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Hybrid control of bifurcation and chaos in stroboscopic model of Internet congestion control system*

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Interaction between transmission control protocol (TCP) and random early detection (RED) gateway in the Internet congestion control system has been modelled as a discrete-time dynamic system which exhibits complex bifurcating and chaotic behaviours. In this paper, a hybrid control strategy using both state feedback and parameter perturbation is employed to control the bifurcation and stabilize the chaotic orbits embedded in this discrete-time dynamic system of TCP/RED. Theoretical analysis and numerical simulations show that the bifurcation is delayed and the chaotic orbits are stabilized to a fixed point, which reliably achieves a stable average queue size in an extended range of parameters and even completely eliminates the chaotic behaviour in a particular range of parameters. Therefore it is possible to decrease the sensitivity of RED to parameters. By using the hybrid strategy, we may improve the stability and performance of TCP/RED congestion control system significantly.

Keywords: chaos control, dynamical systems, congestion control, bifurcation
PACC: 0545, 0547

1. Introduction

As the Internet continues to expand in size, diversity and reach, congestion control mechanism in Internet represents one of the largest deployed artificial feedback system. Congestion control, an extremely important and challenging problem in the Internet, has been the main subject of intensive study over the last decade. The whole Internet congestion and avoidance mechanism is a combination of end-to-end transmission control protocol (TCP) congestion control mechanism[1] at the end hosts and queue management mechanism at the routers. As the most popular active queue management (AQM) mechanism, random early detection (RED)[2] algorithm attempts to control the congestion level at a bottleneck link by monitoring and updating average queue size. It is reported that the interaction between TCP and RED often leads to undesirable behaviours, such as bifurcation and chaos, which may degrade network performance.[3,4]

Recently, Ranjan *et al*[5,6] demonstrated that a TCP/RED system could be approximately modelled as a first-order nonlinear map which exhibits a rich variety of irregular behaviours such as bifurcation and chaos. Then, Chen *et al*[7,8] investigated the possibility of controlling bifurcation and chaos in the TCP/RED system by several time delayed feedback control strategies. In Refs.[9,10], another discrete-time model named stroboscopic model for the TCP/RED system was derived and used it to investigate the bifurcation and chaos in the system. Luo *et al*[11] designed a hybrid control strategy to control the bifurcation and chaos in a discrete nonlinear dynamical system.

In this paper, first we have briefly introduced the stroboscopic model of the TCP/RED system. Then, through theoretical analysis, we have found that a fixed point of the system is positive real solution of a third-order polynomial. Meanwhile, a stable range of a system parameter is analysed and the existence of period-doubling bifurcation and chaos behaviour is confirmed by bifurcation diagram and Lyapunov exponent diagram as well. Finally, in order to extend the stable range of parameter in TCP/RED, efforts are made to control the bifurcation and to stabilize the chaotic orbits to the fixed point in the system through the hybrid control strategy. Theoretical analysis and numerical simulations have shown that it works effectively.

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The remainder of this paper is organized as follows. Presented in Section 2 is the stroboscopic model of TCP/RED system. Then in the third section, the fixed point of the model is analysed and the bifurcation and chaotic behaviours of the system are shown by bifurcation diagram and Lyapunov exponent diagram. In Section 4, the proposed hybrid control strategy is applied to stabilizing the chaotic orbits of the system. Finally, the conclusion of this paper is given in Section 5.

2. Stroboscopic model for TCP/RED

First we briefly introduce the stroboscopic model of a TCP/RED system. The average queue size is employed as the state variable in the model. The network topology and other conditions are the same as those in Ref.[9]. The final dynamic model of the TCP/RED system is as follows:[9]

$$\begin{aligned}\overline{q}_{k+1} =&(1-w)\overline{q}_k + w\max(\frac{NK}{\sqrt{p_k}} - \frac{Cd}{M}, 0), \text{ if } p_k \neq 0,\\ &(1-w)\overline{q}_k + w(\text{rwnd}N - \frac{Cd}{M}), \text{ if } p_k = 0,\end{aligned} \tag{1}$$

where the variables are listed below:
$\overline{q}_{k+1}$: average queue size in round k+1,
$\overline{q}_k$: average queue size in round k,
w: exponential averaging weight in RED,
N: number of TCP connections,
K: constant($\sqrt{3/2}$),
p_k: drop probability in round k,
C: capacity of the link between the two routers,
d: round-trip propagation delay,
M: packet size,
rwnd: receiver's advertised window size.

