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Widely tunable dual-pump parametric amplifiers with photonic crystal fibres*

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This paper theoretically studies the double-pumped fibre-optical parametric amplifiers (FOPAs) in photonic crystal fibres. Two distinct working regimes of FOPAs are researched, which depend on the dispersion at the central wavelength of the two pumps. Extremely broad tuning range can be obtained when the central pump wavelength is in the normal dispersion regime and is insensitive to the wavelength separation between the two pumps, while the tuning range is narrow in the anomalous dispersion regime and can be significantly enhanced by increasing the wavelength separation. Impacts of higher-order dispersions and temporal walk-off on the gain spectra are also discussed.

Keywords: nonlinear optics, parametric amplification, photonic crystal fibres, dispersion
PACC: 4265, 4281, 4255N

1. Introduction

Fibre-optic parametric amplifiers (FOPAs), based on four-wave mixing (FWM),[1] lately have received considerable attention because of their unique properties.[2−8] In addition to provide broadband and high gain in many spectral bands, FOPAs exhibit low noise figures which can be equivalent to those of conventional fibre-optic amplifiers, or even better when they operate in a phase-sensitive configuration. They additionally offer the possibilities to achieve wavelength band conversion and many other applications such as ultrafast all-optical signal sampling, high-repetition-rate pulse train generation, or time-division multiplexing. High and uniform gain over a relatively large bandwidth is, however, an important issue of optical amplifiers of current interest, especially in dense wavelength-division-multiplexed systems.

Because of the relatively weak nonlinearity in traditional silica glass, FOPAs require high peak power and long length before FWM effects can be observed. In such a long fibre, phase-matching is difficult to maintain because of longitudinal variations of zero-dispersion wavelength (ZDWL). In addition, traditional silica glass has pool dispersion properties, and high-order dispersions are generally neglected in the treatment of phase-match condition. As a result, it is difficult for FOPAs to provide good results. With the development of highly nonlinear fibres, these limitations are gradually conquered. Approaches to accomplish broad and flat gain spectrum have been proposed and demonstrated, which include utilizing a highly nonlinear fibre,[9] a multisection dispersion-tailored nonlinear fibre arrangement,[10] and dual-pump scheme.[11] Compared with single-pump FOPAs, dump-pump scheme has more tunable parameters, which may support flat gain over a larger bandwidth.[12,13] Flat gain spectrum over tens of nanometres can be obtained.[14] Recently, newly developed photonic crystal fibres (PCFs) exhibit enhanced nonlinearity,[15] and show significant nonlinear effects with length as short as a few metres, leading to high FWM efficiency. In addition, PCFs also exhibit engineerable dispersion.[16] High-order dispersions can play important roles in phase-match condition. The combination of their unusual dispersive properties and high nonlinearities has proved valuable and has provided a new platform for investigating FOPAs.

In this paper we theoretically study the double-

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pumped FOPAs with PCFs where the higher-order dispersions and temporal walk-off are taken into account. Unlike the single-pumped FOPAs, dual-pump scheme provides several degrees of freedom to design FOPAs, including the parameters such as the centre wavelength and the wavelength separation of two pumps. Results show that the phase-matching features and gain spectra of double-pumped FOPAs strongly depend on the dispersion at the centre wavelength of two pumps in an explicit manner. Two distinct working regimes of FOPAs are observed in terms of the dispersion at the centre pump wavelength. Extremely broad tuning range can be obtained in the normal dispersion regime and is insensitive to the wavelength separation, while the tuning range is narrow in the anomalous dispersion regime and can be significantly enhanced by increasing the wavelength separation. These features may be useful in designing and optimizing broadly tunable FOPAs. In addition, the shape of dispersion curve in PCFs is complex, and the walk-off delay among the interacting waves may be over ten picoseconds in a typical optical device. For fast signals approaching several picoseconds in future optical networks, the effects of temporal walk-off may become a factor to degrade the performance of FOPAs seriously and are thus necessary to be considered.

This paper is organized as follows. In Section 2 we discuss the contributions of the temporal walk-off and the dispersion in FWM process. Section 3 analyses phase-matching condition and shows that widely tunable gain can be obtained by use of high-order dispersion in PCF for the central wavelength of two pumps being located in the normal dispersion regime. Section 4 gives output results of gain spectrum by numerical simulations. Finally, the results are summarized in Section 5.

