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Efficient scheme of quantum SWAP gate and multi-atom cluster state via cavity QED*

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In this paper, we propose a physical scheme to realize quantum SWAP gate by using a large-detuned single-mode cavity field and two identical Rydberg atoms. It is shown that the scheme can also be used to create multi-atom cluster state. During the interaction between atom and cavity, the cavity is only virtually excited and thus the scheme is insensitive to the cavity field states and cavity decay. With the help of our scheme it is very simple to prepare the N-atom cluster state with perfect fidelity and probability. The practical feasibility of this method is also discussed.

Keywords: cluster states, SWAP gate, cavity QED
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1. Introduction

The performance of a quantum computer relies on certain universal one-qubit and two-qubit logic gates. Any quantum computation[1] can be reduced to a sequence of operations of these gates.[2] There have been a number of experimental systems proposed for implementing these logic gates, and many of them have been implemented. We may mention the trapped ions,[3] cavity quantum electrodynamics,[4] nuclear magnetic resonance,[5] quantum dots,[6] and neutral atoms in an optical lattice[7] as examples. The SWAP gate is one of the basic two-qubit logic gates. On the other hand, the entangled state of quantum particles has attracted so much attention for it not only provides a useful tool to test quantum mechanics against local hidden variable theory,[8] but also constitutes a valuable resource in the field of quantum information science, such as quantum teleportation,[9−11] quantum cryptography,[12] quantum computation,[13] quantum dense coding and so on.[14] In the past years, many schemes using optical systems, cavity quantum electrodynamics (QED), ion trap and Heisenberg interaction systems, have been proposed for the preparation of entanglement between separate particles.[15−19]

Recently, Briegel and Raussendorf introduced a class of N-qubit entangled states, namely the cluster states,[20] which have some special properties. The cluster states share the properties both of the Greenberger–Horne–Zeilinger (GHZ) and of the W-class entangled states. But they still have some unique properties: e.g., they have a large persistency of entanglement, that is, they (in the case of $N > 4$) are harder to be destroyed by local operations than GHZ-class states. In addition, they are the key ingredient in measurement-based quantum computing and one-way quantum computing.[21] With a cluster state of an array of qubits, we can realize expected quantum gates by some single-qubit operations and detections. In a lattice structure, for any site a of the lattice, the cluster state $|\phi_{\{k\}}\rangle_{\rm c}$ obeys the set of eigenvalue equations $K^{(a)}|\phi_{\{k\}}\rangle_{\rm c} = (-1)^{k_a}|\phi_{\{k\}}\rangle_{\rm c}$, with the corresponding operator $K^{(a)} = \sigma_x^{(a)} \otimes_{b\in nghb(a)} \sigma_{\rm z}^{(\rm b)}$, where $nghb(a)$ specifies the sites of all the neighbours of the site a and $k_a \in \{0, 1\}$. The operators $K^{(a)}$ with $\{a \in \text{lattice}\}$ form a complete family of commuting operators on the lattice. Some physical systems such as quantum dots, optical photons, and cavity QED are suitable for preparation of cluster states.[22,23]

It has been shown that, the schemes used for QPG (quantum phase gate) can also be used for generating cluster states of particles.[24,25] In this article, we propose an efficient simple scheme of quantum (SWAP) gate which can also be used for generating multi-atom cluster state. We use the large detuned interaction of the atom and the cavity field. The distinct advantage of the proposed scheme is that during the operation the cavity is only virtually excited, there is no en-

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ergy exchange between the atoms and the cavity mode and thus the efficient decoherence time of the cavity is greatly prolonged.

2. Interaction of the atom with the cavity

Consider two identical Rydberg atoms simultaneously interacting with a single-mode cavity field of high Q. The relevant atomic levels and transitions are shown in Fig.1, where the states $|e\rangle$, $|g\rangle$, and $|f\rangle$ represent the three circular levels of the Rydberg atom

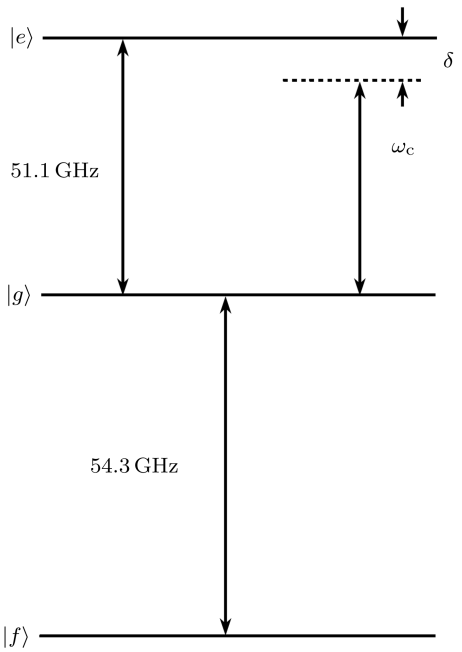


Fig.1. The level structure of the three-level Rydberg atom in the energy representation.

