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Quantum analysis on the four-photon interference*

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We present a theory for quantum interference of four photons generated by spontaneous parametric down-conversion. Detailed investigation of the dependence of fourfold coincidence count rate on time delay between the incident and the reflective pump laser pulses is carried out. Gaussian type dependence is found, and good agreement between our theoretical results and experimental data reported in the literature is achieved.

Keywords: quantum interference, four photons, spontaneous parametric down conversion
PACC: 4250, 0367

1. Introduction

Multi-particle quantum interference is important to multiparty quantum communications.[1−6] Recently Pan *et al*[7] and Zhao *et al*[8] reported experiments in which quantum interference was observed to result from four photons relying on photon pairs produced randomly in the spontaneous parametric down-conversion process. The dependence of fourfold coincidence on time delay between the incident and the reflective pump laser pulses was observed, and a visibility as high as 0.84 ± 0.03 was obtained. The four photon interference was considered in Ref.[9], but the further calculation was not carried out and the contribution of the compensatory crystal was not taken into consideration. In this paper we present a theory for quantum interference of four photons created in spontaneous parametric down-conversion process in which the contribution of the compensators is also considered. The dependence of fourfold coincidence count rate on time delay is investigated in detail, and these theoretical results are in good agreement with experimental data reported in the literature.

2. The experiment

The experimental setup used to observe four-photon interference in Refs.[7, 8] is schematically illustrated in Fig.1. An ultraviolet femtosecond pulsed laser with a central wavelength $\lambda \simeq 394.5\,\text{nm}$ and a Beta Barium Borate (BBO) crystal cut for type-II phase matching are used to produce two correlated photon pairs. Photon pairs in arms 1 and $2'$ are generated by spontaneous parametric down-conversion when a laser pulse passes through the BBO crystal toward the translatable delay mirror, and photon pairs in arms $3'$ and 4 are generated when the laser pulse that is reflected back by the mirror passes again through the BBO crystal. Compensatory crystals are placed in arms 1, $2'$, $3'$ and 4.

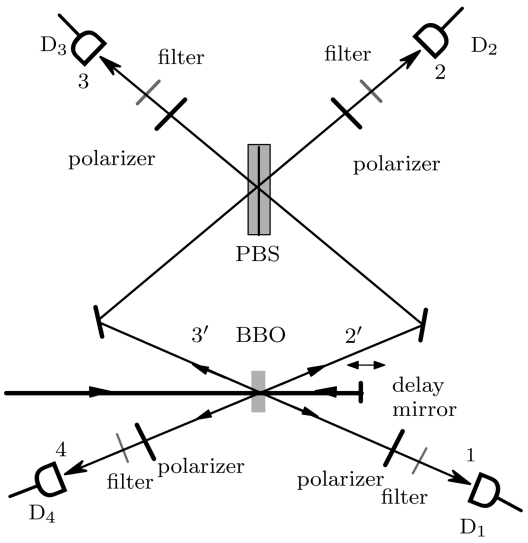


Fig.1. Schematic illustration of the experimental setup for the measurement of four-photon polarization correlations.

Photons in arms 1 and 4 are detected by the single photon detectors D_1 and D_4, while photons in arms $2'$ and $3'$ are reflected to a polarization beam splitter (PBS), which allows horizontally polarized photons to

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be transmitted, but reflects back vertically polarized photons. The arms after the PBS are labelled with 2 and 3 separately. Photons in arms 2 and 3 are detected by single photon detectors D_2 and D_3. Spectral filters and linear polarizers are placed in front of detectors in arms 1, 2, 3 and 4. Signals from single photon detectors D_1, D_2, D_3 and D_4 are analysed for fourfold coincidences.

Pan *et al* found that when directions of polarization in arms 1, 2, 3 and 4 were either horizontal or vertical, the fourfold coincidences were null except for the following polarization combinations: VHHV and HVVH. When the angle between the photon polarization and the vertical direction was set to be $\pi/4$ in arms 1, 2 and 3, but $\pm\pi/4$ in arm 4, time delay dependent fourfold coincidences were observed (see Fig.2).

