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Super energy flows in a waveguide filled with air and left-handed materials*

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The phenomena of super energy flows are studied theoretically and numerically in a parallel-plate waveguide which is filled with two layered equally-thick different media, i.e. air and specific left-handed materials (LHM) with $\epsilon_{r1} = -1/(1+\delta) + i\gamma$ and $\mu_{r1} = -(1+\delta) + i\gamma$. In this special waveguide, two-directional super-energy flows are excited by a three-dimensional horizontal electric dipole at the same time, which has transmission patterns different from those of two-dimensional source and three-dimensional vertical electric dipole. We also show that the retardation and loss in LHM are sensitive to the amplitude of super power densities, and the dimensions of waveguide determine the propagating modes, which makes super energy flows more practical.

Keywords: super energy flows, left-handed materials
PACC: 4110H, 4290, 4270Y

1. Introduction

Forty years ago Veselago proposed the concept of left-handed materials (LHM) which have negative permittivity and negative permeability.[1] But these materials did not receive a great deal of attention until after Pendry *et al* had discovered a microwave-plasma thin-wire structure exhibiting negative permittivity below the electronic plasma frequency and a magnet-free split-ring resonator structure exhibiting negative permeability below the magnetic plasma frequency.[2,3] Because the materials have both negative permittivity and permeability, this feature gives rise to a negative refractive index, and leads to backward-wave propagation. In other words, the wave vector $\bar{\boldsymbol{k}}$ forms a left-handed triplet with the vectors $\bar{\boldsymbol{E}}$ and $\bar{\boldsymbol{H}}$, and wave fronts for propagating waves travel towards the source, opposite to the direction of energy flow. Although these materials are not found in nature, many interesting properties have been predicated theoretically, such as the reversal of Snell's law, Doppler effect, and Cherenkov radiation. In recent years, many artificial materials with positive or negative permittivity or permeability have been explored,[4] and many other novel phenomena have been discovered theoretically, such as perfect lens and energy localizations.[1,5−8] More recently, one of the interesting properties is the super energy flow in a special waveguide filled with two equally thick layers, which are air and LHM.[9,10] In this structure, extremely high power density flows are generated and transmitted along the waveguide. In the above discussion, there is only one polarization mode (TE or TM mode) inside the waveguide. If two modes exist simultaneously, how the waves with two different kinds of polarizations are transmitted and whether super energy flows can be generated or not become very important. In this paper, a horizontal electrical dipole (HED) is placed in the waveguide, and it will produce two kinds of polarizations: TM mode and TE mode. We first theoretically analyse the distributions of fields and energy flows in a planar waveguide, excited by the HED source. Then we give numerical results to validate our conclusions.

2. Theoretical analysis

The proposed system is schematically shown in Fig.1, which includes a planar parallel-plate waveg-

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uide with metallic walls and two equally thick different medium layers. The first layer ($d_0 < z < d_1$) is air and the second layer ($d_1 < z < d_2$) is LHM. The relative electric permittivity and magnetic permeability of such layers are denoted as ϵ_{ri} and μ_{ri} ($i = 0, 1$), respectively. We assume that $\epsilon_{r0} = 1.0$, $\mu_{r0} = 1.0$, $\epsilon_{r1} = -1/(1+\delta)+\mathrm{i}\gamma$, and $\mu_{r1} = -(1+\delta)+\mathrm{i}\gamma$, where δ and γ are the small parameters representing the retardation and loss of LHM, respectively. We also require that the two layers have the same thickness.