with principal quantum numbers 51, 50 and 49, respectively. In the rotating-wave approximation, the interaction Hamiltonian of the system in the interaction picture is (assuming $\hbar = 1$)[16]

$$H_{\rm i} = g \sum_{j=1,2} (\mathrm{e}^{-\mathrm{i}\delta t} a^{+} S_j^{-} + \mathrm{e}^{\mathrm{i}\delta t} a S_j^{+}), \qquad (1)$$

where $S_j^{+} = |e_j\rangle\langle g_j|$ and $S_j^{-} = |g_j\rangle\langle e_j|$, with $|e_j\rangle$ and $|e_j\rangle (j = 1, 2)$ being the excited and ground states of the jth atom, a^{+} and a are the creation and annihilation operators for the cavity mode, respectively, g is the atom cavity coupling strength, and δ is the detuning between the atomic transition ($|e\rangle \leftrightarrow |g\rangle$) frequency ω_0 and cavity frequency $\omega_{\rm c}$. In the case $\delta \ll (\omega_g - \omega_f) - (\omega_e - \omega_g) = 3.2\,\mathrm{GHZ}$, the state $|f\rangle$ will not be affected during the atom–cavity interaction. In the large detuning case $\delta \gg g$, there is no energy exchange between the atomic system and the cavity. Then the effective Hamiltonian is given by

$$H = \lambda \Bigg[\sum_{j=1,2} (|e_j\rangle\langle e_j| a a^{+} - |g_j\rangle\langle g_j| a^{+} a) + (S_1^{+} S_2^{-} + S_1^{-} S_2^{+}) \Bigg], \qquad (2)$$

where $\lambda = g^2/\delta$. The first and second terms describe the photon-number dependent Stark shifts, and the third and fourth terms describe the dipole coupling between the two atoms induced by the cavity mode. If we assume that the cavity field is initially in the vacuum state, the Hamiltonian reduces to

$$H = \lambda \Bigg[\sum_{j=1,2} (|e_j\rangle\langle e_j| + (S_1^{+} S_2^{-} + S_1^{-} S_2^{+}) \Bigg]. \qquad (3)$$

Then when we send two atoms through the cavity, after an interaction time t, by solving the Schrödinger equation, we can obtain the evolution of different initial atomic states during the interaction time:

$$\begin{aligned}
&|g_1\rangle|g_2\rangle \rightarrow |g_1\rangle|g_2\rangle,\\
&|e_1\rangle|g_2\rangle \rightarrow \mathrm{e}^{-\mathrm{i}\lambda t}(\cos\lambda t|e_1\rangle|g_2\rangle - \mathrm{i}\sin\lambda t|g_1\rangle|e_2\rangle),\\
&|g_1\rangle|e_2\rangle \rightarrow \mathrm{e}^{-\mathrm{i}\lambda t}(\cos\lambda t|g_1\rangle|e_2\rangle - \mathrm{i}\sin\lambda t|e_1\rangle|g_2\rangle),\\
&|e_1\rangle|e_2\rangle \rightarrow \mathrm{e}^{-2\mathrm{i}\lambda t}|e_1\rangle|e_2\rangle. \qquad (4)
\end{aligned}$$

3. Realization of quantum SWAP gate and cluster state

In order to realize the quantum SWAP gate, we only need to chose the interaction time such that $t = \dfrac{\pi}{2\lambda}$ is fulfilled. We can obtain:

$$\begin{aligned}
&|g_1\rangle|g_2\rangle \rightarrow |g_1\rangle|g_2\rangle,\\
&|g_1\rangle|e_2\rangle \rightarrow -|e_1\rangle|g_2\rangle,\\
&|e_1\rangle|g_2\rangle \rightarrow -|g_1\rangle|e_2\rangle,\\
&|e_1\rangle|e_2\rangle \rightarrow -|e_1\rangle|e_2\rangle. \qquad (5)
\end{aligned}$$

In this way, we implement a two-qubit quantum SWAP gate operation by using our procedure after a single qubit rotation.

We note that the procedure can be further used to prepare a special type of entangled state of many atoms. First we consider the two-atom case. In order to generate the two-atom cluster state, we first let two

atoms initially prepared in the state $|g_1g_2\rangle$ cross a classical field appropriately so that each atom undergoes the following transition:

$$|g_j\rangle = \frac{1}{\sqrt{2}}(|e_j\rangle - |g_j\rangle), \tag{6}$$

where $j = 1, 2$. After that, we simultaneously send the two atoms through a single-mode cavity initially prepared in the vacuum state. After an interaction time t, the state of the atoms becomes

$$|\psi(t)\rangle_2 = \mathrm{e}^{-2\mathrm{i}\lambda t}(|e_1e_2\rangle - |e_1g_2\rangle - |g_1e_2\rangle) + |g_1g_2\rangle. \tag{7}$$

If we chose the interaction time as before, we can obtain

$$|\psi(t)\rangle_2 = \frac{1}{2}(g_1g_2\rangle + |g_1e_2\rangle + |e_1g_2\rangle - |e_1e_2\rangle). \tag{8}$$

