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Modification of gravitational redshift of x-ray burst produced by pulsar surface magnetoplasma*

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In this paper, the propagation of x-ray bursts in the magnetoplasma of pulsar magnetosphere is discussed. The electromagnetic interaction between x-ray bursts and magnetoplasma is described as some geometry. The electromagnetic effects of surface superstrong magnetic field and dynamic effects of outflowing magnetoplasma of pulsars are treated as an optical metric. The Gordon metric is introduced to represent the gravitational metric and optical metric. So the propagation of x-ray bursts in magnetoplasma of pulsars can be described as x-ray bursts transmitting in an effective space characterized by Gordon metric. The modification of gravitational redshift, attributed to the flowing magnetoplasma of pulsars, is obtained and it is shown that the modification is of redshift and can reach the same magnitude as the gravitational redshift for ordinary pulsars.

Keywords: pulsars, x-ray spectra, relativity and gravitation, redshift
PACC: 9760J, 3220R, 9530S, 9850H

1. Introduction

Since the discovery of pulsar by Hewish in 1967, one has been trying to describe the dynamic model of pulsars. Pulsars are explained as rapidly rotating neutron stars with strong magnetic fields by Goldreich and Julian (1969),[1] and the magnetic field in the polar zones of pulsars is about 10^8T. Neutron stars are the consequence of massive stellar evolution, and they are assumed to be composed of neutrons essentially, although some protons and electrons are also required in order to guarantee stability against the Pauli's exclusion principle for fermions. In recent years, some new and exotic pulsars with a superstrong magnetic field ($10^{10}-10^{11}$T), called "magnetars", attract extensive attention.[2,3] They are said to be the best models to explain the dynamics of soft gamma-ray repeaters and anomalous x-ray pulsars.[4] There are also strange models of pulsar such as quark stars and strange magnetic stars.[5−8] The present models of these compact objects still face many unresolved fundamental problems such as what matter the star core is composed of, neutrons or strange quark matter.[9,10]

The constituent of pulsar core is manifested through the stellar mass-to-radius ratio (M/R), and gravitational redshift of spectral lines of emission created in the pulsar photosphere depends on the ratio (M/R). Measuring redshift z at the pulsar surface provides a unique insight into the mass-to-radius ratio and thus into dense matter equation of state, which reflects the physics of the strong interaction among the particles that the star is comprised of. Hence, it is currently argued that the effective and feasible method of determining fundamental properties of pulsars is to measure the gravitational redshift z of spectral lines of emission produced in the pulsar magnetosphere.[11] In actual observation, Cottam *et al* (2002)[12] reported on the discovery of significant absorption lines in the spectra of 28 bursts of the low-mass x-ray binary EXO 0748-676, and they identified the most significant features with the $F_{\rm e}$ XXVI and $F_{\rm e}$ XXV n=2–3 and O VIII n=1–2 transitions, all with a redshift of z=0.35. Based on this redshift, they agreed with the model in which pulsars were composed of normal nuclear matter, and rejected some models in which the pulsars consisted of some exotic matter.

It is well known that pulsar possesses an extremely strong magnetic field and is surrounded by magnetoplasma which is highly relativistic, flowing out with a Lorentz factor about $\gamma_{\rm p} \sim 10 - 10^3$.[13,14] X-ray bursts of pulsar are produced by thermonuclear

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flashes in the freshly deposited outermost layers of an accreting neutron star[15−19] or attributed to nuclear fusion on the surface following sufficient accretion from its low mass companion.[20] After generating on the surface of pulsar, x-ray bursts will propagate in the outflowing magnetoplasma, so the electromagnetic effects and dynamic effects of outflowing magnetoplasma should be considered to describe the overall physics involved on the pulsar surface. Hence, we argue that the surface redshift of spectral lines obtained by Cottam *et al*[12] is deduced not only from the pulsar surface gravitational field but also attributed to the outflowing magnetoplasma. Therefore, we should distinguish the redshift produced by pulsar magnetoplasma from the one produced by surface gravity of pulsars when we use gravitational redshift z of spectral lines to estimate the mass-to-radius ratio of pulsars and dense matter equations of state.

In this paper, the electromagnetic interaction between x-ray bursts and magnetoplasma is described as some geometry. The electromagnetic effect of pulsar super surface magnetic field and dynamic effect of outflowing magnetoplasma are treated as optical metric. Gordon metric[21,22] is introduced to represent the gravitational metric and optical metric. So the propagation of x-ray bursts in magnetoplasma of pulsars can be treated as x-ray bursts transmitting in an effective space specified by Gordon metric.

