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Influences of decoherence on the generation of a macroscopic superposition state using weak cross-Kerr effect*

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Considering a system in which a single photon and a coherent field propagate through a Kerr medium, when the weak cross-Kerr interaction between the coherent state and the single photon under decoherence is involved, this paper derives analytically a macroscopic superposition state by the superoperator method and investigates the influences of decoherence on the coherence properties of the obtained state. It finds that the macroscopic superposition state will experience evolution from a pure superposition state to a mixed state in a dissipative environment and the Kerr effect makes the field display a periodic revival from decoherence for a short time.

Keywords: Kerr effect, superoperator, decoherence
PACC: 4250

1. Introduction

The Kerr effects, which involve the coupling between two modes of a field or the motion of centre of mass of an atom and a field, are very useful in quantum optics such as squeezing of field and quantum nondemolition detectors (QND). With the development of quantum information and computation, they also have many applications in entanglement purification,[1] teleportation,[2−4] optical Fock state synthesizer,[5] and CNOT gates.[6] However, the conventional materials can only offer a very small nonlinearity, which limits greatly the applications of Kerr effects. In order to bypass this limitation, one scheme is to obtain large Kerr nonlinearities with electromagnetically induced transparency (EIT)[7] and dipole induced transparency (DIT),[8] another is to use coherent fields to realize the coupling of two modes when the nonlinearity is small. It is suggested that the weak Kerr nonlinearity can be used for quantum computation[6] and for the production of macroscopic superpositions and entanglement.[9−11] When this scheme is applied, however, many researches do not involve the dissipation effect in the nonlinear media. Under a real environment, the decoherence is unavoidable in the process to realize quantum information. Jeong considered the Kerr interaction between two modes in which one mode is single photon and another is coherent field and investigated numerically the generation of macroscopic entanglement and quantum gate operations by using weak Kerr nonlinearity only under decoherence of the coherent field.[12,13] In fact, those two modes of light field are both exposed in the real environment, it is not enough that only involves the dissipation for the mode initially preparing in a coherent state but ignores the decay for the mode initially in a single photon state. Furthermore, the calculations in Refs.[12] and [13] are numerical which is not very precise.

In this paper, we consider a system in which a single photon and a coherent field propagate through a Kerr medium to generate a macroscopic superposition state under decoherence. By using superoperator method, we derive analytically the density operator of the system including weak Kerr nonlinear interaction under the consideration of the dissipation of the two modes. Our results show that the Kerr effect competes with decoherence to revive the purity of the field, though the system reaches a mixed state finally. Our research also displays that our analytical results are consistent with those of Refs.[12,13] in which only the dissipation of the coherent field is taken into account. However, it is revealed that the dissipation of the single photon field is also crucial for the generation of the macroscopic superposition state.

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2. Analytical solution of the master equation

The interaction Hamiltonian of cross-Kerr nonlinearity between two modes is $H = \chi a_1^\dagger a_1 a_2^\dagger a_2$, where $a_1^\dagger(a_2^\dagger)$ and $a_1(a_2)$ are the creation and annihilation operators for the two modes, respectively, χ is the interaction strength. The decoherence effects in the Kerr medium can be induced in the master equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\mathrm{i}\left[\chi a_1^\dagger a_1 a_2^\dagger a_2,\ \ \rho\right] + L_{a_1}\rho + L_{a_2}\rho \qquad (1)$$

with

$$\begin{aligned} L_{a_1}\rho &= \kappa_1\left(2a_1\rho a_1^\dagger - a_1^\dagger a_1\rho - \rho a_1^\dagger a_1\right), \\ L_{a_2}\rho &= \kappa_2\left(2a_2\rho a_2^\dagger - a_2^\dagger a_2\rho - \rho a_2^\dagger a_2\right), \end{aligned} \qquad (2)$$

where $L_{a_1}\rho$ and $L_{a_2}\rho$ describe the decay of the two modes in the nonlinear medium, respectively. κ_1 and κ_2 are dissipation rates of the single photon and the coherent field, respectively.