2. Propagation equations

Double-pumped FOPAs, involving the nonlinear interaction of four copropagating waves in a single polarization optical fibre with an arbitrary dispersion profile, can be described by the following nonlinear coupled-wave equations:[1]

$$\frac{\partial A_k}{\partial z}+\frac{1}{v_{gk}}\frac{\partial A_k}{\partial t}+\frac{\mathrm{i}}{2}\beta_{2k}\frac{\partial^2 A_k}{\partial t^2}=\mathrm{i}\gamma\left\{\left[|A_k|^2+2(|A_{3-k}|^2+|A_3|^2+|A_4|^2)\right]A_k+2A_3A_4A_{3-k}^*\exp(\mathrm{i}\Delta\beta z)\right\}, \quad (1)$$

$$\frac{\partial A_j}{\partial z}+\frac{1}{v_{gj}}\frac{\partial A_j}{\partial t}+\frac{\mathrm{i}}{2}\beta_{2j}\frac{\partial^2 A_j}{\partial t^2}=\mathrm{i}\gamma\left\{\left[|A_j|^2+2(|A_1|^2+|A_2|^2+|A_{7-j}|^2)\right]A_j+2A_1A_2A_{7-j}^*\exp(-\mathrm{i}\Delta\beta z)\right\}, \quad (2)$$

where $k=1$ or 2 and $j=3$ or 4, A_1 and A_2 represent pump fields, and A_3 and A_4 are the signal and idler fields, respectively. γ is the nonlinearity parameter; β_{2m} ($m=1$ to 4) is the second-order dispersion coefficient; $\Delta\beta=\beta(\omega_3)+\beta(\omega_4)-\beta(\omega_1)-\beta(\omega_2)$ is the wave-vector mismatch; and v_{gm} is the group velocity. To study the effects of the walk-off and dispersion separately, we compare three length scales, the walk-off length L_{W}, the nonlinear length L_{NL} and the group-velocity dispersion (GVD) length L_{D}. For pump pulse duration T_0 and peak power P_0, these characteristic lengths are defined by

$$L_{\mathrm{NL}}=\frac{1}{\gamma_{\mathrm{p}}P_0},\quad L_{\mathrm{W}m}=\frac{T_0}{\left|v_{gm}^{-1}-v_{g0}^{-1}\right|},$$
$$L_{\mathrm{D}m}=\frac{T_0^2}{|\beta_{2m}|}. \quad (3)$$

Dispersion and walk-off length decrease rapidly with the short pulse duration, and the shorter one plays a dominant role. For pulse duration $T_0=5\,\mathrm{ps}$, the GVD length is generally over ten thousand metres, while the nonlinear length and the walk-off length will be only about tens of metres in a highly nonlinear fibre. In this case, the GVD term in the left of Eqs.(1)–(2) can be neglected. To get more information about the walk-off effects, we introduce a time scale to describe the walk-off delay τ_{w}:

$$\tau_{\mathrm{w}}=\left(\frac{1}{2}\beta_3\Omega^2+\frac{1}{6}\beta_4\Omega^3+\frac{1}{24}\beta_5\Omega^4+\frac{1}{120}\beta_6\Omega^5\right)L, \quad (4)$$

where $\Omega=\omega-\omega_0$, $\omega_0=2\pi c/\lambda_0$, λ_0 is the ZDWL of a fibre, and L is the fibre length.

To theoretically simulate the FOPAs, realistic fibre parameters with large high-order dispersions were

adopted,[9] which will clearly show the effects of temporal walk-off but may not alter the general conclusions obtained in this paper. It is obvious that the walk-off delay has a complex curve as the wavelength is detuned from the ZDWL (dashed line in Fig.1). When FOPAs are working near the ZDWL, the walk-off delay is largely contributed by the third-order dispersion β_3, and the walk-off delay curve is nearly symmetric with respect to the ZDWL. The higher-order dispersions, however, will play important roles and make the walk-off delay curve asymmetry for the working wavelength far from the ZDWL. When the two pump wavelengths are located on the opposite sides of the ZDWL and the wavelength separation is less than 60 nm, the walk-off delay between the pump $\omega_{\rm p}$ and the pulse ω_0 will be very small. In this case, the second term in the left of Eq.(1) can be neglected. Comparatively, when the signal wavelength is longer than 1580 nm or the corresponding idler wavelength is less than 1420 nm, the walk-off delay between the signal $\omega_{\rm s}$ and the pulse ω_0 will be more than 20 ps, and the walk-off delay between idler $\omega_{\rm i}$ and the pulse ω_0 will be about 10 ps in a 100 m fibre. Thus walk-off effect is commonly severe for signal and idler pulses with duration ∼5 ps.