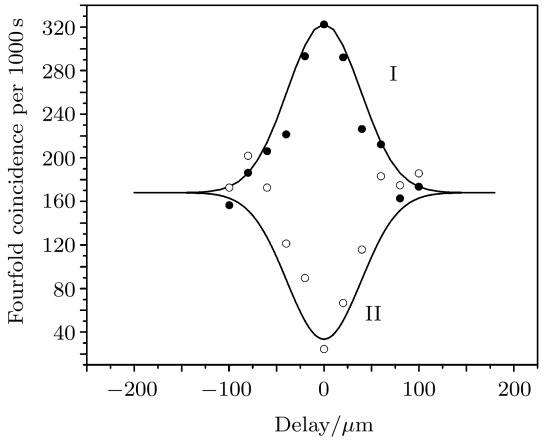


Fig.2. Fourfold coincidences $C(\pi/4, \pi/4, \pi/4, \pm\pi/4)$ as functions of delay $c\tau/2$.

3. Four-photon quantum state and optical fields

The quantum state of the photons produced by the spontaneous parametric down-conversion pumped by a coherent pulse may be expressed as[9]

$$|\Psi\rangle = \left(1 - \frac{|\eta|^2}{2}\right)|0\rangle + \eta|\Phi_1\rangle + \eta^2|\Phi_2\rangle, \tag{1}$$

where $|\eta|^2$ is the possibility of photon conversion in a single pulse, and

$$|\Phi_1\rangle = N\int \mathrm{d}\omega_1 \int \mathrm{d}\omega_2 \Gamma(\omega_1,\omega_2) a_{\mathrm{s}}^{+}(\omega_1) a_{\mathrm{i}}^{+}(\omega_2)|0\rangle, \tag{2}$$

$$|\Phi_2\rangle = \frac{1}{2}N'\int \mathrm{d}\omega_1 \int \mathrm{d}\omega_2 \int \mathrm{d}\omega_1' \int \mathrm{d}\omega_2' \Gamma(\omega_1,\omega_2) \times \Gamma(\omega_1',\omega_2') a_{\mathrm{s}}^{+}(\omega_1) a_{\mathrm{i}}^{+}(\omega_2) a_{\mathrm{s}}^{+}(\omega_1') \times a_{\mathrm{i}}^{+}(\omega_2')|0\rangle \tag{3}$$

with

$$\Gamma(\omega,\omega') = \int_{-L}^{0} \mathrm{d}z \mathrm{e}^{\mathrm{i}\Delta(k_{\mathrm{s}},k_{\mathrm{i}})z} A(\omega+\omega'), \tag{4}$$

where N and N' are the constants of normalization; L is the length of the crystal; $\Delta(k_{\mathrm{s}},k_{\mathrm{i}}) = k_{\mathrm{p}} - k_{\mathrm{s}} - k_{\mathrm{i}}$ is the phase mismatching[10] with k_{p} being the modulus of the wave vector of the component of the pump optical field with $\omega_{\mathrm{p}} = \omega_1 + \omega_2$ or $\omega_{\mathrm{p}} = \omega_1' + \omega_2'$, and k_{s} and k_{i} being z-components of wave vectors of s-mode, and i-mode plane wave components respectively, and $A(\omega)$ being the spectrum of the pump pulse.

Considering two parametric processes, the quantum state that describes the overall system is given by[9]

$$|\Psi\rangle = |\Psi^{(1)}\rangle \otimes |\Psi^{(2)}\rangle, \tag{5}$$

where superscript (1) denotes the quantum state from the incident parametric process and superscript (2) refers to the quantum state from the reverse parametric process. The expression of $|\Psi_1\rangle^{(1,2)}$ has the same form as expression (1), but with $a_{\mathrm{s}}^{\dagger}$ changed into $a_{1\mathrm{s}}^{\dagger}$ or $a_{2\mathrm{s}}^{\dagger}$ and $a_{\mathrm{i}}^{\dagger}$ changed into $a_{1\mathrm{i}}^{\dagger}$ or $a_{1\mathrm{i}}^{\dagger}$.