This paper is organized as follows. In Section 2, a gravitational redshift of emission produced at the pulsar surface is deduced and calculated. In Section 3, by means of the dispersion relation of the x-ray bursts in pulsar superstrong magnetic field, the effective metric experienced by the emission transmitting in the pulsar magnetosphere is deduced. In Section 4, the modification of gravitational redshift is obtained and estimated. In Section 5, we make discussion on the modification of gravitational redshift.

2. Gravitational redshift

It is well known that the redshift of spectral lines of the emission originates from strong gravitational field and is predicted by Einstein's general relativity. The shifting of photon frequency is dependent on the distribution of localized gravitational field, and the redshift z measured by an observer is defined in terms of frequencies

$$1+z=\frac{\nu_{\text{emitter}}}{\nu_{\text{observer}}}, \tag{1}$$

where ν_{emitter} and ν_{observer} are photon frequencies at the emitter and observer respectively. Particles moving in the space–time with static metrics are governed by the localized energy conservation law[23]

$$|g_{00}(r)|^{1/2}E_{\text{local}}=\text{const}, \tag{2}$$

where E_{local} is the localized energy of particles and $|g_{00}(r)|^{1/2}$ denotes the time–time component of metric. As to photons, $|g_{00}(r)|^{1/2}\nu_{\text{local}}=\text{const}$. So expression (1) can be written as

$$1+z=\sqrt{\frac{g_{00}(\text{o})}{g_{00}(\text{e})}}, \tag{3}$$

where $g_{00}(\text{e})$ and $g_{00}(\text{o})$ refer to the time–time components of metrics at the emitter and the observer respectively. In the spherically symmetric Schwarzschild gravitational field, the gravitational redshift of radiation spectra emitted from the stellar surface can be expressed as

$$1+z=\left[1-\frac{2G}{c^2}\left(\frac{M}{R}\right)\right]^{-\frac{1}{2}}, \tag{4}$$

where $M\approx 1.4M_{\odot}$ is pulsar mass($M_{\odot}$ is the mass of the Sun) and R is pulsar radius taken as 10 km. So, the gravitational redshift z is

$$z=\left[1-\frac{2G}{c^2}\left(\frac{M}{R}\right)\right]^{-\frac{1}{2}}-1\approx 0.31. \tag{5}$$

3. The effective metric

Effects of the pulsar magnetic field are neglected in the above analysis. However, pulsar possesses an extremely strong field, and the electromagnetic effects of magnetoplasma should be taken into consideration to describe the overall physics involved on the surface of pulsars. Mosquera and Salim[24] studied the propagation of photons in the background of non-linear electrodynamics (NLED) produced by the superstrong magnetic field of pulsar, and they proved that the redshift induced by NLED effects could be significant as a gravitational effect.

In this paper, the electromagnetic interaction between x-ray bursts and magnetoplasma is described as some geometry. The electromagnetic effects of pulsar super surface magnetic field and dynamic effects of outflowing magnetoplasma are treated as optical metric. Gordon metric is introduced to deal with the gravitational metric and optical metric. So the propagation of x-ray bursts in magnetoplasma of pulsars can be treated as x-ray bursts transmitting in

the free-space determined by Gordon metric. Gordon metric[21,22]

$$G_{\mu\nu} = g_{\mu\nu} + \left(\frac{1}{n^2_{\rm refra}} - 1\right) u_\nu u_\nu \tag{6}$$

is deduced from Maxwell's equations in curved space–time and constitutive relation, which are established on the general space–time manifold(curved or flat). Gordon metric has been widely used to research the electromagnetic phenomenon in curved or flat space–time by many authors.[25−29] Here, $g_{\mu\nu}$ denotes the gravitational metric which can be degenerated into Minkowski space–time metric $\eta_{\mu\nu} = {\rm diag}(1, -1, -1, -1)$ when no gravity exists, $n_{\rm refra}$ is the refractive index of media, and u_μ is the 4-vector velocity of media. It must be pointed out that the Gordon effective metric is derived in the non-dispersive media.

Research shows that the magnetoplasma in the pulsar magnetosphere satisfies the above conditions. The x-ray bursts are generated at the pulsar surface, and they propagate in the outflowing magnetoplasma. We deal with the effects from gravitational field and magnetoplasma of pulsars by using Gordon metric directly. In Gordon metric (6), the electromagnetic effect of surface magnetic field of pulsars and the kinetic effect of magnetoplasma are manifested via the dispersion relation and the 4-vector velocity of the outflowing plasma respectively. Expression (3) of redshift contributed by gravitational field and magnetoplasma of pulsars is corrected as

$$1 + z = \sqrt{\frac{G_{00}({\rm o})}{G_{00}({\rm e})}}, \tag{7}$$

where $G_{00}({\rm o})$ and $G_{00}({\rm e})$ denote the time–time components of effective metrics at the emitter and the observer respectively.