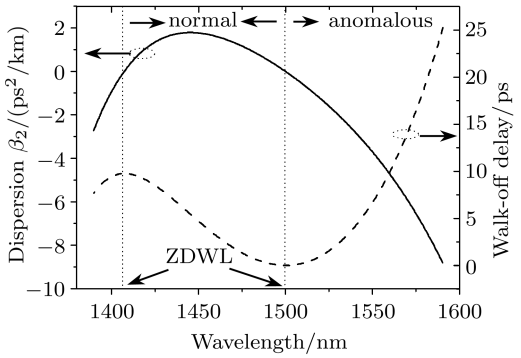


Fig.1. Second-order dispersion (solid line) and walk-off delay (dashed line) versus wavelength. The parameter values are $L = 100$ m, $\beta_3 = 0.066$, $\beta_4 = -1.04 \times 10^{-3}$, $\beta_5 = 3.37 \times 10^{-6}$, $\beta_6 = -1.14 \times 10^{-6}$. The units are ps$^{\rm n}$ km^{-1} for $\beta_{\rm n}$.

3. Phase-matching

According to the above analysis, both the walk-off and dispersion effect can be neglected for the pump pulses. Under the approximation of pump undepletion, Eq.(1) can be simplified to

$$\frac{\partial A_k}{\partial z} = {\rm i}\gamma[(P_k + 2P_{3-k})\sqrt{P_k}],\ (k = 1, 2), \qquad (5)$$

where $P_k = |A_k(0)|^2$. Eq.(5) is easily solved and we obtain

$$A_k(z) = \sqrt{P_k}\exp[{\rm i}\gamma(P_k + 2P_{3-k})z]. \qquad (6)$$

Substituting Eq.(6) into Eq.(2), and introducing the parameter $B_j = A_j \exp[-2{\rm i}\gamma(P_1 + P_2)z]$, $(j = 3, 4)$, we obtain

$$\frac{\partial B_j}{\partial z} + \frac{1}{v_{{\rm g}j}}\frac{\partial B_j}{\partial T} = 2{\rm i}\gamma\sqrt{P_1 P_2}B^*_{7-j}\exp(-{\rm i}\kappa z), \qquad (7)$$

where $\kappa = \Delta\beta + \gamma(P_1 + P_2)$ escribes the total phase mismatch. Using Taylor expansion and taking terms up to the sixth-order dispersion, we get the phase-matching condition among the four waves

$$\begin{aligned}\kappa &= \Delta\beta + \gamma(P_1 + P_2)\\ &= \beta_{2c}[\Omega^2 - (\Delta\omega_{\rm p})^2] + \frac{1}{12}\beta_{4c}[\Omega^4 - (\Delta\omega_{\rm p})^4]\\ &\quad + \frac{1}{360}\beta_{6c}[\Omega^6 - (\Delta\omega_{\rm p})^6] + \gamma(P_1 + P_2)\\ &= 0, \end{aligned} \qquad (8)$$

where $\Omega = \omega_3 - \omega_{\rm c} = -(\omega_4 - \omega_{\rm c})$ is frequency shift. In particular, the central frequency of the two pumps $\omega_{\rm c} = (\omega_1 + \omega_2)/2$ and the frequency separation $\Delta\omega_{\rm p} = (\omega_1 - \omega_2)/2$ are defined, which are alternatives to the two pump wavelengths (ω_1 and ω_2). The two parameters $\omega_{\rm c}$ and $\Delta\omega_{\rm p}$ contribute only when two pumps are used and are independent of the signal and idler frequencies. In the following, we will see that the two parameters play important roles in controlling the phase mismatch. β_{2c}, β_{4c} and β_{6c}, the even-order dispersions at the central frequency $\omega_{\rm c}$, are related to the dispersion values at the ZDWL λ_0 as

$$\begin{aligned}&\beta_{2c}(\omega) = \sum_{m=2}^{6}\beta_m(\omega_0)(\omega - \omega_0)^{m-2}/(m-2)!,\\ &\beta_{4c}(\omega) \approx \beta_4(\omega_0), \quad \text{and} \quad \beta_{6c}(\omega) \approx \beta_6(\omega_0). \end{aligned} \qquad (9)$$