The positive frequency parts of the electric fields at detectors in arms 1, 2, 3 and 4 can be expressed respectively as

$$E_1^{+}(t)\vec{e}_{\theta_1} = \mathcal{A}\int \mathrm{d}\omega F(\omega)[\mathrm{e}^{-\mathrm{i}\omega(t+l_{\mathrm{c}}/u_{\mathrm{i}})} a_{1\mathrm{s}}(\omega)\sin\theta_1 + \mathrm{e}^{-\mathrm{i}\omega(t+l_{\mathrm{c}}/u_{\mathrm{s}})} a_{1\mathrm{i}}(\omega)\cos\theta_1]\vec{e}_{\theta_1}, \tag{6}$$

$$E_2^{+}(t)\vec{e}_{\theta_2} = \mathcal{A}\int \mathrm{d}\omega F(\omega)[\mathrm{e}^{-\mathrm{i}\omega(t+\tau+l_{\mathrm{c}}/u_{\mathrm{i}})} a_{2\mathrm{s}}(\omega)\sin\theta_2 + \mathrm{e}^{-\mathrm{i}\omega(t+l_{\mathrm{c}}/u_{\mathrm{s}})} a_{1\mathrm{i}}(\omega)\cos\theta_2]\vec{e}_{\theta_2}, \tag{7}$$

$$E_3^{+}(t)\vec{e}_{\theta_3} = \mathcal{A}\int \mathrm{d}\omega F(\omega)[\mathrm{e}^{-\mathrm{i}\omega(t+l_{\mathrm{c}}/u_{\mathrm{i}})} a_{1\mathrm{s}}(\omega)\sin\theta_3 + \mathrm{e}^{-\mathrm{i}\omega(t+\tau+l_{\mathrm{c}}/u_{\mathrm{s}})} a_{2\mathrm{i}}(\omega)\cos\theta_3]\vec{e}_{\theta_3}, \tag{8}$$

$$E_4^{+}(t)\vec{e}_{\theta_4} = \mathcal{A}\int \mathrm{d}\omega F(\omega)[\mathrm{e}^{-\mathrm{i}\omega(t+\tau+l_{\mathrm{c}}/u_{\mathrm{i}})} a_{2\mathrm{s}}(\omega)\sin\theta_4 + \mathrm{e}^{-\mathrm{i}\omega(t+\tau+l_{\mathrm{c}}/u_{\mathrm{s}})} a_{2\mathrm{i}}(\omega)\cos\theta_4]\vec{e}_{\theta_4}, \tag{9}$$

where $\mathcal{A}'$ is a constant; τ is the time delay; $l_{\rm c}$ is the thickness of the compensatory crystal; $u_{\rm i}$, $u_{\rm s}$ are the group speeds of the idler and the signal respectively; $\vec{e}_{\theta_1}$, $\vec{e}_{\theta_2}$, $\vec{e}_{\theta_3}$ and $\vec{e}_{\theta_4}$ are, respectively, vectors of unit in the directions of polarization of polarizers in arms 1, 2, 3 and 4; θ_l is the angle between the vertical direction and the direction of polarization of polarizer in arm l; $F(\omega)$ is the transmittance of filter.

4. Fourfold coincidence counting rate

According to quantum transition theory, the fourfold coincidence count C is proportional to[11]

$$C \propto \langle\psi| \int {\rm d}t_1 \int {\rm d}t_2 \int {\rm d}t_3 \int {\rm d}t_4 E_4(t_4)E_3(t_3)E_2(t_2)E_1(t_1)E_1^+(t_1)E_2^+(t_2)E_3^+(t_3)E_4^+(t_4)|\psi\rangle. \tag{10}$$

By substituting expressions (6)–(9) into expression (10) and using expression (5) for the quantum state, neglecting the events where more than two pairs are created simultaneously, we obtain

$$\begin{aligned} C(\theta_1,\theta_2,\theta_3,\theta_4) \;=\; & C_1[\sin^2\theta_1\cos^2\theta_2\cos^2\theta_3\sin^2\theta_4 + \cos^2\theta_1\sin^2\theta_2\sin^2\theta_3\cos^2\theta_4] \\ & +2{\rm Re}[C_2(\tau)]\sin\theta_1\cos\theta_1\sin\theta_2\cos\theta_2\sin\theta_3\cos\theta_3\sin\theta_4\cos\theta_4, \end{aligned} \tag{11}$$

where

$$\begin{aligned} C_1 \;=\; & C_0 \int {\rm d}\omega_1 \int {\rm d}\omega_2 \int {\rm d}\omega_3 \int {\rm d}\omega_4 |F(\omega_1)|^2|F(\omega_2)|^2|F(\omega_3)|^2|F(\omega_4)|^2|A(\omega_1+\omega_2)|^2 \\ & \times|A(\omega_3+\omega_4)|^2|S(\omega_1,\omega_2)|^2|S(\omega_3,\omega_4)|^2 \end{aligned} \tag{12}$$