In order to solve the master equation (1), we define superoperators

$$\begin{aligned} K_{10}\rho &\equiv \frac{1}{2}\left(a_1^\dagger a_1\rho + \rho a_1^\dagger a_1\right), \\ K_{20}\rho &\equiv \frac{1}{2}\left(a_2^\dagger a_2\rho + \rho a_2^\dagger a_2\right), \\ K_{1-}\rho &\equiv a_1\rho a_1^\dagger, \quad K_{2-}\rho \equiv a_2\rho a_2^\dagger, \\ N_1\rho &\equiv \left(a_1^\dagger a_1\rho - \rho a_1^\dagger a_1\right), \\ N_2\rho &\equiv \left(a_2^\dagger a_2\rho - \rho a_2^\dagger a_2\right). \end{aligned}$$

It can be proved easily that

$$\begin{aligned} &[N_1,\ K_{20}]\rho = 0, \quad [N_2,\ K_{10}]\rho = 0, \\ &[K_{10},\ K_{20}]\rho = 0, \quad [K_{10},\ K_{2-}]\rho = 0, \\ &[K_{20},\ K_{1-}]\rho = 0, \quad [K_{1-},\ K_{2-}]\rho = 0, \end{aligned}$$

but

$$\begin{aligned} [K_{1-},\ K_{10}]\rho &= K_{1-}\rho, \\ [K_{2-},\ K_{20}]\rho &= K_{2-}\rho. \end{aligned}$$

The master equation now becomes

$$\begin{aligned} \frac{\mathrm{d}\rho}{\mathrm{d}t} =\ & -(2\kappa_1 + \mathrm{i}\chi N_2)K_{10}\rho - (2\kappa_2 + \mathrm{i}\chi N_1)K_{20}\rho \\ & + 2\kappa_1 K_{1-}\rho + 2\kappa_2 K_{2-}\rho. \end{aligned} \qquad (3)$$

Using the identity relation

$$\begin{aligned} &\exp[\theta(A + xB)]\rho \\ &= \exp(x\theta B)\exp\left[\frac{1}{x}(\mathrm{e}^{x\theta} - 1)A\right]\rho, \end{aligned} \qquad (4)$$

providing that the operator A and B obey the commutation relation $[A,\ B] = A$, where θ and x are parameters independent of A and B, respectively, we obtain the time evolution of the density matrix $\rho(t)$ which is written explicitly as

$$\begin{aligned} \rho(t) =\ & \{\exp[-(2\kappa_1 + \mathrm{i}\chi N_2)K_{10} + 2\kappa_1 K_{1-}]t\}\{\exp[-(2\kappa_2 + \mathrm{i}\chi N_1)K_{20} + 2\kappa_2 K_{2-}]t\}\rho(0) \\ =\ & \exp[-2(\kappa_1 + \mathrm{i}\lambda N_2)t\,K_{10}]\exp\left[-\frac{\kappa_1}{\kappa_1 + \mathrm{i}\lambda N_2}\left(\mathrm{e}^{-2(\kappa_1 + \mathrm{i}\lambda N_2)t} - 1\right)K_{1-}\right] \\ & \times \exp[-2(\kappa_2 + \mathrm{i}\lambda N_1)t\,K_{20}]\exp\left[-\frac{\kappa_2}{\kappa_2 + \mathrm{i}\lambda N_1}\left(\mathrm{e}^{-2(\kappa_2 + \mathrm{i}\lambda N_1)t} - 1\right)K_{2-}\right]\rho(0), \end{aligned} \qquad (5)$$

where $\lambda \equiv \chi/2$.

3. The effect of dissipation of the single photon on macroscopic superpositions

Based on Eq.(5), if the initial state is given, then the time evolution of the system can be obtained and the decoherence effect can be revealed analytically. Recently, Jeong proposed a scheme by combination of Gerry's scheme with Nemoto and Munro's idea so that the weak cross-Kerr nonlinearity can be used with a single photon, a coherent state, two photodetectors, and two beam splitters to generate a macroscopic superposition state.[12] Neglecting the dissipation of the mode 1, we numerically study the decoherence effects on the generation of a macroscopic superposition state. But here through solving the master equation (1), we may discuss the decoherence analytically. In order to compare our analytical results with Jeong's investigation, we take the same initial density matrix with Ref.[12] as

$$\rho(0)=\frac{1}{2}(|0,\ \alpha\rangle\langle 0,\ \alpha|+|0,\ \alpha\rangle\langle 1,\ \alpha|+|1,\ \alpha\rangle\langle 0,\ \alpha|+|1,\ \alpha\rangle\langle 1,\ \alpha|)\,. \tag{6}$$

