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Implementing remote controlled-NOT gates and entanglement swapping via geometric phase gates in ion-trap systems*

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We propose a scheme for the implementation of remote controlled-NOT gates and entanglement swapping via geometric phase gates in ion-trap systems. The proposed scheme uses the two ground states of the Λ-type ions as memory instead of the vibrational mode. And the system is robust against the spontaneous radiation and the dephasing.

Keywords: remote controlled-NOT gates, entanglement swapping, geometric phase gates, ion-trap systems

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1. Introduction

Using entanglement as a resource is a common feature of many tasks in quantum information processing, such as quantum teleportation,[1,2] quantum secret sharing,[3,4] quantum cloning,[5,6] and distributed quantum computation.[7,8] Distributed quantum computation, firstly proposed by Grover[7] and then developed by Cirac *et al*,[8] is a multi-processor device where each processor contains only a small number of qubits and acts as a node of a quantum communications network. To realize this computation, one must transfer not only quantum states, but also quantum operations between the nodes of the network efficiently. Recently, most of research has been devoted to transferring quantum operations,[9−14] which is more challenging than teleportation of quantum states. In the realm of transferring quantum operations, remote controlled-NOT (CNOT) gates have attracted much attention.[9−12] The reason is that CNOT gates together with a set of general one-qubit gates are sufficient to implement any multi-qubit unitary transformation. In 2000, Eisert *et al*[9] proposed a scheme for optimally implementing remote CNOT gates, which was then experimentally realized by Huang *et al*[10] in the photonic system.

Besides the photonic systems, the ion-trap system is also a candidate for quantum information processor. Since the first scheme with trapped ions was proposed by Cirac and Zoller,[15] many groups have devoted themselves to studying the ion-trap processor.[16−22] In Ref.[16], Leibfried *et al* experimentally demonstrated a geometric phase operation between two two-level ions. Following this idea, Zheng demonstrated a geometric phase gate with two Λ-type three-level ions to implement Grover search algorithm[17] and to prepare cluster states.[18] In this paper, we propose a scheme for implementing remote CNOT gates and entanglement swapping via geometric phase gates in ion-trap systems.

This paper is outlined as follows. In Section 2, a model for realizing geometric phase gates is demonstrated. In Section 3, a remote controlled not gate is implemented. And a proposal for the realization of entanglement swapping is described in Section 4. Section 5 is the conclusion.

2. Model for implementing geometric phase gates

We consider two identical three-level ions (ions m and n), each having two ground states $|g_1\rangle$ and $|g_2\rangle$ and an excited state $|e\rangle$, trapped in a linear potential.

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We now follow the ideas of the Refs.[17] and [18] to realize a geometric phase gate. The transition $|g_2\rangle \to |e\rangle$ is driven by two classical laser fields with detunings Δ and $\Delta+\omega_0-\delta$, respectively. Here ω_0 is the frequency of the centre-of-mass (c.m.) vibrational mode and δ denotes the difference between ω_0 and the relative detuning of the two lasers. We suppose that $\Delta \gg \Omega_i$ and ω_0 and $\omega_0 \gg \delta$, where Ω_i $(i=m,\ n\ )$ is the Rabi frequency of the ith laser; and the ions initially have no probabilities of being populated in the state $|e\rangle$. In this case the Hamiltonian of the system is[17,18]

$$H = \omega_0 a^+ a - \sum_{i=m}^{n}\left(\frac{\Omega}{2}\mathrm{e}^{-\mathrm{i}\left[(\omega_0-\delta)\,t-\eta\left(a+a^+\right)\right]} + \frac{\Omega}{2}\mathrm{e}^{\mathrm{i}\left[(\omega_0-\delta)t-\eta\left(a+a^+\right)\right]} + \frac{\Omega_1^2+\Omega_2^2}{4\Delta}\right) \times |g_2\rangle_{i\,i}\,\langle g_2|, \tag{1}$$

where $\Omega = \dfrac{\Omega_1\,\Omega_2}{2\Delta}$.