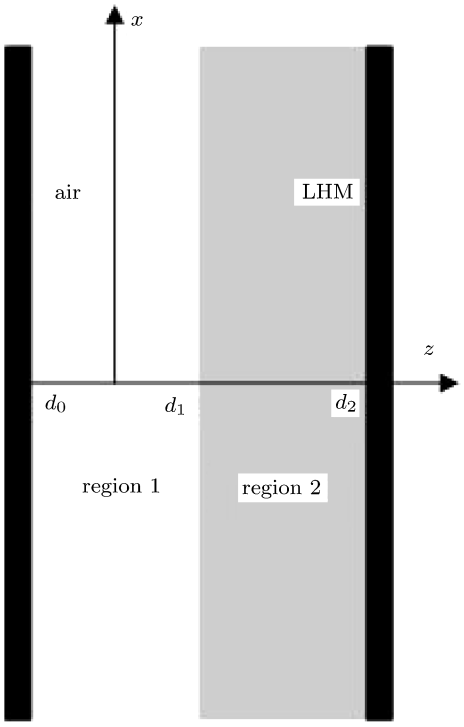


Fig.1. A planar parallel-plate waveguide filled with two equally thick medium layers. The first layer ($d_0 < z < d_1$) is air and the second layer ($d_1 < z < d_2$) is LHM. A 3D HED along the x-direction is located at $z = 0$.

If a three-dimensional(3D) HED along the $+x$ direction is placed at $z = 0$ in the air (see Fig.1), it will produce TM- and TE-mode electromagnetic fields.[11] The z-components of electromagnetic fields in such two regions can be written in the following forms:

$$\bar{E}_{iz} = e_0 \frac{x}{\rho} \int_0^\infty k_\rho^2 J_1(k_\rho \rho) \tilde{E_{iz}} \mathrm{d}k_\rho, \tag{1}$$

$$\bar{H}_{iz} = e_0 \frac{y}{\rho} \int_0^\infty \frac{k_\rho^2}{k_{0z}} J_1(k_\rho \rho) \tilde{H_{iz}} \mathrm{d}k_\rho, \tag{2}$$

in which J_1 is the first order Bessel function, and $\tilde{E_{iz}}$ and $\tilde{H_{iz}}$ are rewritten as

$$\tilde{E_{1z}} = \mathrm{e}^{\mathrm{i}k_{0z}|z|} + R^{\mathrm{TM}_+} \mathrm{e}^{\mathrm{i}k_{0z}z} + R^{\mathrm{TM}_-} \mathrm{e}^{-\mathrm{i}k_{0z}z}, \tag{3}$$

$$\tilde{E_{2z}} = C^{\mathrm{TM}_+} \mathrm{e}^{\mathrm{i}k_{1z}z} + C^{\mathrm{TM}_-} \mathrm{e}^{-\mathrm{i}k_{1z}z}, \tag{4}$$

$$\tilde{H_{1z}} = \mathrm{e}^{\mathrm{i}k_{0z}|z|} + R^{\mathrm{TE}_+} \mathrm{e}^{\mathrm{i}k_{0z}z} + R^{\mathrm{TE}_-} \mathrm{e}^{-\mathrm{i}k_{0z}z}, \tag{5}$$

$$\tilde{H_{2z}} = C^{\mathrm{TE}_+} \mathrm{e}^{\mathrm{i}k_{1z}z} + C^{\mathrm{TE}_-} \mathrm{e}^{-\mathrm{i}k_{1z}z}, \tag{6}$$

where k_ρ is the longitudinal wave vector; k_{0z} and k_{1z} are z-components of the wave vector in layers 1 and 2 with $k_{iz} = \sqrt{\omega^2 \epsilon_i \mu_i - k_\rho^2}$; the coefficients $R^{\mathrm{TM}_\pm}$, $R^{\mathrm{TE}_\pm}$, $C^{\mathrm{TM}_\pm}$, and $C^{\mathrm{TE}_\pm}$ of the forward and backward propagating components are determined by boundaries of stratified media:

$$\begin{aligned}
R^{\mathrm{TM}_-} &= -1 - R^{\mathrm{TM}_+} \mathrm{e}^{2\mathrm{i}k_{0z}d_0}, \\
C^{\mathrm{TM}_-} &= C^{\mathrm{TM}_+} \mathrm{e}^{2\mathrm{i}k_{1z}d_2}, \\
R^{\mathrm{TM}_+} &= (p_2 d^+ a^+ + p_1 d^- a^-)/d_{01}, \\
C^{\mathrm{TM}_+} &= (a^+ b^+ - a^- b^-)/d_{01}, \\
R^{\mathrm{TE}_-} &= -1 - R^{\mathrm{TE}_+} \mathrm{e}^{2\mathrm{i}k_{0z}d_0}, \\
C^{\mathrm{TE}_-} &= -C^{\mathrm{TE}_+} \mathrm{e}^{2\mathrm{i}k_{1z}d_2}, \\
R^{\mathrm{TE}_+} &= (p_3 d^- a^+ + p_1 d^+ a^-)/d_{02}, \\
C^{\mathrm{TE}_+} &= (a^+ b^- - a^- b^+)/d_{02},
\end{aligned}$$

