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Controlling chaos in RCL-shunted Josephson junction by delayed linear feedback

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The resistively–capacitively–inductively-shunted (RCL-shunted) Josephson junction (RCLSJJ) shows chaotic behaviour under some parameter conditions. Here a scheme for controlling chaos in the RCLSJJ is presented based on the linear feedback theory. Numerical simulations show that this scheme can be effectively used to control chaotic states in this junction into stable periodic states. Moreover, the different stable period states with different period numbers can be obtained by appropriately adjusting the feedback intensity and delay time without any pre-knowledge of this system required.

Keywords: chaos control, RCL-shunted Josephson junction, maximum Lyapunov exponent, delayed linear feedback

PACC: 0545, 7450

1. Introduction

With the rapid development in fabrication technology and high-temperature superconducting materials, the investigations on Josephson junctions have attracted much attention due to its potential applications, such as voltage standards and microwave devices.[1−3] Moreover, microwave devices have been used in spaceflight and communication systems. While high-critical current Josephson junctions are preferred in many applications such as high-frequency Josephson oscillators,[4] they would lead to various inductive effects which were traditionally negligible due to the limited current flow in the circuits. In order to further investigate Josephson junctions with high critical currents, the RCL-Shunted Josephson junction (RCLSJJ) model was proposed by Whan *et al*[5] based on previous investigations of the resistively–capacitively shunted junction (RCSJ) model.[6−10] Moreover, RCLSJJ model has avoided the shortage of RCSJ model, and shown the agreement between the experimental and simulated I–V curves when the shunt of the junction contains an inductive component.[5,11] Following this, there have been several reports on dynamics of RCLSJJ.[12−14] Whan and Lobb[12] studied chaotic behaviour in this system through the maximum Lyapunov exponents (MLEs) and bifurcation diagrams where the shunt inductance was taken as the control parameter, and also analysed the effects of thermal noise; Cawthorne *et al* investigated complex dynamics of RCLSJJ[13] and reported synchronized oscillations in RCLCJJ array.[14] Recently, chaotic control and synchronization in RCLSJJs have been an interesting area of research[15] because of extensive applications of this kind of junctions,[4,16,17] such as parametric amplifiers and mixers. Since the chaotic control method was first proposed by Ott *et al*[18] in 1990, chaos control has been a main subject in this field. Thus some other methods, weak periodic perturbations,[19,20] delayed self-controlling feedback,[21,22] etc., were subsequently introduced. Lately many experts have successfully completed chaos control in various nonlinear systems.[23−27] Here we propose a scheme for chaos control in the RCLSJJ via a resistor and a delay device, and the validity of this scheme is numerically demonstrated.

This paper is organized as follows: Section 2 studies chaotic behaviour in RCLSJJ, including a description of the RCLSJJ model, dynamical equations, and chaotic attractor. Section 3 presents a scheme and numerical simulations. Finally, Section 4 gives our conclusion.

2. Chaos in RCL-shunted Josephson junction

The RCLSJJ used here is a current biased RCSJ with a series resistance–inductance shunt (This

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RCLSJJ model is shown in Fig.2). If all noise terms are neglected, its complete circuit equations are:[12]

$$\begin{aligned} &C\frac{\mathrm{d}V}{\mathrm{d}t}+\frac{V}{R_V}+I_\mathrm{c}\sin(\gamma)+I_\mathrm{s}=I,\\ &\frac{\hbar}{2e}\frac{\mathrm{d}\gamma}{\mathrm{d}t}=V,\\ &L\frac{\mathrm{d}I_\mathrm{s}}{\mathrm{d}t}+I_\mathrm{s}R_\mathrm{s}=V, \end{aligned} \tag{1}$$

where C, I_c, and γ are the junction capacitance, the junction critical current, and the phase difference of the superconducting order parameters across this junction, respectively. V is the junction voltage and R_V is the nonlinear resistance. I is the external bias dc current. R_s and L are, respectively, the shunt resistance and the shunt inductance. The current in the shunt branch is denoted as I_s.