We consider the ions to be confined in the Lamb–Dicke regime ($\eta\sqrt{\bar{n}+1} \ll 1$), where $\bar{n}$ is the mean photon number of the centre-of-mass mode and assume that $\eta\Omega \ll \omega_0$, then the interaction Hamiltonian, in the interaction picture, is given by

$$H_{\mathrm{I}} = -\sum_{i=m}^{n}\left(\frac{\Omega_1^2+\Omega_2^2}{4\Delta} + \frac{\Omega}{2}\left[2\cos\left(-\delta\,t+\omega_0 t\right) + \mathrm{i}\eta\left(a^+\mathrm{e}^{\mathrm{i}\delta\,t} - a\mathrm{e}^{-i\delta\,t}\right)\right]\right)|g_2\rangle_{i\,i}\,\langle g_2|. \tag{2}$$

Suppose that the two ions are initially prepared in the states $|g_1\rangle_m\,|g_1\rangle_n$, $|g_1\rangle_m\,|g_2\rangle_n$, $|g_2\rangle_m\,|g_1\rangle_n$ or $|g_2\rangle_m\,|g_2\rangle_n$ and the initial state of the vibrational mode is $|\phi_{\mathrm{v}}\,(0)\rangle$. By applying the Hamiltonian (2), the whole system state evolves to[17]

$$|g_1\rangle_m\,|g_1\rangle_n\,|\phi_{\mathrm{v}}\,(0)\rangle \to |g_1\rangle_m\,|g_1\rangle_n\,|\phi_{\mathrm{v}}\,(0)\rangle\,; \tag{3a}$$

$$|g_1\rangle_m\,|g_2\rangle_n\,|\phi_{\mathrm{v}}\,(0)\rangle \to \mathrm{e}^{\mathrm{i}\theta}\mathrm{e}^{\mathrm{i}\phi}\,|g_1\rangle_m\,|g_2\rangle_n\,|\phi_{\mathrm{v}}\,(0)\rangle\,; \tag{3b}$$

$$|g_2\rangle_m\,|g_1\rangle_n\,|\phi_{\mathrm{v}}\,(0)\rangle \to \mathrm{e}^{\mathrm{i}\theta}\mathrm{e}^{\mathrm{i}\phi}\,|g_2\rangle_m\,|g_1\rangle_n\,|\phi_{\mathrm{v}}\,(0)\rangle\,; \tag{3c}$$

$$|g_2\rangle_m\,|g_2\rangle_n\,|\phi_{\mathrm{v}}\,(0)\rangle \to \mathrm{e}^{2\mathrm{i}\theta}\mathrm{e}^{4\mathrm{i}\phi}\,|g_2\rangle_m\,|g_2\rangle_n\,|\phi_{\mathrm{v}}\,(0)\rangle\,, \tag{3d}$$

where $\theta = \dfrac{\Omega_1^2+\Omega_2^2}{4\Delta}\tau + \dfrac{\Omega}{\omega_0-\delta}\sin\left(\omega_0-\delta\right)\tau$ and we have chosen the interaction time τ to satisfy $\delta\,\tau = 2\pi$, leading to $\phi = \left(\eta\dfrac{\Omega}{2}\right)^2\dfrac{2\pi}{\delta^2}$, controlled only by the Rabi frequencies of the classical laser fields.

3. Implementation of remote CNOT gates

In this section, we describe the scheme for the implementation of remote CNOT gates. Suppose that Alice wants to implement a CNOT transformation with the ion 1 (at her side) acting as controlled qubit and the ion 4 (at Bob's side, far from Alice's) as target qubit. The ion 1 is initially prepared in the state $\left(\alpha\,|g_1\rangle+\beta\,|g_2\rangle\right)_1$ and the ion 4 in the state $\left(\mu\,|+\rangle+\nu\,|-\rangle\right)_4$, where $|\alpha|^2+|\beta|^2=1$, $|\mu|^2+|\nu|^2=1$ and $|\pm\rangle = \dfrac{1}{\sqrt{2}}\left(|g_1\rangle \pm |g_2\rangle\right)$. After the CNOT transformation, the initial state of the system composed of ions 1 and 4

$$\left(\alpha\,|g_1\rangle+\beta\,|g_2\rangle\right)_1\left(\mu\,|+\rangle+\nu\,|-\rangle\right)_4 \tag{4}$$

evolves to

$$\alpha\,|g_1\rangle_1\left(\mu\,|+\rangle+\nu\,|-\rangle\right)_4 + \beta\,|g_2\rangle_1\left(\mu\,|-\rangle+\nu\,|+\rangle\right)_4, \tag{5}$$

which shows that only if the ion 1 is in the state $|g_2\rangle_1$, the ion 4 will flip its state. The whole process for the implementation of the CNOT gates can be described as follows.