It is clear from Eq.(8) that the two atoms are in the state of the following form:

$$|\psi(t)\rangle_2 = \frac{1}{2}[(|g_1\rangle + |e_1\rangle\sigma_z^2)(|g_2\rangle + |e_2\rangle)], \tag{9}$$

where $\sigma_z^2 = |g\rangle_{22}\langle g| - |e\rangle_{22}\langle e|$. The state is a standard two-atom cluster state. Next, we will consider the preparation of N-atom cluster states. The $N(N \geq 3)$ atom are in the ground state $|g\rangle$, they are sent to a classical field before sent to a cavity. The amplitude and the phase of the classical field are chosen appropriately so that each atom undergoes the following transition:

$$|g_j\rangle = \frac{1}{\sqrt{2}}(|e_j\rangle + |g_j\rangle), \tag{10}$$

where $j = N(N \geq 3)$. After we have prepared the atoms $1, 2$ in the cluster state, we simultaneously send one of them, i.e., atom 2 and atom 3 to the cavity, after the interaction time as before we obtain

$$|\psi(t)\rangle_3 = -\frac{1}{2}[(|g_1\rangle + |e_1\rangle\sigma_z^2)(|g_2\rangle + |e_2\rangle\sigma_z^3)(|g_3\rangle + |e_3\rangle)]. \tag{11}$$

Then we sequentially send atom $N-1$ and atom N to the cavity as before, the state of the atoms becomes

$$|\psi(t)\rangle_N = \frac{(-1)^N}{2^{N/2}}\bigotimes_{j=1}^{N}(|g\rangle_j + |e\rangle_j\sigma_z^{j+1}), \tag{12}$$

where $\sigma_z^{N+1} \equiv 1$.

4. Discussion and conclusion

We have proposed a scheme to realize the quantum swap gate and generate multi-atom cluster state via cavity QED. Here we briefly discuss the feasibility of the current scheme. Firstly, the cavity is only virtually excited, there is no energy exchange between the atoms and the cavity mode and thus the efficient decoherence time of the cavity is greatly prolonged. Secondly, the atomic spontaneous emission is the main obstacle for the experimental realization of our scheme, but on one hand, for the experiment dealing with Rydberg atoms with principal quantum numbers $49, 50$ and 51, the radiation time is about $T_\mathrm{r} = 3 \times 10^{-2}\,\mathrm{s}$, and the coupling constant is $g = 2\pi \times 24\,\mathrm{KHz}$,[16] with the choice of $\delta = 10g$, the required atom-cavity-field interaction time is of the order of $\pi\delta/2g^2 \simeq 5 \times 10^{-5}\,\mathrm{s}$. So the condition can be satisfied by choosing Rydberg atoms. On the other hand, taking the atomic spontaneous emission into consideration, the Hamiltonian of the system of Eq.(3) can be rewritten as

$$H = \lambda\left[\sum_{j=1,2}(|e_j\rangle\langle e_j| + (S_1^+S_2^- + S_1^-S_2^+)\right] - \mathrm{i}\frac{\tau}{2}\sum_{j=1}^{2}|e\rangle_{jj}\langle e|, \tag{13}$$

where τ is the atomic spontaneous emission rate. If we send the atoms 1 and 2 through the vacuum cavity, choose the interaction time $t = \dfrac{\pi}{2\lambda}$, and set $\tau = 0.1\lambda$, the state of the atoms 1 and 2 thus becomes

$$|\psi(t)\rangle_2 = \frac{1}{2}(g_1g_2\rangle + |g_1e_2\rangle + |e_1g_2\rangle - \mathrm{e}^{-\pi/20}|e_1e_2\rangle). \tag{14}$$

The fidelity of the state in Eq.(14) relative to the standard two-atom cluster state in Eq.(9) is $\dfrac{(3+\mathrm{e}^{-\pi/20})^2}{16} \simeq 0.9286$ and the probability of success is $\dfrac{3+\mathrm{e}^{-\pi/10}}{4} \simeq 0.9326$. The fidelity and probability approach perfection. While the fidelity of the obtained multi-atom cluster state relative to the standard multi-atom cluster state in Eq.(12) will be $\left(\dfrac{(3+\mathrm{e}^{-\pi/20})^2}{16}\right)^{N-1}$, and the successful probability is $\left(\dfrac{3+\mathrm{e}^{-\pi/10}}{4}\right)^{N-1}$. It is shown that the fidelity and the successful probability decrease exponentially with the increase of N. Thirdly, our scheme requires that two atoms be simultaneously sent through a cavity; otherwise, there will be an error. Assume that during the generation of a two-atom cluster state one atom enters the cavity $0.01t$ sooner than another atom, with t being the time of each atom staying in the cavity. We can obtain the fidelity $F = 0.999$ for the generation of a two-atom cluster state. Obviously the effect of the errors on the fidelity of the resulting state is slight.

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