The phase-matching curves as a function of the central wavelength $\lambda_{\rm c}$ are calculated by using Eq.(8) (Fig.2). we can see from Fig.1 (solid line) that adjusting the central wavelength of the two pumps implies different dispersion β_{2c}. When pumping in the anomalous dispersion regime ($\beta_{2c} < 0$, the right part of Fig.2), a tuning range of tens of nanometres can be obtained. From Eq.(8), it is easy to understand that $\Delta\beta$ must be negative to realize the phase-matching condition. Thus negative β_{2c} implies that the contributions of other higher-order dispersion terms should be small. As a result, the FWM is generally confined to a narrow spectral region, resulting in a limited tunable

bandwidth. In this case, to broaden the gain bandwidth Ω, one effective approach is to utilize a highly nonlinear fibre and/or enhance the pump power.[15] It realizes wide bandwidth by enhancing the nonlinear phase mismatch $\gamma(P_1 + P_2)$. Another approach is to increase the pump wavelength separation,[11] which in effect adds a bias to $\Delta\beta$ (see Eq.(8)), so that linear phase mismatch is large enough to compensate for nonlinear phase mismatch, and flat gain over a wide spectral range can be achieved. However, both the two approaches have some limits. For the first one, high nonlinearity is limited by the performance of a fibre, and high pump power requires more advanced pump technique. For the last one, the core diameter of any fibre exhibits random variations related to manufacturing, resulting in a randomly varying ZDWL along the fibre. The uniformity of gain reduces because of ZDWL fluctuations, which can be solved by reducing the wavelength separation between the two pumps at the expense of a reduced gain bandwidth.[13]

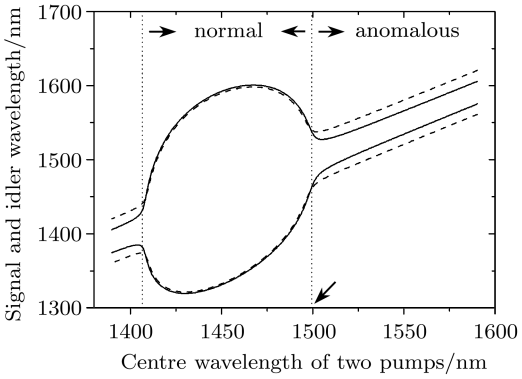


Fig.2. Phase-matching curve in terms of the central wavelength of two pumps. Solid line: the wavelength separation between the two pumps is 30 nm, $P_1 = 1$ W, $P_2 = 1$ W. Dot line: the wavelength separation between the two pumps is 30 nm, $P_1 = 10$ W, $P_2 = 10$ W. Dashed line: the wavelength separation between the two pumps is 60 nm, $P_1 = 1$ W, $P_2 = 1$ W. Vertical dotted lines indicate the ZDWLs. The parameter $\gamma = 30\,\mathrm{W}^{-1}/\mathrm{km}$, and other parameters are the same with that in Fig.1.

Contrary, when the central wavelength λ_c of the two pumps is located in the normal dispersion regime ($\beta_{2c} > 0$, the left part of Fig.2), and depart from ZDWL a very small distance, wider gain spectrum can be achieved. Signal and idler outputs between 1400 and 1600 nm can be covered if the central wavelength λ_c is tuned only 20-nm from the ZDWL. The tunable range can reach several hundred nanometres. This phenomenon can also be understood from Eq.(8). $\Delta\beta$ must be negative to realize the phase-matching condition. The positive β_{2c} implies a new balance among the different orders of dispersion, and the higher-order dispersions begin to play important roles in phase-matching. A small but positive β_{2c} associated with the nonlinear phase mismatch needs to be compensated by a large but negative higher-order dispersion terms. As a result large Ω can be obtained and a large bandwidth of phase matching can be realized. It is noted that the co-compensation between the second-order dispersion and higher-order dispersions terms play more important roles in phase-matching, and phase mismatch induced by nonlinear effects and the bias $\Delta\omega_p$ becomes unimportant when Ω is large. So the phase-matching condition is not so sensitive to the wavelength separation between the two pumps and the pump power in the normal dispersion regime. With such a steep phase-matching curve, a widely tuning parametric gain of FOPA can be resulted from a small change in the central wavelength near the ZDWL.

4. Parametric gain

Equation (4) governs the evolution of the signal and idler waves including the effect of walk-off, which can be numerically solved to obtain the gain of the FOPA. The split-step Fourier method was adopted to solve the equations in this paper.[1] The small-signal power gain at the output can be expressed as

$$\begin{aligned} G &= 10\log[P_3(L)/P_3(0)] \\ &= 20\log[|B_3(L)|\,/\,|B_3(0)|]. \end{aligned} \tag{10}$$