The drop probability in round k is

$$p_k = \begin{cases} 0, & \text{if } \overline{q}_k \leqslant q_{\min}, \\ 1, & \text{if } \overline{q}_k \geqslant q_{\max}, \\ \dfrac{\overline{q}_k - q_{\min}}{q_{\max} - q_{\min}} p_{\max}, & \text{otherwise}, \end{cases} \tag{2}$$

where $q_{\min}$ is the minimum threshold and $q_{\max}$ is the maximum threshold.

3. Fixed point and bifurcation

Equation (1) is denoted as

$$\overline{q}_{k+1} = f(\overline{q}_k, \rho) \tag{3}$$

where ρ summarizes the system parameters, including the exponential averaging weight w, the maximum packet drop probability $p_{\max}$, the minimum threshold $q_{\min}$ and the maximum threshold $q_{\max}$.

If an average queue size satisfies the equation

$$q^* = f(q^*, \rho) \tag{4}$$

then q^* is a fixed point of mapping $f(*)$. We should choose the system parameter appropriately in order that the fixed point of system remains between $q_{\min}$ and $q_{\max}$. Therefore the fixed point q^* is given as the positive real solution of the following third-order polynomial:

$$\begin{aligned} &y^3 + \left(\frac{2Cd}{M} - q_{\min}\right) y^2 + \left(\frac{C^2d^2}{M^2} - \frac{2Cd}{M} q_{\min}\right) y \\ &- \left(\frac{C^2d^2}{M^2} q_{\min} + \frac{N^2K^2}{v}\right) = 0, \end{aligned} \tag{5}$$

where $v = p_{\max}/(q_{\max} - q_{\min})$. From Eq.(5), we know that the fixed point does not depend on the exponential averaging weight w.

In order to analyse the stability of the fixed point q^* , we consider the eigenvalue of the system

$$\begin{aligned} &\frac{\partial}{\partial \overline{q}_k} f(\overline{q}_k, \rho)\Big|_{\overline{q}_k = q^*} \\ &= 1 - w - \frac{wNK}{2\sqrt{v}(q^* - q_{\min})^{3/2}}. \end{aligned} \tag{6}$$

The linear stability of the fixed point q^* is

$$\left|1 - w - \frac{wNK}{2\sqrt{v}(q^* - q_{\min})^{3/2}}\right| < 1. \tag{7}$$

Changing the parameter setting can lead to the loss of stability of the fixed point if inequality (7) is not satisfied. In order to demonstrate the existence of such a bifurcation leading to the chaotic behaviour of system, we focus on the exponential averaging weight w as the distinguished bifurcation parameter. The critical value of w can be deduced from which the eigenvalue given by Eq.(6) is –1. The fixed point of system is stable when w is less than its critical value as equation Eq.(6) is a descending function of w. The critical value can be expressed as

$$w_{\text{c}} = \frac{2}{1 + \dfrac{NK}{2\sqrt{v}(q^* - q_{\min})^{3/2}}}. \tag{8}$$

For example, we set system parameters as follows:[9] C=1.54 Mbps, $K=\sqrt{3/2}$, d=0.0228, M=4000 bits, rwnd=1000 packets, N=1, $p_{\max} = 0.1, q_{\min} = 5$ packets, and $q_{\max} = 15$ packets. Solving Eq.(5), we obtain the fixed point of system, $q^* = 5.7142$. From Eq.(8), we obtain $w_C = 0.17944$. When we vary the exponential averaging weight w in $0 < w < w_c$, the system is stable at the fixed point q^*. A period-double bifurcation occurs from the fixed point when w=0.17944. We vary w from 0.14 to 0.27, with steps of 0.0002. The bifurcation diagram is shown in Fig.1. From the diagram we know that the simulation result at the fixed point and the critical value of w are consistent with those in the theoretical

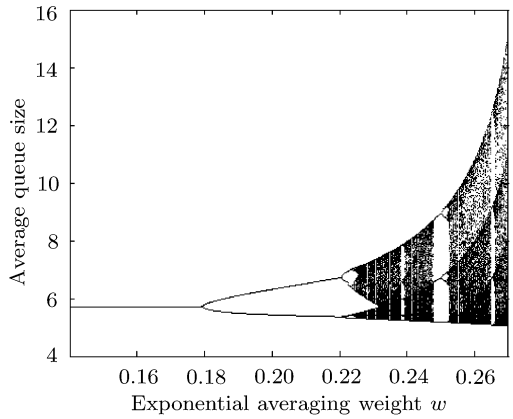


Fig.1. Bifurcation diagram with as a parameter.