To make it convenient for numerical analysis, Eq.(1) is expressed in dimensionless form as follows:[12]

$$\begin{aligned} &\frac{\mathrm{d}\gamma}{\mathrm{d}\tau}=\upsilon,\\ &\frac{\mathrm{d}\upsilon}{\mathrm{d}\tau}=\frac{1}{\beta_\mathrm{c}}(i-g\upsilon-\sin(\gamma)-i_\mathrm{s}),\\ &\frac{\mathrm{d}i_\mathrm{s}}{\mathrm{d}\tau}=\frac{1}{\beta_L}(\upsilon-i_\mathrm{s}), \end{aligned} \tag{2}$$

where

$$\begin{aligned} &\upsilon=V/(I_\mathrm{c}R_\mathrm{s}),\\ &i_\mathrm{s}=I_\mathrm{s}/I_\mathrm{c}, \end{aligned} \tag{3}$$

and the normalized time $\tau=\omega_\mathrm{c}t$, with $\omega_\mathrm{c}=(2eI_\mathrm{c}R_\mathrm{s})/\hbar$. Here all currents are normalized to I_c, and all voltages to $I_\mathrm{c}R_\mathrm{s}$. Equation (2) contains four dimensionless parameters, which are the following:[12]

$$\beta_C=(2eI_\mathrm{c}R_\mathrm{s}^2C)/\hbar,$$

$$\beta_L=(2eI_\mathrm{c}L)/\hbar,$$

$$i=I/I_\mathrm{c},$$

$$g=\begin{cases} R_\mathrm{s}/R_\mathrm{N} & |\upsilon|>V_\mathrm{g}/(I_\mathrm{c}R_\mathrm{s}),\\ R_\mathrm{s}/R_\mathrm{sg} & |\upsilon|\le V_\mathrm{g}/(I_\mathrm{c}R_\mathrm{s}), \end{cases}$$

Where β_C and β_L are reduced capacitance and reduced inductance, respectively; $\hbar$ is Planck constant. R_N and R_sg are the junction normal state resistance and the subgap resistance, respectively. The gap voltage is: $V_\mathrm{g}=2\Delta/e$, 2Δ is the width of the energy gap, and e is the electron charge.

To obtain the chaotic attractor of RCLSJJ, we solve Eq.(2) numerically. In this work, the differential equations are solved using a fourth-order Runge-Kutta method with a fixed step $\Delta\tau=0.01$ and total steps $N_\mathrm{step}=10^5$, the Lyapunov exponents are computed using the algorithms presented by Wolf *et al*[28] and the voltage bifurcation diagrams are plotted by recording the local maxima in the voltage time series.[29] The parameter values we used here are obtained from Refs.[5] and [12] at a temperature $T=7.60\,\mathrm{K}$ as follows: $\beta_C=0.707$, $\beta_L=2.68$, $g=0.0478$, and $i=1.2$. Then, a strange attractor and the time series of voltage υ are shown in Figs.1(a) and 1(b), respectively. Meanwhile, its Lyapunov exponent spectrum is also calculated, the result of which is: $\lambda_1=8.06307\times10^{-2}$, $\lambda_2=-2.98260\times10^{-3}$, $\lambda_3=-7.13507\times10^{-1}$, which correspond to the directions of γ, υ and i_s, respectively. Thus this system is in chaotic state because only one Lyapunov exponent is positive.

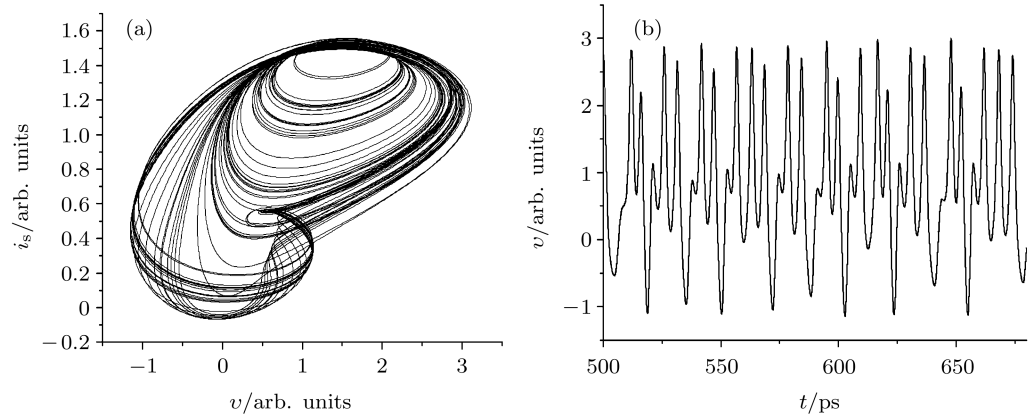


Fig.1. (a) The strange attractor on plane $\upsilon-i_\mathrm{s}$; (b) The time series of υ. The parameters are: $\beta_C=0.707$, $\beta_L=2.68$, $g=0.0478$, and $i=1.2$.