Now, we consider the electromagnetic effects of the pulsar magnetic field. Tsai and Erber (1975)[30] examined properties of photons in a super magnetic field and presented the refractive index for x-rays in superstrong magnetic field of pulsars. Under the condition of $B << B_{\rm cr}$, the indices of refraction for radiation polarizing parallel ($n_{||}$) and perpendicular ($n_\perp$) to the plane formed by directions of propagation of the radiation and the magnetic field are different and dependent on the field strength, and they are given by

$$n_{||} \approx 1 + \frac{\alpha}{4\pi}\sin^2\theta\left[\frac{14}{45}\left(\frac{B}{B_{\rm cr}}\right)^2 - \frac{13}{315}\left(\frac{B}{B_{\rm cr}}\right)^4\right], \tag{8}$$

and

$$n_\perp \approx 1 + \frac{\alpha}{4\pi}\sin^2\theta\left[\frac{8}{45}\left(\frac{B}{B_{\rm cr}}\right)^2 - \frac{379}{5040}\left(\frac{B}{B_{\rm cr}}\right)^4\right], \tag{9}$$

where $B_{\rm cr} = m^2c^3/e\hbar \approx 4.4\times10^9{\rm T}$ is the critical magnetic field, α is the fine structure constant and θ is the angle between the propagation of the photons and the magnetic field. Thus for a field of $B \sim 3\times10^8{\rm T}$, we have $n_{||} - n_\perp \sim 4\times10^{-7}$.[31]

Then, we consider kinetic effects of the outflowing magnetoplasma. Goldreich and Julian (1969)[1] gave electrodynamics of pulsars, and they found that rotating pulsars with axisymmetric strong magnetic fields must be surrounded by dense plasma magnetospheres. The electric fields in the magnetospheres produce the plasma corotating with pulsars. The corotation of plasma cannot be extended from the light cylinder. In particular, relativistic magnetoplasma mainly consisting of electron–positron pairs plays a central role in dynamic process of pulsar magnetospheres. The pair plasma is created in two-staged process: primary particles are accelerated by an electric field parallel to the magnetic field near the poles up to extremely high energies with the Lorentz factor $\gamma_{\rm pr} \sim 10^7$, and they produce a secondary denser pair plasma via an avalanche or cascade process.[32−36] The number density $N_{\rm p}$ of the secondary pair plasma exceeds the Goldreich–Julian density $N_{\rm GJ} = \Omega B/2\pi ec$ (where Ω is the angular speed and B is the magnetic field of pulsars) by several orders of magnitude called the multiplicity factor $\kappa = N_{\rm p}/N_{\rm GJ}$, which is introduced by Sturrock (1971).[33] The plasma is highly relativistic, flowing with a Lorentz factor about $\gamma_{\rm p} \sim 10 - 10^3$.[13,14]

By the above analysis and under the approximation of $n^2_{\rm refra} \approx n^2_{||}$, we can obtain the effective metric of the magnetoplasma medium by inserting dispersion relation (8) into Gordon metric (6)

$$G_{\mu\nu} = g_{\mu\nu} + \left(\frac{1}{n_{\rm refra}^2} - 1\right) u_\mu u_\nu$$
$$\approx g_{\mu\nu} + \left(\frac{1}{\left(1 + \frac{\alpha}{4\pi}\sin^2\theta \left[\frac{14}{45}\left(\frac{B}{B_{\rm cr}}\right)^2 - \frac{13}{315}\left(\frac{B}{B_{\rm cr}}\right)^4\right]\right)^2} - 1 \right) u_\mu u_\nu, \tag{10}$$

where $B << B_{\rm cr}$ and $\theta \approx 0.4°$, so $\frac{\alpha}{4\pi}\sin^2\theta \left[\frac{14}{45}\left(\frac{B}{B_{\rm cr}}\right)^2 - \frac{13}{315}\left(\frac{B}{B_{\rm cr}}\right)^4\right] << 1$ and expression(10) can be approximated as

$$G_{\mu\nu} \approx g_{\mu\nu} - \frac{\alpha}{2\pi}\sin^2\theta \left[\frac{14}{45}\left(\frac{B}{B_{\rm cr}}\right)^2 - \frac{13}{315}\left(\frac{B}{B_{\rm cr}}\right)^4\right] u_\mu u_\nu. \tag{11}$$

4. Modification of the gravitational redshift

Inserting the effective metric (11) into expression (7), we have

$$1 + z = \left(g_{00} - \frac{\alpha}{2\pi}\sin^2\theta \left[\frac{14}{45}\left(\frac{B}{B_{\rm cr}}\right)^2 - \frac{13}{315}\left(\frac{B}{B_{\rm cr}}\right)^4\right] u_0 u_0 \right)^{-\frac{1}{2}}. \tag{12}$$

