the adaptive mechanism.

It is worth mentioning that in this paper only a simple example of the adaptive network is presented. Richer dynamics may be expected in more complex and strict models. For instance, irrespective of the kind of object to be transmitted, a realistic diffusive model should take into account whether the object passed from one individual to another is modified during the process. In particular, information might be continuously revised while flowing throughout the network. Some recent works considered the degradation of information during transmission processes and investigated the information spreading on the static networks.[19,41] In this aspect, more complicated variations of the microscopic rules of adaptive rewiring may be implemented to reveal important qualitative and quantitative effects on both the dynamics of networks and information spreading.

In addition, the information, such as public opinion, technological innovation, or new idea, can spread by replication rather than transference, i.e., they could be copied with some freedom and without conservation of the total number. However, in some spreading systems, the objects to be transmitted might be conservative, i.e., they spread by transference rather than replication.[42] Therefore, one can consider the spreading of the objects with conservation on an adaptive network, which possibly enrich both the topological properties and the dynamical features.

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