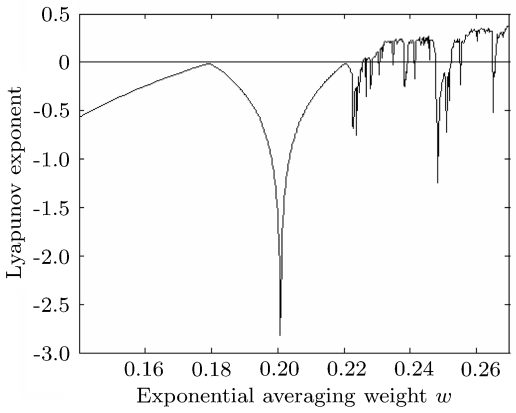


Fig.2. Lyapunov exponent.

analysis respectively. Lyapunov exponents are plotted in Fig.2 for the bifurcation scenario in Fig.1, as a positive Lyapunov exponent indicates chaos.[12] From Fig.2 we can see that w when is small the exponent is negative, which corresponds to the single fixed point. As w increases, the exponent slowly increases to zero and then becomes negative again due to a stable period-doubling orbit. Finally, it becomes positive and indicates that the system is in chaotic orbits. In the chaotic region, there are some small period windows where the exponent is negative. Both of the two diagrams show that the system enters into chaos state through a period-doubling route and some small-periodic windows are embedded in the chaotic region.

4. Control bifurcation and chaos

Over the past decade, there has been tremendous interest in controlling bifurcation and chaos in dynamical system.[13–24] In this section, we employ a hybrid control strategy proposed in Ref.[11] for controlling bifurcation and chaos in a stroboscopic model of TCP/RED. The objective is to extend the range of RED parameters, in which the system remains its stable dynamical behaviour, and to delay or even eliminate the bifurcation completely. Now we apply both parameter perturbation and state feedback to the system; therefore we acquire the following controlled form of the map (3):

$$\overline{q}_{k+m} = \alpha f^{(m)}(q_k, \rho) + (1-\alpha)q_k, \tag{9}$$

where α is control parameter ($0 < \alpha < 1$), m is a positive integer, and $f^{(m)}$ is the mth iteration of $f(\cdot)$. Here we set $m = 1$ since we want to stabilize the average queue size to a fixed point. According to Theorem 1 in Ref.[11], we can see that the fixed point of controlled system (9) is equal to that in original system (1).

As an example, we investigate the controlled results when different parameters, such as the exponential averaging weight w, the maximum packet drop probability $p_{\max}$, the minimum and maximum thresholds $q_{\min}$ and $p_{\max}$, are varied.

4.1. Exponential averaging weight as bifurcation parameter

Now we focus on the stable parameter range of controlled system. We represent the controlled system as

$$\overline{q}_{k+1} = F(\overline{q}_k, \alpha) = \alpha f(\overline{q}_k, \alpha) + (1-\alpha)\overline{q}_k. \tag{10}$$

The eigenvalue of the controlled system is described as

$$\frac{\partial}{\partial \overline{q}_k} F(\overline{q}_k, \rho)\big|_{\overline{q}_k = q^*} = \frac{\partial}{\partial \bar{q}_k} f(\overline{q}_k, \rho) + 1 - \alpha\big|_{\overline{q}_k = q^*}. \tag{11}$$

Substituting Eq.(6) into Eq.(11) yields

$$\left.\frac{\partial}{\partial \overline{q}_k}F(\overline{q}_k,\rho)\right|_{\overline{q}_k=q^*} = \alpha\left(1-w-\frac{wNK}{2\sqrt{v}(q^*-q_{\min})^{3/2}}\right)+1-\alpha. \quad (12)$$

If the controlled system is stabilized to the fixed point, the linear stability criterion should be satisfied, i.e.