with

$$\begin{aligned}
a^\pm &= \mathrm{e}^{\mathrm{i}k_{0z}d_1} \pm \mathrm{e}^{-\mathrm{i}k_{0z}d_1}, \\
b^\pm &= \mathrm{e}^{\mathrm{i}k_{0z}d_1} \pm \mathrm{e}^{-\mathrm{i}k_{0z}(d_1-2d_0)},
\end{aligned}$$

and

$$d^\pm = \mathrm{e}^{\mathrm{i}k_{1z}d_1} \pm \mathrm{e}^{-\mathrm{i}k_{1z}(d_1-2d_2)}.$$

In the above expressions, $p_1 = k_{1z}/k_{0z}$, $p_2 = \epsilon_1/\epsilon_0$, $p_3 = \mu_1/\mu_0$, and $d_{01,02}$ are denominators of $\tilde{E}_{iz}$ and $\tilde{H}_{iz}$, which are written as

$$d_{01} = -(p_2 d^+ b^- + p_1 d^- b^+), \tag{7}$$

$$d_{02} = -(p_3 d^+ b^- + p_1 d^- b^+). \tag{8}$$

For a conventional air-filled waveguide ($\epsilon_{r1} = \mu_{r1} = 1.0$), the denominators d_{01} and d_{02} in coefficients have the same forms:

$$d_{01} = d_{02} = \mathrm{e}^{\mathrm{i}2k_{0z}d_0}(1 - \mathrm{e}^{\mathrm{i}2k_{0z}h}), \tag{9}$$

where $h = d_2 - d_0$ is the thickness of the waveguide. From expression (9), we clearly know that TM and TE modes have the same poles located at $k_{0z} = n\pi/h \leq \omega\sqrt{\epsilon_0 \mu_0}$ ($n = 1, 2, \cdots$), which correspond to the propagation constants of guided modes. In our system, we remark that $k_{0z} = 0$ is not a pole of the waveguide according to expressions (1), (2), and (9) for both TM and TE modes. This is similar to the case of two-dimensional (2D) line source,[9] but different from the case of 3D vertical electric dipole(VED).[10] Therefore $n = 1$ is the dominant mode, and the values of $n > 1$ correspond to high-order modes. According to k_{0z}, we can easily know that no propagating waves can be

excited and guided in the waveguide if $2h$ is less than the wavelength.

In a lossless air-LHM-filled waveguide, we have $\epsilon_{r1} = -1/(1+\delta)$ and $\mu_{r1} = -(1+\delta)$. For propagating waves, $k_{0z} = -k_{1z}$. Then the denominators $d_{01,02}$ in the coefficients can be simplified into

$$d_{01} = \delta e^{i2k_{0z}d_0}(1 - e^{i2k_{0z}h}), \tag{10}$$

$$d_{02} = \frac{\delta}{1+\delta} e^{i2k_{0z}d_0}(1 - e^{i2k_{0z}h}). \tag{11}$$

It is obvious that there are the same poles for two modes in the super waveguide, and the pole positions are the same as those in the air-filled waveguide. Hence the condition of propagating modes in the super waveguide is the same as that in the conventional air-filled waveguide. However, the amplitudes of fields are quite different since $\bar{E}_{iz}$ and $\bar{H}_{iz}$ are proportional to δ^{-1}. Based on the Maxwell's equations, other transverse components of electromagnetic fields are also proportional to δ^{-1}. Because δ is very small, the fields can be greatly amplified.