If one performs a measurement onto the basis states $(|0\rangle\pm|1\rangle)/\sqrt{2}$ as Jeong[12] did, then the macroscopic superposition state obtained by the measurement is

$$\begin{aligned}\rho_{\pm}(t)\ =\ &\frac{1}{4}\Big\{|\ \mathrm{e}^{-\kappa_2 t}\alpha\rangle\langle \mathrm{e}^{-\kappa_2 t}\alpha|\pm C|\mathrm{e}^{-\kappa_2 t}\alpha\rangle\langle \mathrm{e}^{-\kappa_2 t}\alpha\mathrm{e}^{-\mathrm{i}\chi t}|\pm C^{*}|\mathrm{e}^{-\kappa_2 t}\alpha\mathrm{e}^{-\mathrm{i}\chi t}\rangle\langle \mathrm{e}^{-\kappa_2 t}\alpha|\\ &+\mathrm{e}^{-2\kappa_1 t}|\mathrm{e}^{-\kappa_2 t}\alpha\mathrm{e}^{-\mathrm{i}\chi t}\rangle\langle \mathrm{e}^{-\kappa_2 t}\alpha\mathrm{e}^{-\mathrm{i}\chi t}|+\exp\left[-|\alpha|^2\,\mathrm{e}^{-2\kappa_2 t}\right]\sum_{n,m=0}^{\infty}\frac{\alpha^n\alpha^{*m}}{\sqrt{n!m!}}z\\ &\times\exp[-\kappa_2(n+m)t]|n\rangle\langle m|\Big\},\end{aligned} \tag{7}$$

where $C=\mathrm{e}^{-\kappa_1 t}\exp[-|\alpha|^2(1-\mathrm{e}^{-2\kappa_2 t})]\exp\left[-\dfrac{\kappa_2|\alpha|^2}{\kappa_2-\mathrm{i}\lambda}\left(\mathrm{e}^{-2(\kappa_2-\mathrm{i}\lambda)t}-1\right)\right]$ and $z=\dfrac{\kappa_1}{\kappa_1+\mathrm{i}\lambda(n-m)}\{1-\exp[-2[\kappa_1+\mathrm{i}\lambda(n-m)]t\}$.

We can see from Eq.(7) that the weak Kerr nonlinearity effect makes the coherent field experience a phase shift $\theta=\chi t$, but the amplitude of the coherent field decreases with time due to the dissipative environment. In order to gain a large separation between the two coherent component states in Eq.(7) when the single photon and the coherent field propagate through the weak Kerr nonlinearity medium, the strong coherent field is required. In this case, the coherence time may be reduced under the dissipation of the two modes. This can be revealed analytically from the parameter C in Eq.(7), which determines the degree of decoherence. Obviously, both the dissipation of the single photon and the coherent field lead to decrease of $|C|$ with time, therefore the obtained superposition state $\rho_{\pm}(t)$ is a mixed one.

As a special example of our calculation, let us discard the decay of the single photon just as Ref.[12] did, that means $\kappa_1=0$ in Eq.(7). When γ is taken to be $2\kappa_2$, we can obtain the explicit expression of the coherence parameter as

$$\begin{aligned}C'\ =\ &\exp\left[-|\alpha|^2\left(1-\mathrm{e}^{-\gamma t}\right)\right]\\ &\times\exp\left[-\frac{\gamma|\alpha|^2}{\gamma-\mathrm{i}\chi}\left(\mathrm{e}^{-(\gamma-\mathrm{i}\chi)t}-1\right)\right].\end{aligned} \tag{8}$$

According to our analytical calculation, the decrease of the coherence parameter $|C'|$ against time when only considering that the dissipation of the coherent state is plotted in Fig.1, the curves are very similar to those curves in Fig.3 of Ref.[12], but Jeong's calculation is numerical. We also calculate coherence parameters under various conditions, for example, when $\alpha=300$, if $\chi/\gamma=0.0125$, $\theta=0.0105$ and $\chi/\gamma=0.0303$, $\theta=0.0105$, respectively, our results are 0.4733 and 0.6456, in comparison with 0.474 and 0.644 of Ref.[13], they are very close, however, our calculations are very convenient and we did not have any assumptions for the calculation.