The pulsar magnetoplasma is highly relativistic, flowing with a Lorentz factor about $\gamma_{\rm p} \sim 10 - 10^3$. Observations indicate that the space speed of pulsars ranges from zero to 900km/s with an average value of $v_{\rm space}$=450±50km/s.[37] In a corotating medium frame, the rotation speed of magnetoplasma is estimated at $v_{\rm rotation} = 2\pi R\Omega \approx 400 \times 10^3$km/s (for $R \sim 10$km and $P \sim 1$s). Hence, comparing with the outflowing speed of the magnetoplasma, the space speed of pulsars can be ignored. Then substituting $\gamma_{\rm p}$ for u_0 in expression(12) and inserting Schwarzschild metric $g_{00} = 1 - 2{\rm GM}/(c^2R)$ into expression(12), we have

$$1 + z = \left(1 - 2{\rm GM}/(c^2R) - \frac{\alpha}{2\pi}\sin^2\theta \left[\frac{14}{45}\left(\frac{B}{B_{\rm cr}}\right)^2 - \frac{13}{315}\left(\frac{B}{B_{\rm cr}}\right)^4\right] \gamma_{\rm p}^2 \right)^{-\frac{1}{2}}. \tag{13}$$

The redshift z is

$$z = \left(1 - 2{\rm GM}/(c^2R) - \frac{\alpha}{2\pi}\sin^2\theta \left[\frac{14}{45}\left(\frac{B}{B_{\rm cr}}\right)^2 - \frac{13}{315}\left(\frac{B}{B_{\rm cr}}\right)^4\right] \gamma_{\rm p}^2 \right)^{-\frac{1}{2}} - 1, \tag{14}$$

and so modification of the gravitational redshift δ is expressed as

$$\delta = \left(1 - 2{\rm GM}/(c^2R) - \frac{\alpha}{2\pi}\sin^2\theta \left[\frac{14}{45}\left(\frac{B}{B_{\rm cr}}\right)^2 - \frac{13}{315}\left(\frac{B}{B_{\rm cr}}\right)^4\right] \gamma_{\rm p}^2 \right)^{-\frac{1}{2}} - \left[1 - \frac{2G}{c^2}\left(\frac{M}{R}\right)\right]^{-\frac{1}{2}}. \tag{15}$$

Now we make discussion on the parameters of pulsar. Most ('ordinary') pulsars have a conventional surface magnetic field of $B \sim 10^8$T[14] and Millisecond pulsars (MSPs) have a short period and weak field $B \sim 10^6$T.[38] There is also a sort of new and exotic pulsars called "magnetars" with a superstrong magnetic field $(10^{10} - 10^{11}\text{T})$.[2,3] The pair plasma is intrinsically highly relativistic, flowing out with a Lorentz factor about $\gamma_{\rm p} \sim 10 - 10^3$.[13,14] The angle θ between the direction of propagation of the photons and the magnetic field is assumed to be 0.4° in this paper.[31] Hence, the modification of gravitational redshift of MSPs is

$$\delta < 2.3 \times 10^{-5}, \tag{16}$$

and the modification of gravitational redshift of ordinary pulsars is

$$0.0023 \leq \delta \leq 0.33. \tag{17}$$

Because the surface magnetic field of magnetars is much larger than the critical magnetic field, the dispersion relationship (8) and (9) are invalid for magnetars. Hence here we do not make discussion on the magnetars.

From the above results, we can find that the modification of gravitational redshift z produced by pulsar magnetoplasma is of redshift, and it is irrelevant to the frequency of x-ray, so the effect is non-dispersive.

5. Discussion and conclusion

In this paper, the propagation of x-ray bursts in the magnetoplasma of pulsar surface magnetosphere is studied, and the electromagnetic interaction between x-ray bursts and magnetoplasma is described as some geometry. Gordon metric is introduced to deal with the gravitational metric and optical metric together. So the propagation of x-ray bursts in magnetoplasma of pulsars can be treated as x-ray bursts transmitting in the free-space characterized by Gordon metric. The modification of gravitational redshift is obtained, and calculation shows that the magnetoplasma can produce spectrum of emission red-shifting. The modification is inconspicuous for those pulsars with a weak surface magnetic field and low Lorentz factor, and it becomes obvious with the increase of the surface magnetic field and out-flowing Lorentz factor $\gamma_{\rm p}$. Especially for an ordinary pulsar with a surface magnetic field $B \sim 10^8$T and out-flowing Lorentz factor $\gamma_{\rm p} \sim 10^3$, the modification can reach the same magnitude as the gravitational redshift.

Finally, extending the research on interaction between emission and magnetoplasma of pulsars to optical, radio and γ-ray range and comparing values obtained via calculations with the ever-growing data will be useful to examining the problems about pulsar dynamics such as pulsar radiation mechanisms, pulsar's mass-to-radius ratio, and the structure of pulsar magnetosphere.

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