$$\left|\alpha\left(1-w-\frac{wNK}{2\sqrt{v}(q^*-q_{\min})^{3/2}}\right)+1-\alpha\right|<1. \quad (13)$$

Therefore, the critical value of the controlled system is expressed in the following closed form according to inequality (13):

$$w_{c1}=\frac{2}{\alpha\left(1+\dfrac{NK}{2\sqrt{v}(q^*-q_{\min})^{3/2}}\right)}=\frac{1}{\alpha}w_c. \quad (14)$$

Since the control parameter α is valued in a range of $0<\alpha<1$, the critical value w_{c1} of controlled system is larger than w_c of original system. That is to say, the stable range is extended in the controlled system. We vary w from 0.14 to 0.64, with steps of 0.001. Other parameters are the same as those in Section 3 respectively. Here we set the control parameter $\alpha=0.4$. The control result is shown in Fig.3. We conclude that the first period-doubling bifurcation is delayed from $w_c=0.17944$ (in Fig.1) to $w_{c1}=0.4486$ (in Fig.3). Meanwhile, the average queue size of controlled system is stabilized to the fixed point $q^*=5.7142$ which is equal to that in original system. From the diagram we can see that the numerical simulation is consistent with the theoretical analysis.

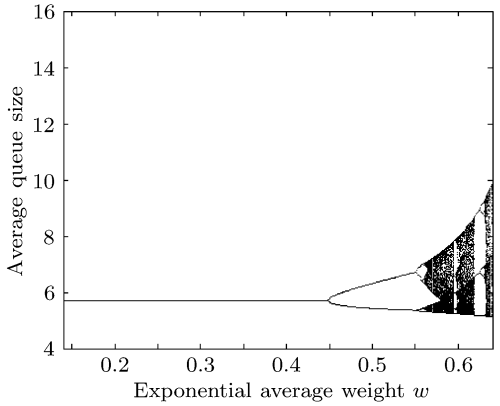


Fig.3. Control bifurcation with w ($\alpha=0.4$).

According to inequality (13), we can also determine the stable range of control parameter in which the controlled system is stabilized to the fixed point. This range is expressed as

$$0<\alpha<\min\left\{1,\frac{2}{w\left(1+\dfrac{NK}{2\sqrt{v}\,(q^*-q_{\min})^{3/2}}\right)}\right\}. \quad (15)$$

We set $w=0.26$ as an example for controlling chaos. From Fig.2 we know that the system is in chaos when $w=0.26$. According to inequality (15), we obtain a stable range of control parameter $0<\alpha<0.6902$. Here we choose $\alpha=0.5$ which is in its stable range (0, 0.6902). The control result is shown in Fig.4. The hybrid control strategy is applied to the system at time 400. It shows that the system is stabilized to the fixed point rapidly.

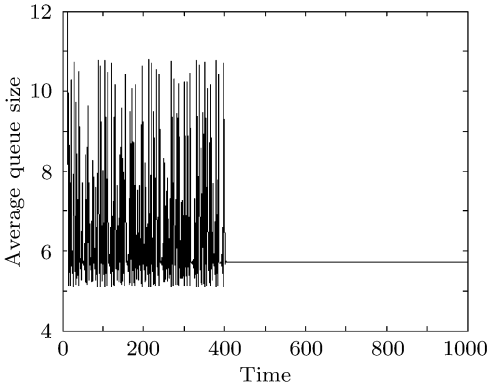


Fig.4. Control result of chaos ($\alpha=0.5$ and $w=0.26$).

4.2. Maximum packet drop probability as bifurcation parameter

We also choose the maximum packet drop probability $p_{\max}$ as the bifurcation parameter. $p_{\max}$ is varied from 0.1 to 0.9, with step of 0.001. The exponential

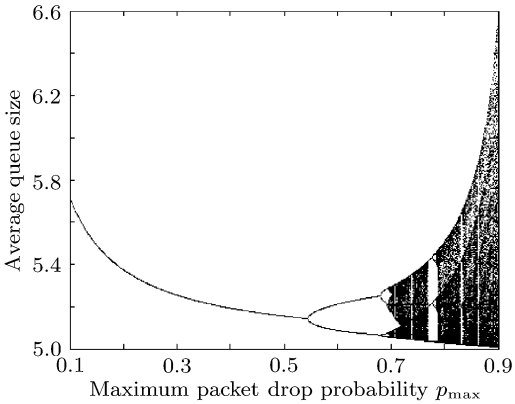


Fig.5. Bifurcation diagram with $p_{\max}$ ($w=0.046$).

averaging weight w is 0.04. The corresponding bifurcation diagram is shown in Fig.5. The control result of bifurcation with $\alpha = 0.5$ is shown in Fig.6. Because the fixed point depends on the maximum packet drop probability $p_{\max}$, the controlled stable value varies with the bifurcation parameter. From Fig.6 we see that the chaotic behaviour of system is eliminated completely when $p_{\max}$ is in the range [0.1, 0.9].