3. Scheme and numerical simulations

3.1. Scheme for chaos control

Here we propose a scheme for chaos control in the RCLSJJ based on the linear feedback theory,[30] where chaos control is realized by feeding back a proper intensity output signal of a chaotic system to this system and disturbing it, the perturbation term is $F_{(t)} = \alpha y_{(t)}$, where α is the feedback intensity, and $y_{(t)}$ expresses the output variables. For our scheme, its schematic diagram is shown in Fig.2, where a delay device D[31] and a resistor $R_{\rm m}$ form control circuit, a part of the delayed junction output voltage is taken

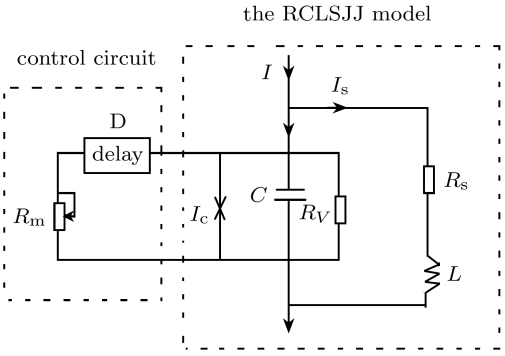


Fig.2. The scheme for chaos control.

as the feedback signal. So a perturbation term $\left[F_{(t)} = \alpha y_{(t-\tau)} = \dfrac{V_{(t-t_{\rm d})}}{R_{\rm m}}\right]$ is added in the dynamic equation, τ and $t_{\rm d}$ are delay times. Hence, the first one of Eq.(1) can be expressed as follows:

$$C\frac{{\rm d}V}{{\rm d}t} + \frac{V}{R_V} + I_{\rm c}\sin(\gamma) + I_{\rm s} = I - \frac{V_{(t-t_{\rm d})}}{R_{\rm m}}. \tag{4}$$

So, Eq.(2) can be changed as follows:

$$\begin{aligned}
&\frac{{\rm d}\gamma}{{\rm d}\tau} = \upsilon,\\
&\frac{{\rm d}\upsilon}{{\rm d}\tau} = \frac{1}{\beta_C}(i - g\upsilon - \sin(\gamma) - i_{\rm s} - m\upsilon_{(\tau-\tau_{\rm d})}),\\
&\frac{{\rm d}i_{\rm s}}{{\rm d}\tau} = \frac{1}{\beta_L}(\upsilon - i_{\rm s}),
\end{aligned} \tag{5}$$

where

$$m = \frac{R_{\rm s}}{R_{\rm m}}, \tag{6}$$

$$\tau_{\rm d} = \omega_{\rm c} t_{\rm d}. \tag{7}$$

Here, m is the normalized feedback strength and $\tau_{\rm d}$ is the normalized delay time; m and $\tau_{\rm d}$ can be taken as the control parameters and can be adjusted by changing $R_{\rm m}$ and $t_{\rm d}$, respectively, thus chaos control can be achieved.

3.2. Numerical simulations and analysis

To illustrate the process of chaos control in this scheme, firstly the RCLSJJ system is let to remain in the chaotic state, as shown in Fig.1. Secondly numerical simulations are carried out by solving Eq.(5) in two cases: case 1 is to regard parameter m as the control parameter; case 2 is to use parameter $\tau_{\rm d}$ as the control parameter.