Step 1 To achieve this goal, an ancillary EPR pair

$$|\psi\rangle_{23} = \frac{1}{\sqrt{2}}\left(|g_1g_2\rangle + |g_2g_1\rangle\right)_{23}, \tag{6}$$

where the ion 2 belongs to Alice and the ion 3 belongs to Bob, should be appended to the initial state (4), thus the total system state will be

$$|S_0\rangle = \left(\alpha\,|g_1\rangle+\beta\,|g_2\rangle\right)_1\frac{1}{\sqrt{2}}\left(|g_1g_2\rangle+|g_2g_1\rangle\right)_{23} \times\left(\mu\,|+\rangle+\nu\,|-\rangle\right)_4. \tag{7}$$

Step 2 Alice performs a Hadamard transformation on the ion 2, i.e.

$$|g_1\rangle \to \frac{1}{\sqrt{2}}\left(|g_1\rangle+|g_2\rangle\right), \qquad |g_2\rangle \to \frac{1}{\sqrt{2}}\left(|g_1\rangle-|g_2\rangle\right). \tag{8}$$

The system state $|S_0\rangle$ becomes

$$\begin{aligned}|S_1\rangle \;=\; & \frac{1}{2\sqrt{2}}\left(\alpha\,|g_1\rangle+\beta\,|g_2\rangle\right)_1\\ & \times\left(|g_1g_1\rangle+|g_1g_2\rangle-|g_2g_1\rangle+|g_2g_2\rangle\right)_{23}\\ & \times\left[(\mu+\nu)\,|g_1\rangle+(\mu-\nu)\,|g_2\rangle\right]_4. \end{aligned}\tag{9}$$

Step 3 Alice performs a geometric phase gate depicted in Eqs.(3) on her ions 1, and Bob does the same on his ions 3 and 4, leading the whole system state $\left|S_1\right\rangle$ to

$$\begin{aligned}|S_2\rangle \;=\; & \frac{1}{2\sqrt{2}}\Big[\alpha(\mu+\nu)\,\Big(|g_1g_1g_1g_1\rangle+\mathrm{e}^{\mathrm{i}\theta_2}\mathrm{e}^{\mathrm{i}\phi_2}|g_1g_1g_2g_1\rangle-\mathrm{e}^{\mathrm{i}\theta_1}\mathrm{e}^{\mathrm{i}\phi_1}|g_1g_2g_1g_1\rangle\\ & +\mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}\mathrm{e}^{\mathrm{i}(\phi_1+\phi_2)}|g_1g_2g_2g_1\rangle\Big)+\alpha(\mu-\nu)\,\Big(\mathrm{e}^{\mathrm{i}\theta_2}\mathrm{e}^{\mathrm{i}\phi_2}|g_1g_1g_1g_2\rangle+\mathrm{e}^{2\mathrm{i}\theta_2}\mathrm{e}^{4\mathrm{i}\phi_2}|g_1g_1g_2g_2\rangle\\ & -\mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}\mathrm{e}^{\mathrm{i}(\phi_1+\phi_2)}|g_1g_2g_1g_2\rangle+\mathrm{e}^{\mathrm{i}(\theta_1+2\theta_2)}\mathrm{e}^{\mathrm{i}(\phi_1+4\phi_2)}|g_1g_2g_2g_2\rangle\Big)+\beta(\mu+\nu)\,\Big(\mathrm{e}^{\mathrm{i}\theta_1}\mathrm{e}^{\mathrm{i}\phi_1}|g_2g_1g_1g_1\rangle\\ & +\mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}\mathrm{e}^{\mathrm{i}(\phi_1+\phi_2)}|g_2g_1g_2g_1\rangle-\mathrm{e}^{2\mathrm{i}\theta_1}\mathrm{e}^{4\mathrm{i}\phi_1}|g_2g_2g_1g_1\rangle+\mathrm{e}^{\mathrm{i}(2\theta_1+\theta_2)}\mathrm{e}^{\mathrm{i}(4\phi_1+\phi_2)}|g_2g_2g_2g_1\rangle\Big)\\ & +\beta(\mu-\nu)\,\Big(\mathrm{e}^{\mathrm{i}(\theta_1+\theta_2)}\mathrm{e}^{\mathrm{i}(\phi_1+\phi_2)}|g_2g_1g_1g_2\rangle+\mathrm{e}^{\mathrm{i}(\theta_1+2\theta_2)}\mathrm{e}^{\mathrm{i}(\phi_1+4\phi_2)}|g_2g_1g_2g_2\rangle\\ & -\mathrm{e}^{\mathrm{i}(2\theta_1+\theta_2)}\mathrm{e}^{\mathrm{i}(4\phi_1+\phi_2)}|g_2g_2g_1g_2\rangle+\mathrm{e}^{\mathrm{i}(2\theta_1+2\theta_2)}\mathrm{e}^{\mathrm{i}(4\phi_1+4\phi_2)}|g_2g_2g_2g_2\rangle\Big)\Big]_{1234}, \end{aligned}\tag{10}$$

where the subscript $i=1$ (2) represents the transformation that Alice (Bob) has done.