In the proposed two-layer system, TE and TM modes produce not only the ρ-directional energy flow, but also the φ-directional energy flow. We can calculate the two directional time-averaged Poynting vectors: $\bar{S}_\rho = \frac{1}{2}\mathrm{Re}(\bar{E}_\varphi \times \bar{H}_z^* - \bar{E}_z \times \bar{H}_\varphi^*)$, and $\bar{S}_\varphi = \frac{1}{2}\mathrm{Re}(\bar{E}_z \times \bar{H}_\rho^* - \bar{E}_\rho \times \bar{H}_z^*)$. For simplicity, if we analyse the energy flow in the yoz plane with $x = 0$ and xoz plane with $y = 0$, the formulas of time-averaged Poynting vectors can be simplified into

$$\bar{S}_y = \frac{1}{2}\mathrm{Re}(\bar{E}_z \times \bar{H}_x^*), \tag{12}$$

$$\bar{S}_x = \frac{1}{2}\mathrm{Re}(\bar{E}_y \times \bar{H}_z^*), \tag{13}$$

which are corresponding to TM and TE modes, respectively. Due to all the components of fields are proportional to δ^{-1}, the time-averaged Poynting vectors are proportional to δ^{-2}. Hence the energy flow will be greatly amplified along the two directions, which is similar to the cases of 2D line source[9] and 3D VED.[10]

3. Numerical results

In order to clearly show the super-high energy flows in the two layers, we consider simulation results from the super waveguide and compare them with those from the conventional air-filled waveguide, because the conventional waveguide with the same dimensions as those of the super waveguide has the same guided condition as the super waveguide. According to earlier analysis, if the thickness of waveguide is larger than twice the wavelength, both dominant mode and higher modes will be excited and guided. If we choose the size of waveguide as $d_0 = -0.05$ m, $d_1 = 0.05$ m, and $d_2 = 0.15$ m, only the dominant mode is excited and guided. In the following simulation results, the 3D HED is located at $z = 0$ in region 1 (see Fig.1), $Il = 1$ mA·m, and the frequency is $f = 1$ GHz. For the TE mode, the computational region is along the xoz plane with y from 3m ($10\lambda_0$) to 3.6m ($12\lambda_0$) and $x = 0$. For the TM mode, the computational region is along yoz plane with x from 3m ($10\lambda_0$) to 3.6m ($12\lambda_0$) and $y = 0$.

The simulation results of time-averaged power densities propagating in the air-filled waveguide along the xoz and yoz planes are shown in Fig.2. We clearly see that the maximum values of the power densities for TE and TM modes are about 2.5mW/m^2 at $x = 0$ (see Fig.2(a)) and 1.4mW/m^2 at $y = 0$ (see Fig.2(b)), respectively.

If region 2 is filled with a lossless LHM with $\delta = -10^{-4}$, under the same conditions, the time-averaged power density distributions in the super waveguide are illustrated in Fig.3. Although the propagating modes are similar to those in the conventional waveguide, their averaged power densities are quite different in value. In the x-direction, the averaged power density reaches 1×10^9mW/m^2 at y=3m, and decays slowly along the y-direction. In the x-direction, the averaged power density reaches 5×10^8mW/m^2.

The simulation results are consistent with the earlier theoretical analyses. That is to say, the extremely high power generation and transmission can be realized in the super waveguide for both TM and TE modes excited by the 3D HED. However, we notice that there are different transmission patterns along the two planes in the waveguide from those in the cases of 2D line source[9] and 3D VED.[10] We also remark that the super high power flows do not violate the energy conservation[9,10] because the power flows in regions 1 and 2 have similar amplitudes but opposite directions.