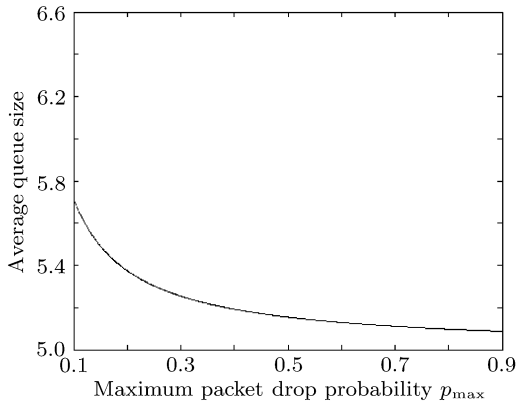


Fig.6. Control bifurcation with $p_{\max}$ ($\alpha = 0.5$).

4.3. Minimum and maximum threshold as bifurcation parameter

Now we take the minimum threshold $q_{\min}$ as the bifurcation parameter. The minimum threshold is varied from 0 to 12 with steps of 0.02. The maximum threshold is set to be three times the minimum threshold, i.e. $q_{\max} = 3q_{\min}$. The corresponding bifurcation diagram is shown in Fig.7. The control result of chaos with $\alpha = 0.3$ is shown in Fig.8. From Fig.8, we know that the chaotic behaviour of system is also eliminated when $q_{\min}$ is less than 12 packets.

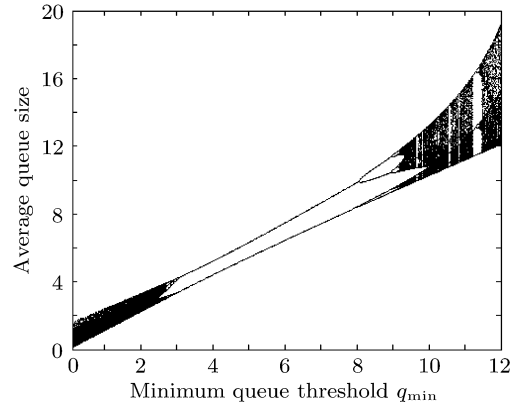


Fig.7. Bifurcation diagram with $q_{\min}$ ($w = 0.2$).

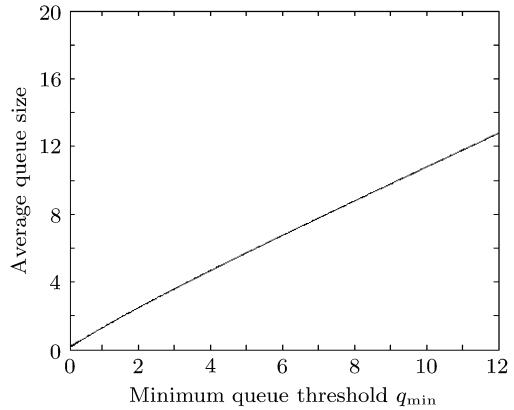


Fig.8. Control bifurcation with $q_{\min}$ ($\alpha = 0.3$).

5. Conclusion

In this paper, we have introduced a stroboscopic model of the TCP/RED system and analysed its period-doubling bifurcation and chaotic behaviour by bifurcation diagram and Lyapunov exponent diagram. Moreover, we have employed a hybrid control strategy by combining state feedback and parameter perturbation for controlling bifurcation and chaos in this model to extend the stable parameter range of TCP/RED. This hybrid control strategy does not use the exact fixed point or any system parameters for controlling. Theoretical analysis and numerical simulation results have demonstrated that it works effectively. Therefore, it is possible to extend the stable region in parameter space and improve the stability and performance of TCP/RED congestion control system.

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