If both Alice and Bob choose the appropriate Rabi frequencies of the classical laser fields to make ϕ_1 and $\phi_2=\dfrac{\pi}{2}$, and perform the transformations $|g_2\rangle\to\mathrm{e}^{-\mathrm{i}\theta_1}\mathrm{e}^{-\mathrm{i}\phi_1}\,|g_2\rangle$ on ions 1 and 2 and $|g_2\rangle\to\mathrm{e}^{-\mathrm{i}\theta_2}\mathrm{e}^{-\mathrm{i}\phi_2}\,|g_2\rangle$ on ions 3 and 4, respectively, the system state described in Eq.(10) evolves to

$$\begin{aligned}|S_3\rangle \;=\; & \frac{1}{2\sqrt{2}}\left[\alpha\,(\mu+\nu)\,(|g_1g_1g_1g_1\rangle+|g_1g_1g_2g_1\rangle-|g_1g_2g_1g_1\rangle+|g_1g_2g_2g_1\rangle)\right.\\ & +\alpha\,(\mu-\nu)\,(|g_1g_1g_1g_2\rangle-|g_1g_1g_2g_2\rangle-|g_1g_2g_1g_2\rangle-|g_1g_2g_2g_2\rangle)\\ & +\beta\,(\mu+\nu)\,(|g_2g_1g_1g_1\rangle+|g_2g_1g_2g_1\rangle+|g_2g_2g_1g_1\rangle-|g_2g_2g_2g_1\rangle)\\ & \left.+\beta\,(\mu-\nu)\,(|g_2g_1g_1g_2\rangle-|g_2g_1g_2g_2\rangle+|g_2g_2g_1g_2\rangle+|g_2g_2g_2g_2\rangle)\right]_{1234}. \end{aligned}\tag{11}$$

The above equation (11) can be rewritten as

$$\begin{aligned}|S_3\rangle \;=\; & \frac{1}{2}\Big\{|{+}{+}\rangle_{23}\left[\alpha\,|g_1\rangle_1\,(\mu\,|-\rangle+\nu\,|+\rangle)_4+\beta\,|g_2\rangle_1\,(\mu\,|+\rangle+\nu\,|-\rangle)_4\right]\\ & -|{+}{-}\rangle_{23}\left[\alpha\,|g_1\rangle_1\,(\mu\,|-\rangle+\nu\,|+\rangle)_4-\beta\,|g_2\rangle_1\,(\mu\,|+\rangle+\nu\,|-\rangle)_4\right]\\ & +|{-}{+}\rangle_{23}\left[\alpha\,|g_1\rangle_1\,(\mu\,|+\rangle+\nu\,|-\rangle)_4+\beta\,|g_2\rangle_1\,(\mu\,|-\rangle+\nu\,|+\rangle)_4\right]\\ & +|{-}{-}\rangle_{23}\left[\alpha\,|g_1\rangle_1\,(\mu\,|+\rangle+\nu\,|-\rangle)_4-\beta\,|g_2\rangle_1\,(\mu\,|-\rangle+\nu\,|+\rangle)_4\right]\Big\}. \end{aligned}\tag{12}$$

Step 4 Alice (Bob) measures the ion 2 (3) in the basis $\{|+\rangle\,,\,|-\rangle\}$. From the above formula (12), we can see only if they communicate one cbit (bit of classical communication) of their respective measurement results each other, and perform a corresponding local unitary operation $U_1\otimes U_4(U_{\mathrm{i}}\in\{I,\,\sigma_z\}\,,\;i=1,\,4,$ $I=|g_1\rangle\langle g_1|+|g_2\rangle\langle g_2|$ and $\sigma_z=|g_1\rangle\langle g_1|-|g_2\rangle\langle g_2|)$ on their ions 1 and 4 respectively. After that, the CONT gates have been implemented, namely, Alice will obtain the state depicted by Eq.(5) all the time. The relation of the measurement results (R_{13}), the reduced state ($|\psi\rangle_{14}$) of the ions 1 and 4 and the local unitary operation (U_{14}) is depicted in Table 1.