In case 1, we also discuss two different conditions cases. One is taking delay time $\tau_{\rm d} = 0$, which is a special example of the delayed linear feedback; the other is taking $\tau_{\rm d} \neq 0$. First, we solve Eq.(5) with changing the feedback strength m at $\tau_{\rm d} = 0$ and other parameter values are the same as in Fig.1. The MLE and corresponding bifurcation diagram are presented in Fig.3, where one can find six ranges of m values, for which the MLE is zero or negative, and periodic windows occur. The six ranges are: $0.008 < m < 0.013$, $0.026 < m < 0.033$, $0.053 < m < 0.058$, $m > 0.063$, and near 0.042, 0.048, respectively. One of these ranges is relatively broad, i.e., $m > 0.063$. In order to further clarify that the controlled system can be in the different period states with different m values, according to Figs.3(a) and 3(b), $m = 0.105$ and $m = 0.030$ are chosen. The corresponding attractors and the time series of υ are presented in Fig.4, where (a), (b), and (c), (d) show that this controlled system are in period-2 (P2) state and period-5 (P5) state, respectively. It is remarkable that the RCLSJJ system was in a chaotic state before it is controlled, which is shown in Fig.1. Hence, chaotic states can be controlled into stable different period states, such as P2 and P5 states, by selecting appropriate m values in the six ranges above and keeping $\tau_{\rm d} = 0$, $\beta_C = 0.707$, $\beta_L = 2.68$, $g = 0.0478$, and $i = 1.2$.

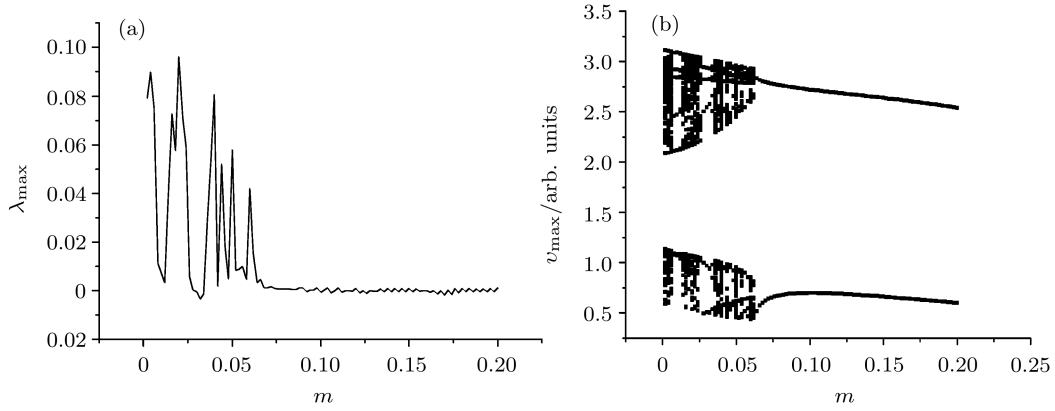


Fig.3. (a) The maximum Lyapunov exponent $\lambda_{max} - m$; (b) The bifurcation diagram of the controlled system via m. The parameter $\tau_d = 0$ and other parameter values are the same as in Fig.1.