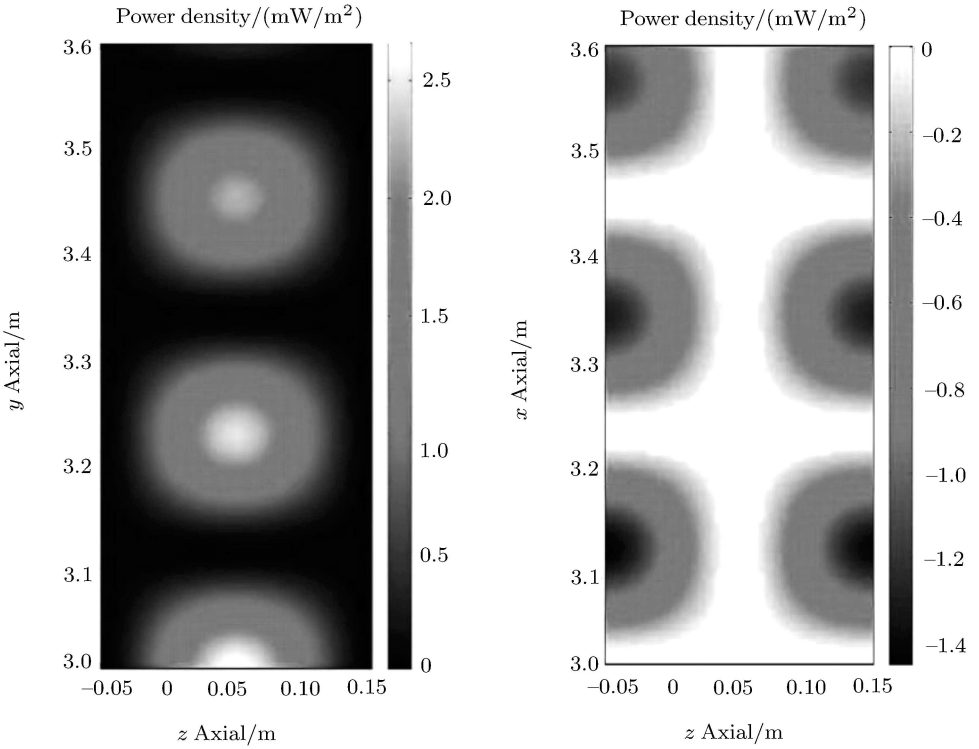


Fig.2. Distribution of power density in the conventional air-filled waveguide. Subfigure (a) (left) is for the yoz section with y from 3 to 3.6m and $x = 0$. Subfigure (b) (right) is for the xoz section with x from 3 to 3.6m and $y = 0$.