Table 1. Relation of the measurement results (R_{13}), the reduced state ($|\psi\rangle_{14}$) of the ions 1 and 4 and the local unitary operation (U_{14}).

R_{13}	$\|\psi\rangle_{14}$	$U_1 \otimes U_4$
$\|++\rangle_{23}$	$\alpha\|g_1\rangle_1(\gamma\|-\rangle+\delta\|+\rangle)_4+\beta\|g_2\rangle_1(\gamma\|+\rangle+\delta\|-\rangle)_4$	$I_1\otimes\sigma_{4z}$
$\|+-\rangle_{23}$	$\alpha\|g_1\rangle_1(\gamma\|-\rangle+\delta\|+\rangle)_4-\beta\|g_2\rangle_1(\gamma\|+\rangle+\delta\|-\rangle)_4$	$\sigma_{1z}\otimes\sigma_{4z}$
$\|-+\rangle_{23}$	$\alpha\|g_1\rangle_1(\gamma\|+\rangle+\delta\|-\rangle)_4+\beta\|g_2\rangle_1(\gamma\|-\rangle+\delta\|+\rangle)_4$	$I_1\otimes I_4$
$\|--\rangle_{23}$	$\alpha\|g_1\rangle_1(\gamma\|+\rangle+\delta\|-\rangle)_4-\beta\|g_2\rangle_1(\gamma\|-\rangle+\delta\|+\rangle)_4$	$\sigma_{1z}\otimes I_4$

4. Implementation of entanglement swapping

In Section 3, we use a remote maximal EPR pair to achieve remote CNOT gates. However, in real experiment, the actual available quantum channels are usually nonmaximally entangled states (pure nonmaximally entangled state or mixed entangled states) and the distance between two qubits cannot be very long. Fortunately, Zukowski *et al*[23] proposed a method, called entangled swapping, which can effectively solve these problems and generate a maximally entangled state between two distant parties that have never interacted before. Since then, many theoretical and experimental schemes for (via) entanglement swapping are proposed.[19,24–28] In this section, we will describe our scheme for entanglement swapping. We suppose that there are two nonmaximally entangled ionic states, initially prepared in

$$|\psi\rangle_{56}=(a|g_1g_1\rangle+b|g_2g_2\rangle)_{56}\,, \tag{13}$$

$$|\psi\rangle_{78}=(c|g_1g_1\rangle+d|g_2g_2\rangle)_{78}\,, \tag{14}$$

where $|a|^2+|b|^2=1$, $|c|^2+|d|^2=1$; the ion 5 was held by Alice; the ions 6 and 7 held by Charlie and the ion 8 held by Bob. So the whole system state is

$$\begin{aligned}|S\rangle_0 \;=\; &(a|g_1g_1\rangle+b|g_2g_2\rangle)_{56}\\ &\times(c|g_1g_1\rangle+d|g_2g_2\rangle)_{78}\,.\end{aligned} \tag{15}$$

Firstly, Charlie performs a Hadamard transformation (8) on the ion 7, which leads the whole system state (15) to

$$\begin{aligned}|S\rangle_1 \;=\; &\frac{1}{\sqrt{2}}\left[ac\left(|g_1g_1g_1g_1\rangle+|g_1g_1g_2g_1\rangle\right)+ad\left(|g_1g_1g_1g_2\rangle-|g_1g_1g_2g_2\rangle\right)\right.\\ &\left.+bc\left(|g_2g_2g_1g_1\rangle+|g_2g_2g_2g_1\rangle\right)+bd\left(|g_2g_2g_1g_2\rangle-|g_2g_2g_2g_2\rangle\right)\right]_{5678}\,.\end{aligned} \tag{16}$$