and

$$\begin{aligned} C_2(\tau) \;=\; & C_0 \int {\rm d}\omega_1 \int {\rm d}\omega_2 \int {\rm d}\omega_3 \int {\rm d}\omega_4 |F(\omega_1)|^2|F(\omega_2)|^2|F(\omega_3)|^2|F(\omega_4)|^2 A(\omega_1+\omega_2)A(\omega_3+\omega_4) \\ & \times A^*(\omega_1+\omega_3)A^*(\omega_2+\omega_4)S(\omega_1,\omega_2)S(\omega_4,\omega_3)S^*(\omega_3,\omega_1)S^*(\omega_2,\omega_4){\rm e}^{{\rm i}(\omega_2-\omega_3)\tau} \end{aligned} \tag{13}$$

with C_0 being a constant,

$$S(\omega_l,\omega_m) = {\rm sinc}\left[\frac{1}{2}L\Delta(k_{{\rm s}z}(\omega_l),k_{{\rm i}z}(\omega_m))\right], \tag{14}$$

$(l,m=1,2,3,4)$, and

$${\rm sinc}(x) = \frac{\sin x}{x}. \tag{15}$$

If the polarizers are set such that the polarizations of photons in each arm are either in the horizontal direction or in the vertical direction, then, according to expression (11), the fourfold coincidence is different from zero only in two cases:

$$C(0,\pi/2,\pi/2,0) \neq 0 \text{ and } C(\pi/2,0,0,\pi/2) \neq 0. \tag{16}$$

This is the experimental result reported by Pan *et al.*[7]

To make a detailed comparison between our theoretical results and experimental data, we carry out the calculations for the fourfold coincidences $C(\pi/4,\pi/4,\pi/4,\pm\pi/4)$ as functions of delay τ. From expression (11), we have

$$C(\pi/4,\pi/4,\pi/4,\pm\pi/4) = \frac{1}{8}\{C_1 \pm {\rm Re}[C_2(\tau)]\}. \tag{17}$$

For a Gaussian pulse, we have

$$A(\omega) \propto \exp\left[-\frac{\tau_c^2(\omega-2\omega_0)^2}{4\pi}\right] \tag{18}$$

with $\tau_{\rm c}$ being the coherence time of the pump beam.

We use the following expression for the transmittance $F(\omega)$,

$$F(\omega) = \exp\left[-4\ln 2\left(\frac{\omega-\omega_0}{\Delta\omega}\right)^2\right] \tag{19}$$

with $\Delta\omega$ being the band width of the filter.

Suppose that the phase matching condition is satisfied by photon pairs with $\omega_{\rm s}=\omega_0$ and $\omega_{\rm i}=\omega_0$, that means

$$\Delta(k_{{\rm s}z}(\omega_0),\quad k_{{\rm i}z}(\omega_0)) = 0, \tag{20}$$

then we may expand the phase mismatching $\Delta(k_{{\rm s}z}(\omega_l),k_{{\rm i}z}(\omega_m))$ to the first order of $\omega_l-\omega_0$ and

$\omega_m - \omega_0$:

$$\begin{aligned} &\Delta(k_{\mathrm{s}z}(\omega_l),\ k_{\mathrm{i}z}(\omega_m)) \\ &= D_{\mathrm{s}}(\omega_l - \omega_0) + D_{\mathrm{i}}(\omega_m - \omega_0) \end{aligned} \tag{21}$$

with $l, m = 1, 2, 3, 4$, where

$$D_{\mathrm{s}} = \frac{\mathrm{d}}{\mathrm{d}\omega}\left[k_{\mathrm{p}}(\omega + \omega_0) - k_{\mathrm{s}z}(\omega)\right]_{\omega=\omega_0}, \tag{22}$$

and

$$D_{\mathrm{i}} = \frac{\mathrm{d}}{\mathrm{d}\omega}\left[k_{\mathrm{p}}(\omega + \omega_0) - k_{\mathrm{i}z}(\omega)\right]_{\omega=\omega_0}. \tag{23}$$