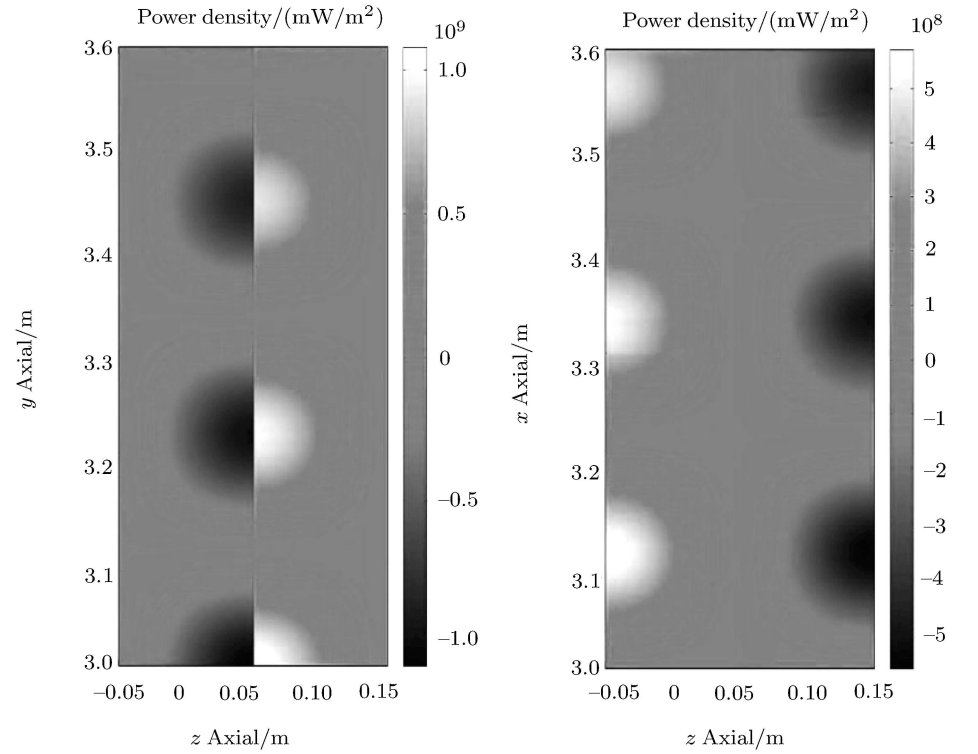


Fig.3. Distribution of power density in the super waveguide with $\delta = -10^{-4}$ and $\gamma = 0$. Subfigure (a) (left) is for the yoz section with y from 3 to 3.6m and $x = 0$. Subfigure (b) (right) is for the xoz section with x from 3 to 3.6m and $y = 0$.

In reality, it is difficult to make ideal lossless LHM and all artificial LHM are always lossy. Now we consider a slightly lossy LHM slab with $\epsilon_{r1} = -1/(1+\delta)+i\gamma$ and $\mu_{r1} = -(1+\delta)+i\gamma$ where $\delta = -10^{-4}$ and $\gamma = 10^{-6}$, and other conditions remain the same. Figure 4 illustrates the energy flows propagating along the y-direction and x-direction. Although there is no obvious transmitting pattern because of complicated propagation constants, the high power flows still exist in the waveguide along two directions. The averaged power densities are also approximately proportional to $1/\delta^2$, and only the amplitudes decrease due to the loss. For the TE mode, the maximum energy flow reaches $1.5 \times 10^8 \mathrm{mW/m^2}$ and distributes mainly along the interface between two layers (see Fig.4(a)). For the TM mode, the maximum energy flow reaches $8 \times 10^7 \mathrm{mW/m^2}$ and distributes mainly along sides of waveguide (see Fig.4(b)). They are both consistent with the lossless case shown in Fig.3.

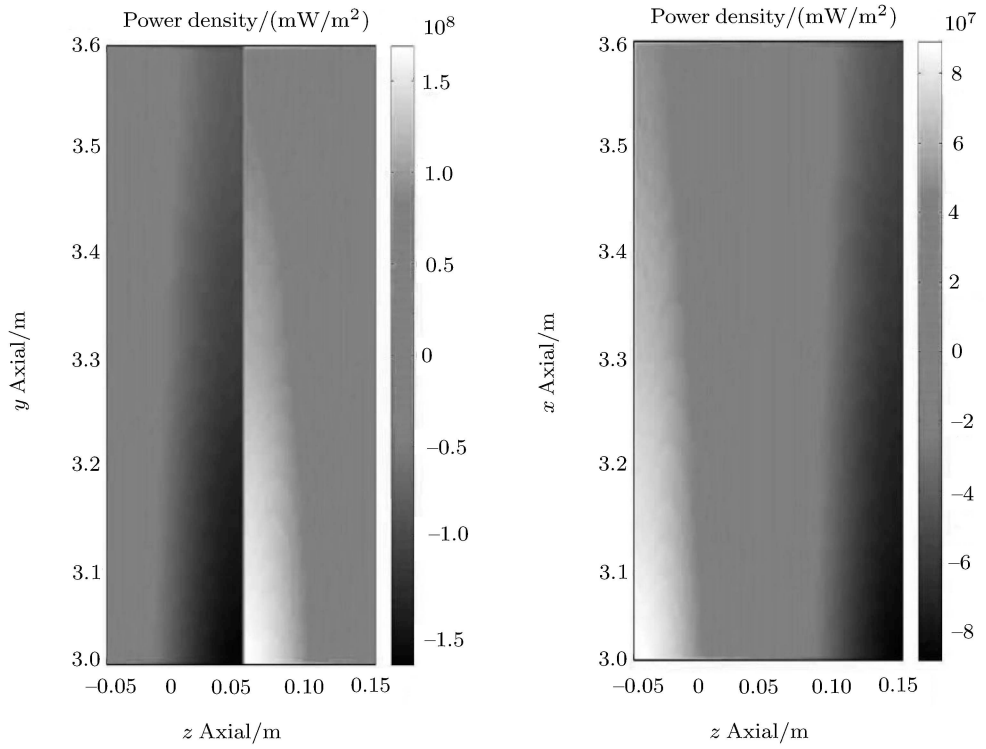


Fig.4. The distribution of power density in the super waveguide with $\delta = -10^{-4}$ and $\gamma = 10^{-6}$. Subfigure (a) (left) is for the yoz section with y from 3 to 3.6m and $x = 0$. Subfigure (b) (right) is for the xoz section with x from 3 to 3.6m and $y = 0$.