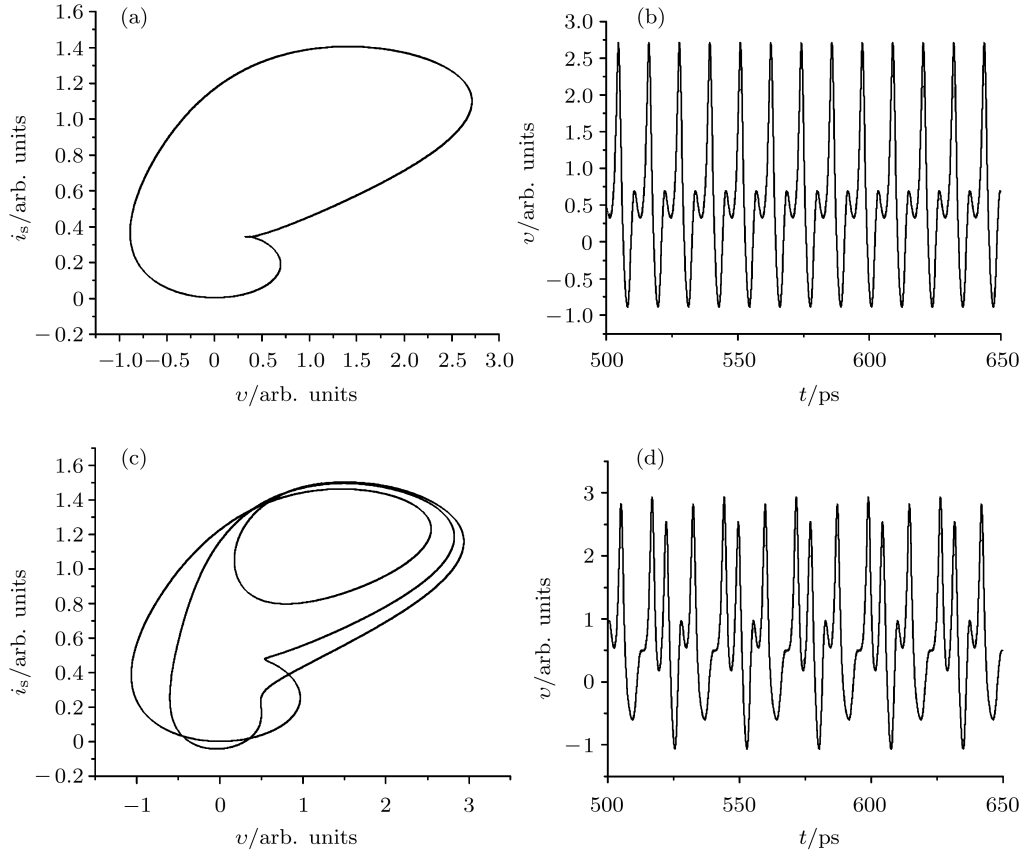


Fig.4. The attractors on the phase plane v–i_s and the time series of v at different feedback intensities m and other parametric values are the same as in Fig.3. (a), (b) The P2 attractor and the time series of v, $m = 0.105$; (c), (d) The P5 attractor and the time series of v, $m = 0.030$.

Second, we solve Eq.(5) with varying the feedback strength m at fixed $\tau_d = 200$ step and other parametric values are the same as in Fig.1. Then bifurcation diagram is presented in Fig.5, where one can see four ranges of m values, for which periodic windows occur, namely, the controlled system can be in stable periodic states when m is chosen in the four ranges, which are $0.052 < m < 0.303$ and near 0.015, 0.335, 0.345, respectively. Similarly, to further indicate that different period states with different period numbers

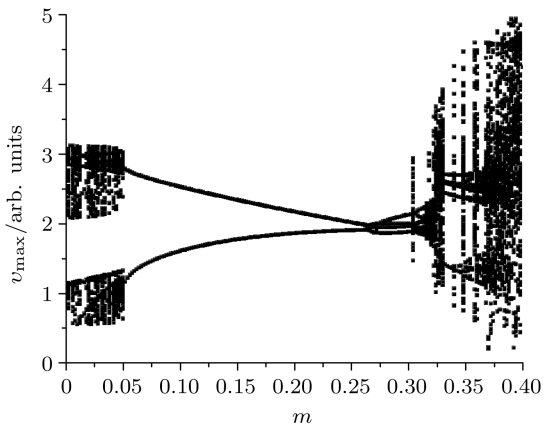


Fig.5. The bifurcation diagram of the controlled system via m. The parameter $\tau_d = 200$ step and other parametric values are the same as in Fig.1.

can exist in the controlled system, according to Fig.5, $m = 0.10$ and $m = 0.28$ are selected. Then, attractors and the time series of v are presented in Fig.6, where (a), (b), and (c), (d) respectively show this controlled system are in P2 state and P4 state. Therefore, chaotic states in the RCLSJJ system can be controlled into different stable period states, such as P2 and P4 states, by choosing proper m values in the four ranges above and holding $\tau_d = 200$ step, other parameter values are the same as in Fig.1.

In case 2, we change the control parameter τ_d within the range 20 step $\leq \tau_d \leq$ 2000 step and hold $m = 0.11$, other parametric values being the same as in Fig.1. Numerical results of Eq.(5) are shown in Fig.7, where one can also observe four regions of τ_d values, for which periodic windows appear. This means that the dynamical behaviour of the controlled system is periodic motion when τ_d is selected in the four regions, which are 20 step $\leq \tau_d <$ 610 step, 640 step $< \tau_d <$ 730 step, 1050 step $< \tau_d <$ 1420 step, and 1630 step $< \tau_d <$ 1670 step, respectively. Similarly, the chaotic motion in the RCLSJJ can also be controlled into different stable periodic motions by appropriately choosing τ_d in the above four regions. According to Fig.7, $\tau_d = 250$ step and $\tau_d = 1250$ step are chosen, so p2 and p4 states are obtained, respectively. The corresponding periodic attractor projections and the time series of v are similar to Fig.6.