Secondly, Charlie implements the geometric phase gate (3) on ions 6 and 7, the system state $|S\rangle_1$ becomes

$$\begin{aligned}|S\rangle_2 \;=\; &\frac{1}{\sqrt{2}}\left[ac\left(|g_1g_1g_1g_1\rangle+\mathrm{e}^{\mathrm{i}\theta}\mathrm{e}^{\mathrm{i}\phi}|g_1g_1g_2g_1\rangle\right)+ad\left(|g_1g_1g_1g_2\rangle-\mathrm{e}^{\mathrm{i}\theta}\mathrm{e}^{\mathrm{i}\phi}|g_1g_1g_2g_2\rangle\right)\right.\\ &\left.+bc\left(\mathrm{e}^{\mathrm{i}\theta}\mathrm{e}^{\mathrm{i}\phi}|g_2g_2g_1g_1\rangle+\mathrm{e}^{2\mathrm{i}\theta}\mathrm{e}^{4\mathrm{i}\phi}|g_2g_2g_2g_1\rangle\right)+bd\left(\mathrm{e}^{\mathrm{i}\theta}\mathrm{e}^{\mathrm{i}\phi}|g_2g_2g_1g_2\rangle-\mathrm{e}^{2\mathrm{i}\theta}\mathrm{e}^{4\mathrm{i}\phi}|g_2g_2g_2g_2\rangle\right)\right]_{5678}\,.\end{aligned} \tag{17}$$

If he chooses $\phi=\dfrac{\pi}{2}$ and performs the transformation $|g_2\rangle\rightarrow\mathrm{e}^{-\mathrm{i}\theta}\mathrm{e}^{-\mathrm{i}\phi}|g_2\rangle$ on ions 6 and 7, the system state $|S\rangle_2$ evolves to

$$\begin{aligned}|S\rangle_3 \;=\; &\frac{1}{\sqrt{2}}\left[ac\left(|g_1g_1g_1g_1\rangle+|g_1g_1g_2g_1\rangle\right)+ad\left(|g_1g_1g_1g_2\rangle-|g_1g_1g_2g_2\rangle\right)\right.\\ &\left.+bc\left(|g_2g_2g_1g_1\rangle-|g_2g_2g_2g_1\rangle\right)+bd\left(|g_2g_2g_1g_2\rangle+|g_2g_2g_2g_2\rangle\right)\right]_{5678}\,.\end{aligned} \tag{18}$$

Assume that $a=c$ and $b=d$, the system state (18) becomes

$$\begin{aligned}|S\rangle_4 \;=\; &\frac{1}{2}\left\{\left(a^2|g_1\rangle_5|g_1\rangle_6|g_1\rangle_8+b^2|g_2\rangle_5|g_2\rangle_6|g_2\rangle_8\right)|+\rangle_7\right.\\ &\left.+\frac{ab}{\sqrt{2}}\left[\left(|g_1\rangle_5|g_2\rangle_8+|g_2\rangle_5|g_1\rangle_8\right)|+\rangle_6+\left(|g_1\rangle_5|g_2\rangle_8-|g_2\rangle_5|g_1\rangle_8\right)|-\rangle_6\right]|-\rangle_7\right\}.\end{aligned} \tag{19}$$

Thirdly, Charlie measures both of the ions 7 and 6 in sequence in the basis $\{|+\rangle, |-\rangle,\}$. If the measurement result is $|-\rangle_7 |\pm\rangle_6$, the system state $|S\rangle_4$ reduces to

$$\frac{1}{\sqrt{2}} \left(|g_1\rangle_5 |g_2\rangle_8 \pm |g_2\rangle_5 |g_1\rangle_8 \right). \tag{20}$$

In these two cases, Charlie informs Alice and Bob of having prepared a maximal EPR pair via entanglement swapping using classical channels.

But if Charlie detects the ion 7 in the state $|+\rangle_7$, the system reduces to

$$\frac{1}{\sqrt{|a|^4 + |b|^4}} \left(a^2 |g_1 g_1\rangle_{56} |g_1\rangle_8 + b^2 |g_2 g_2\rangle_{56} |g_2\rangle_8 \right), \tag{21}$$

which is still a nonmaximally entangled state of three ions. Then Charlie tells Alice and Bob the entanglement swapping fails and they must repeat preparing a maximally entangled state via entanglement swapping. The success probability is $P = 2|a|^2 \left(1 - |a|^2\right)$.

5. Summary

In conclusion, we have proposed a scheme for implementing remote controlled-NOT gates and entanglement swapping via geometric phase gates in ion-trap systems. In the present protocol, the distinct advantages of our protocol are as follows. One is that geometric phase gates, determined only by the path area, are used, making the systems robust against dephasing and the fidelity significantly higher than that of the dynamical ones used in the previous schemes. The other one is that only the ionic qubits instead of vibrational mode are used as memory. In addition, the interaction between the ions and vibrational modes may be implemented via the way of the Ref.[16]. Therefore, the scheme may be realized by the current techniques.

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