Finally, we consider the case with relatively large retardation and loss: $\delta = 10^{-2}$ and $\gamma = 10^{-4}$, and other conditions keep unchanged. In such a case, the distributions of averaged power density are shown in Fig.5. For the TE mode, the maximum energy flow reaches $1.5 \times 10^4 \mathrm{mW/m^2}$, and for the TM mode, the maximum energy flow reaches $8 \times 10^3 \mathrm{mW/m^2}$. Therefore the high energy flows are still produced.

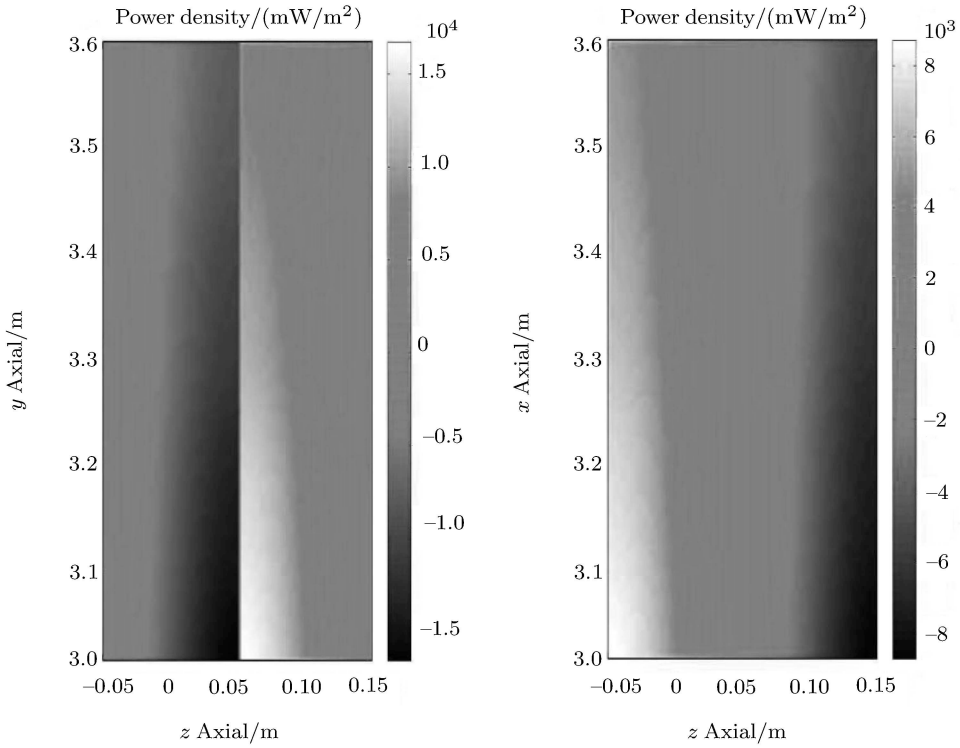


Fig.5. The distribution of power density in the super waveguide with $\delta = -10^{-2}$ and $\gamma = 10^{-4}$. Subfigure (a) (left) is for the yoz section with y from 3 to 3.6m and $x = 0$. Subfigure (b) (right) is for the xoz section with x from 3 to 3.6m and $y = 0$.

4. Conclusions

In this paper, we discuss the super high energy transmitting in a planar waveguide excited by a 3D HED source, where both TE and TM modes exist. If electromagnetic parameters are appropriately chosen in the air-LHM planar waveguide, super high power density flows can be produced along two directions in the waveguide. Theoretical and numerical analyses prove that there occur propagating modes different from those for the cases of 2D line source and 3D VED. The retardation and loss in LHM can affect the amplitude of average power density.

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