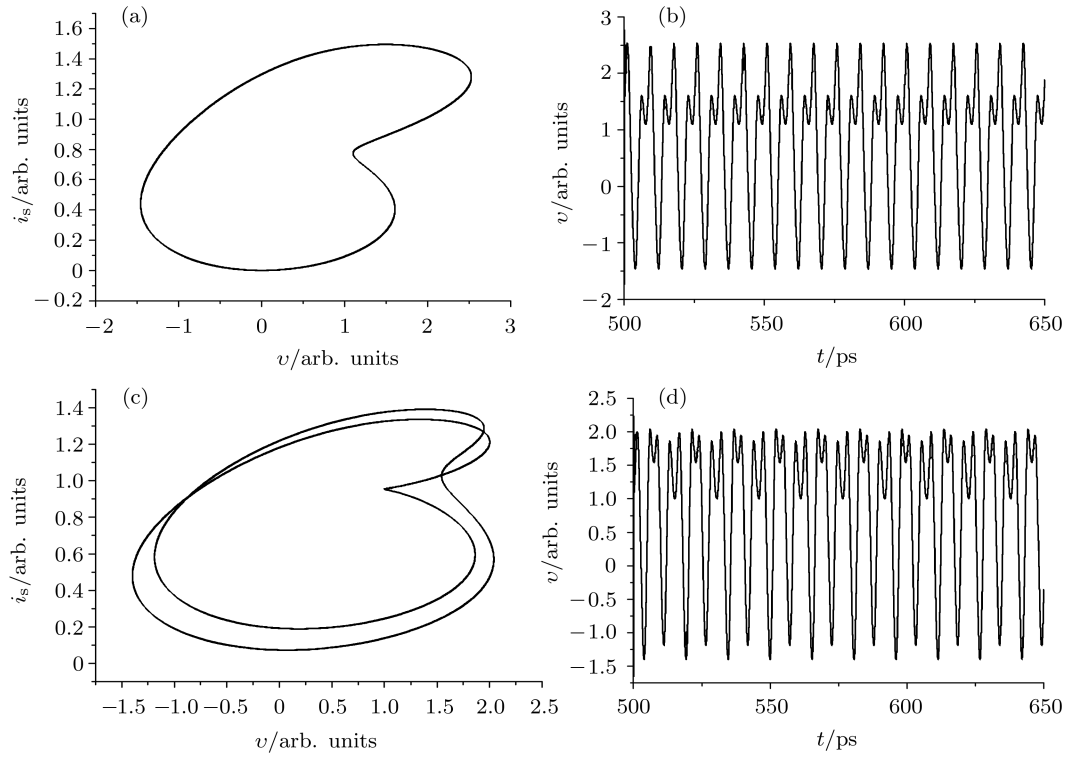


Fig.6. The attractors on the phase plane $v - i_s$ and the time series of v at different feedback intensities m with other parametric values being the same as in Fig.5. (a), (b) The P2 attractor and the time series of v, $m = 0.10$; (c), (d) The P4 attractor and the time series of v, $m = 0.28$.

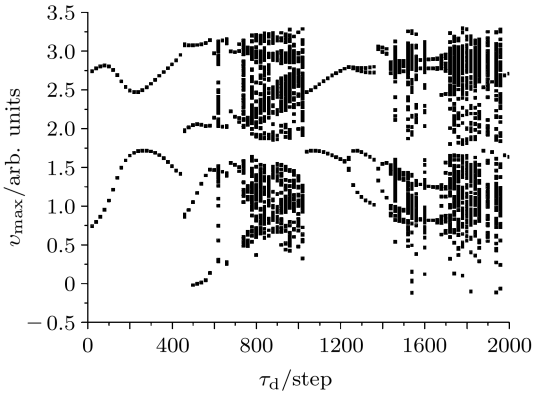


Fig.7. The bifurcation diagram of the controlled system via τ_d. The parameter $m = 0.11$ and other parameter values are the same as in Fig.1.

4. Conclusion

The RCL-shunted Josephson junction shows chaotic behaviours at the parameter values we have chosen. A scheme for chaos control in this system have been proposed, and numerical results have demonstrated that this scheme can be effectively used to control the chaotic states in this system into stable periodic states. Moreover, the different periodic states, such as p2, p4, and p5 states, can be obtained by properly adjusting the feedback intensity m and the delay time τ_d, which can be defined theoretically according to Eqs.(6) and (7), respectively. This scheme is easier to implement by appropriately choosing m value in the experiment.

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