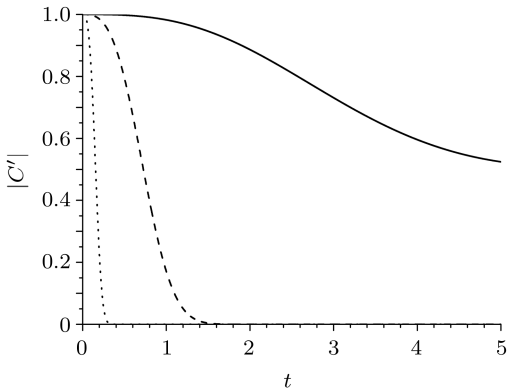


Fig.1. The time evolution of coherence parameter $|C'|$ under different probe fields. $\alpha=3$ (solid line), $\alpha=30$ (dashed line), $\alpha=300$ (dotted line), respectively and $\chi/\gamma=0.0125$.

Since the purity $\mathrm{Tr}(\rho_{+}^2)$ is another good physical quantity to measure the influence of the dissipation of both modes on the macroscopic superposition state, so we plot the time evolution of purity $\mathrm{Tr}(\rho_{+}^2)$ in Fig.2. One can find that when χ increases, i.e., the Kerr interaction gets relatively stronger, although the purity decreases totally because of decoherence process, it exhibits periodic revival within a short time under Kerr effect. As time goes on, the system loses purity in an irreversible way due to the decay of the single photon

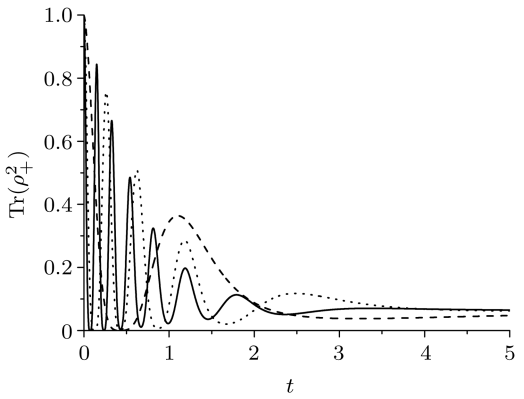


Fig.2. The time evolution of purity $\mathrm{Tr}(\rho_+^2)$ with different Kerr interaction strength χ. $\chi/\gamma = 0.05$ (solid line), $\chi/\gamma = 0.03$ (dotted line), $\chi/\gamma = 0.01$ (dashed line), respectively and $\alpha = 30$, $\kappa_1 = \kappa_2 = \gamma/2$.

and the coherent field, which means that the system is a mixed state. For comparing the dissipation of the single photon and that of the coherent field, we also plot the evolution of the purity with time for different single photon decay rates in Fig.3. We can see that

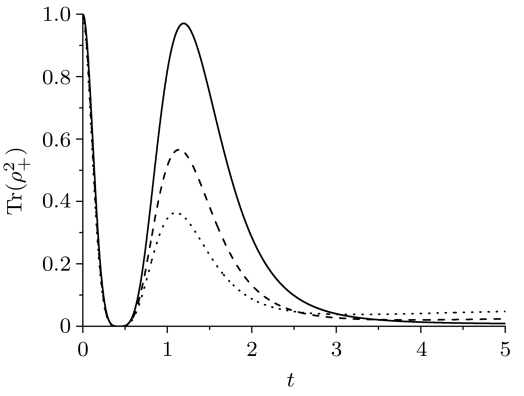


Fig.3. The time evolution of purity $\mathrm{Tr}(\rho_+^2)$ but with different single photon decay rates. $\kappa_1 = 0$ (solid line), $\kappa_1 = 0.5\kappa_2$ (dashed line), $\kappa_1 = \kappa_2$ (dotted line), respectively and $\alpha = 30$, $\chi/\gamma = 0.01$.

the purity decreases rapidly in the beginning for a short time and the effect of the decay of the single photon is not so obvious, however, when the decay rate of the single photon is larger, the effect of its dissipation becomes evident when the interaction time is relatively longer. Because the two modes expose in the same dissipative environment, so both of their loss in the Kerr medium should be included, which means that the loss of the single photon field could not be underestimated, and this causes that the obtained state $\rho_\pm(t)$ in Eq.(7) is deviated from the macroscopic superposition state obtained in Ref.[12]. In the following, we will use the average fidelity to measure the deviation of $\rho_\pm(t)$ from that given by Jeong[12] when $\kappa_1 = 0$.