Because

$$\mathrm{sinc}(x) = 1 - \frac{1}{6}x^2 + O(x^4)$$

and

$$\mathrm{e}^{-x^2} = 1 - x^2 + O(x^4), \tag{24}$$

we will approximate $\mathrm{sinc}(x)$ with $\exp(-x^2/6)$. The integrals in Eq.(11) can be carried out now. Parameters used for numerical calculations are cited from Ref.[8]: $\tau_c = 200\,\mathrm{fs}$, $\omega_0 = 2.387 \cdot 10^{15}\,\mathrm{s}^{-1}(\lambda_0 = 789\,\mathrm{nm})$, and $\Delta\omega = 1.059 \times 10^{13}\,\mathrm{s}^{-1}$ ($\Delta\lambda = 3.5\,\mathrm{nm}$ at λ_0). D_{s} and D_{i} can be computed from dispersion relations of the nonlinear optical crystal.[12] For a BBO crystal, we obtain $D_{\mathrm{s}} = 0.28 \times 10^{-9}\mathrm{s} \cdot \mathrm{m}^{-1}$, and $D_{\mathrm{i}} = 0.08 \times 10^{-9}\mathrm{s} \cdot \mathrm{m}^{-1}$ at $\lambda_0 = 789\,\mathrm{nm}$. The thicknesses of the BBO crystal and the compensatory crystal are 2 mm and 1 mm respectively. By using these parameters after a lengthy calculation the normalized fourfold coincidence becomes

$$\begin{aligned} &\frac{C(\pi/4, \pi/4, \pi/4, \pm\pi/4, \tau)}{C(\pi/4, \pi/4, \pi/4, \pm\pi/4, \infty)} \\ &= 1 \pm 0.94\mathrm{e}^{-7.57\times10^{24}\tau^2}. \end{aligned} \tag{25}$$

So far, we have considered only the case in which two photon pairs are created simultaneously. But the fourfold coincidence may also occur when three photon pairs are created simultaneously, two in one down-conversion process and one in the other. All the fourfold events can be detected except the HHHH and VVVV components if the contribution of three photon pairs is considered. Similar to Riedmatten's analysis,[13] after a simple and lengthy calculation we find that this contribution to the fourfold coincidence reduces the visibility by the factors

$$F(\pi/4, \pi/4, \pi/4, -\pi/4) = \frac{1 - 2P}{1 + 7P} = 0.86, \tag{26}$$

and

$$F(\pi/4, \pi/4, \pi/4, +\pi/4) = \frac{1 + 6P}{1 + 7P} = 0.98. \tag{27}$$

From Ref.[8], the ratio between any of the desired fourfold events HVVH and VHHV to any of other nondesired ones is better than 1/60. The ratio becomes 1/58 which is the value of P if only the contribution of the two photon pairs to the detected events HVVH and VHHV is considered.

Then the normalized fourfold coincidences are

$$\begin{aligned} &\frac{C(\pi/4, \pi/4, \pi/4, -\pi/4, \tau)}{C(\pi/4, \pi/4, \pi/4, -\pi/4, \infty)} \\ &= 1 - 0.81\mathrm{e}^{-7.57\times10^{24}\tau^2}, \end{aligned} \tag{28}$$

and

$$\begin{aligned} &\frac{C(\pi/4, \pi/4, \pi/4, \pi/4, \tau)}{C(\pi/4, \pi/4, \pi/4, \pi/4, \infty)} \\ &= 1 + 0.92\mathrm{e}^{-7.57\times10^{24}\tau^2}. \end{aligned} \tag{29}$$

Our results are presented in Fig.2, and they are compared with experimental data. Experimental data ($\bullet$ and $\circ$) are cited from Ref.[8]. Curve I corresponds to $C(\pi/4, \pi/4, \pi/4, \pi/4)$, and curve II corresponds to $C(\pi/4, \pi/4, \pi/4, -\pi/4)$. The agreement between our theoretical results and the reported experimental results is found.

5. Conclusion

In this paper we present a theory for quantum interference of four photons created freely in a spontaneous parametric down-conversion process. We investigate the dependence of fourfold coincidence count rate on time delay between the incident and the reflective pump laser pulses. We find that the fourfold coincidence count rate is Gaussian function of delay time. Our theoretical results are in good agreement with experimental data reported in the literature.

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