Figure 3 shows the gain spectra of the FOPA as the central wavelength λ_c is tuned in the vicinity of the ZDWL. Dispersion β_{2c} aries from $0.2159\,\mathrm{ps}^2/\mathrm{km}$ to $-0.1119\,\mathrm{ps}^2/\mathrm{km}$ when λ_c is tuned from 1496 nm to 1502 nm (see Fig.1). Like the situation in a single-pumped FOPA,[17] the gain spectra are also broad when the central wavelength λ_c is tuned in the normal dispersion regime, which is in agreement with the bandwidth of phase-matching. The most uniform gain spectrum is observed in the normal regime close to the ZDWL ($\lambda_c = 1499$ nm, $\beta_{2c} = 0.05496\,\mathrm{ps}^2/\mathrm{km}$ in our simulations). As the central wavelength is apart from ZDWL, nonuniform gain spectra with sidebands will be resulted and are also sensitive to the central pump wavelength. Therefore, to design broadly tunable FOPAs for the practical use, the central pump

wavelength should be close to the ZDWL and in the normal dispersion regime, which corresponds to uniform broad gain spectra. These features can be easily understood from the phase-matching condition given by Eq.(8). When the signal and the idler are close to the central frequency ω_c, or the frequency shift Ω is small, the terms of the higher-order dispersions become small and have small impacts on the gain. The higher-order dispersions, however, are proved to be important for the gain uniformity by compensation for the central dip of the gain spectrum. When the signal and the idler are far from the central frequency, or the frequency shift is large, the higher-order dispersions play an important role in the gain. As a result, the phase mismatch changes very rapidly close to the boundary of phase-matching condition. This correspondingly yields narrow peaks of parametric gain and makes the gain spectra nonuniform. Finally, the walk-off effect is proved to have great influence on the gain spectra of FOPAs (Fig.3). Compared with the case without walk-off, the gain spectra decrease at least 25 dB by the effect of walk-off.

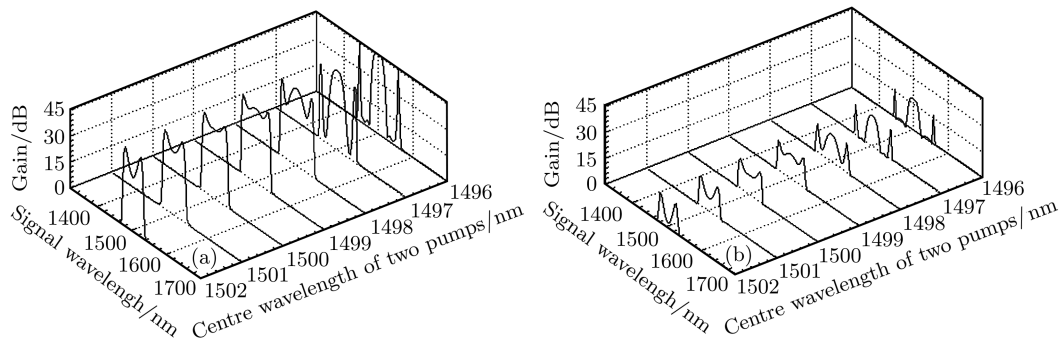


Fig.3. Gain spectra as the central wavelength of the two pumps is tuned in the case without the effects of walk-off (a) and the case with the effects of walk-off (b). The parameters $T_0 = 5\,\mathrm{ps}$, $P_1 = 1\,\mathrm{W}$, $P_2 = 1\,\mathrm{W}$, and the wavelength separation of 30 nm are used in the simulations, and other parameters are the same with that in Fig.2.

5. Conclusion

We have theoretically studied the doubled-pumped FOPAs where the higher-order dispersions and temporal walk-off are taken into account. Strong control over the dispersion and nonlinearity in PCFs may improve the performance of FOPAs greatly. Dual-pump scheme leaves us an extra freedom to design FOPAs with broad and flat gain spectrum. In terms of the dispersion at the central wavelength of the two pumps, two distinct working regimes of FOPAs are found. When the central wavelength is in the normal-dispersion regime, the phase-matching can be obtained in an extremely wide range of wavelength and is not sensitive to the wavelength separation. While in the anomalous-dispersion regime, the bandwidth of phase-matching is narrow and significantly enhanced by increasing wavelength separation. Higher-order dispersions play important role in the gain spectrum. The most uniform gain spectrum occurs in the normal dispersion regime close to the ZDWL. As the central wavelength is apart from the ZDWL, the gain spectra are with sidebands and become highly nonuniform. The effects of walk-off on dual-pump FOPAs greatly decrease the gain for picosecond signal pulses. Conclusions may apply to the single-pump case as well. The results obtained in this paper could be a useful guideline in designing and optimizing broadly tunable FOPAs.

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