In order to examine the effect of energy loss of the single photon on the coherent field further, we use the average fidelity[14] to assess the difference between the state for which only the decoherence of the coherent field is considered and the state involving both the decoherence effect of the single photon and the coherent field. The average fidelity is defined as

$$F = \frac{\mathrm{Tr}(\rho_1 \otimes \rho_2)}{\mathrm{Tr}(\rho_1 \otimes \rho_1)}, \tag{9}$$

where ρ_1 is the density operator in which only the dissipation of the mode 2 is considered, i.e., κ_1 is taken to be 0 in Eq.(7); ρ_2 is the density operator of both the decoherence effect involved, i.e. $\kappa_1 \neq 0$. From Eq.(7), we find that the dissipation of the single photon not only affects the coherence parameter C, but also contributes the state density an extra term, it is the decay of the single photon that leads to the difference between ρ_1 and ρ_2. Figure 4 shows the time evolution of the average fidelity with various dissipation rates of the mode 1. One can see that the average fidelity decreases when the decay rate of the mode 1 increases, this means that there is large deviation from the state in which only decoherence of the mode 2 is considered, so the dissipation of the single photon can not be discarded.

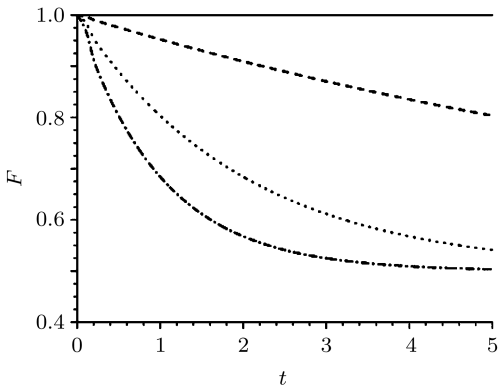


Fig.4. The time evolution of average fidelity with various loss rates of single photon. $\kappa_1 = 0$ (solid line), $\kappa_1 = 0.1\,\kappa_2$ (dashed line), $\kappa_1 = 0.5\,\kappa_2$ (dotted line), $\kappa_1 = \kappa_2$ (dashed dotted line), respectively and $\gamma = 2\kappa_2$, $\chi/\gamma = 0.0125$.

4. Conclusions

In this paper, we discuss analytically the decoherence effect on the generation of a macroscopic superposition state in a system consisted of a single pho-

ton and a coherent field propagating through a Kerr medium by use of the superoperator method. We found that the Kerr effect can revive the system from decoherence in the beginning for a short time. It is worthwhile to emphasize that the dissipation of the single photon affects the coherent parameter and the state of the system, so it should be taken into account under realistic experimental conditions for the realization of the macroscopic entanglement generation and quantum computation.

References

[1] Duan L M, Giedke G, Cirac J I and Zoller P 2000 *Phys. Rev. Lett.* **84** 4002

[2] Vitali D, Fortunato M and Tombesi P 2000 *Phys. Rev. Lett.* **85** 445

[3] Zhang J X, Xie C D and Peng K C 2005 *Chin. Phys. Lett.* **22** 3005

[4] Liao J Q and Kuang L M 2006 *Chin. Phys.* **15** 2246

[5] D'Ariano G M, Maccone L, Paris M G A and Sacchi M F 2000 *Phys. Rev.* A **61** 053817

[6] Nemoto K and Munro W J 2004 *Phys. Rev. Lett.* **93** 250502

[7] Schmidt H and Imamough A 1996 *Opt. Lett.* **21** 1936

[8] Waks E and Vuckovic J 2006 *Phys. Rev.* A **73** 041803 (R)

[9] Van Enk S J 2003 *Phys. Rev. Lett.* **91** 017902

[10] Jeong H, Kim M S, Ralph T C and Ham B S 2004 *Phys. Rev.* A **70** 061801 (R)

[11] Huang Y X, Zhao P Y, Huang X and Zhan M S 2004 *Acta Phys. Sin.* **53** 75 (in Chinese)

[12] Jeong H 2005 *Phys. Rev.* A **72** 034305

[13] Jeong H 2006 *Phys. Rev.* A **73** 052320

[14] Li F L and Li H R 2004 *Chin. Phys. Lett.* **21** 991