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Passive adaptive control of chaos in synchronous reluctance motor*

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The performance of synchronous reluctance motor (SynRM) degrades due to chaos when its systemic parameters fall into a certain area. To control the undesirable chaos in SynRM, a passive control law is presented in this paper, which transforms the chaotic SynRM into an equivalent passive system. It is proved that the equivalent system can be asymptotically stabilized at the set equilibrium point, namely, chaos in SynRM can be controlled. Moreover, in order to eliminate the influence of undeterministic parameters, an adaptive law is introduced into the designed controller. Computer simulation results show that the proposed controller is very effective and robust against the uncertainties in systemic parameters. The present study may help to maintain the secure operation of industrial servo drive system.

Keywords: chaos control, passive control, adaptive control, synchronous reluctance motor
PACC: 0545

1. Introduction

The synchronous reluctance motors (SynRMs) are widely used in industries. The SynRM has advantages over other type of AC motor in simple mechanical construction, no slip ring and no permanent magnet, and over other servomotors in high efficiency, high power density and low manufacturing cost. In recent years, the secure and stable operation of SynRM has been actively researched, for it is an essential requirement for industrial automation manufacturing. Some investigations have indicated that, with systemic parameters falling into a certain area, the SynRM can exhibit chaotic behaviours (see, for instance, Ref.[1] by Gao and Chau). Chaotic behaviours in SynRM, which appear mainly as intermittent ripples of torque, low-frequency oscillations of rotational speed of motor, etc. can extremely destabilize the motor and even cause the drive system to fail. Thus, it is indispensable to study the method of controlling or suppressing the chaos in SynRM. Up to now, there have been suggested a large number of methods to control the chaos theoretically and technologically.[2−6] But, to the best of our knowledge, no analytical investigation into controlling chaos in SynRM can be found in the literature. The main aim of the present paper is to find an appropriate and applicable method of controlling the chaos in SynRM.

Passivity is part of a broader and general theory of dissipativity, which can be found in Refs.[6, 7] and various others references therein. The main idea of passivity theory is that the passive properties of system can keep the system internally stable. So, to make the system stable, one can design a controller which renders the closed loop system passive by using passivity theory. During the last decade the passivity theory has played an important role in designing asymptotically stabilizing controller for nonlinear system.[8−11] These papers show that the main feature of the passive control approach lies in the fact that the controller design includes at a fundamental level the system structural properties that can be exploited to solve a given control problem. It is also shown that the passive control has many advantages, e.g. clear physical interpretation, less control effort required or ease in implementation. On the other hand, the adaptive control of various nonlinear systems containing constant but undeterministic parameters has attracted many researchers' attention recently.[12,13] One usually utilizes the adaptive control technique to eliminate the influence of uncertainties in systemic parameters. To control the undesirable chaos in SynRM with unknown parameters, a passive control law combined with the adaptive control algorithm is first proposed in the present paper, which transforms the chaotic SynRM into an equivalent passive system. And then, it is

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proved that the equivalent system can be asymptotically stabilized at the set equilibrium point, namely, chaos in SynRM can be controlled. Since the feasibility has been taken into account throughout the design process, the designed controller is more applicable to the practical systems. Our simulation results indicate that the presented control law is very effective and robust against the uncertainties in system parameters.

2. Chaos in the synchronous reluctance motor

The dimensionless mathematical model of SynRM drive is given by[1]

$$\begin{aligned}\frac{\mathrm{d}i_\mathrm{d}}{\mathrm{d}t} &= v'_\mathrm{d} - Bi_\mathrm{d} + i_\mathrm{q}\omega,\\ \frac{\mathrm{d}i_\mathrm{q}}{\mathrm{d}t} &= -i_\mathrm{q} - i_\mathrm{d}\omega + C(\omega - \omega_\mathrm{ref}),\\ \frac{\mathrm{d}\omega}{\mathrm{d}t} &= i_\mathrm{d}i_\mathrm{q} - A\omega + T'_L,\end{aligned} \tag{1}$$

where, ω, i_q and i_d are state variables, denoting angle speed, quadrature- and direct-axial current of the motor, respectively; A, B and C are system operating parameters, which are positive; v'_d, ω_ref and T'_L represent the direct-axial stator voltage components, reference mechanical rotor speed and external load torque, respectively. In this paper, we consider only the case in which the system is unforced. This case can be thought of as that, after an operating period of the system, the external inputs are set to zero, namely, $v'_\mathrm{d} = \omega_\mathrm{ref} = T'_L = 0$.[1] Then, system (1) becomes

$$\begin{aligned}\frac{\mathrm{d}i_\mathrm{d}}{\mathrm{d}t} &= -Bi_\mathrm{d} + i_\mathrm{q}\omega,\\ \frac{\mathrm{d}i_\mathrm{q}}{\mathrm{d}t} &= -i_\mathrm{q} - i_\mathrm{d}\omega + C\omega,\\ \frac{\mathrm{d}\omega}{\mathrm{d}t} &= i_\mathrm{d}i_\mathrm{q} - A\omega.\end{aligned} \tag{2}$$

In a realistic model of SynRM, the systemic parameters A, B and C have uncertainties, which are influenced by working conditions. The nonlinear dynamical behaviours of system (2) have been investigated fully in Ref.[1] by using a modern nonlinear theory such as bifurcation and chaos. The results show that, with the operating parameters A, B and C falling into a certain area, SynRM can exhibit chaotic behaviours. For example, when system operating parameters are taken to be the following values: $A = 1.58$, $B = 0.18$ and $C = 10$, the SynRM becomes chaotic. The typical chaotic attractor is shown in Fig.1. Chaotic behaviour in SynRM, which appears as aperiodic, random, sudden or intermittent morbid oscillations of the motor angle speed, can seriously destroy the stable operation of the motor and even cause industrial drive system to fail. Therefore, how to control the chaos in SynRM is a significant job.

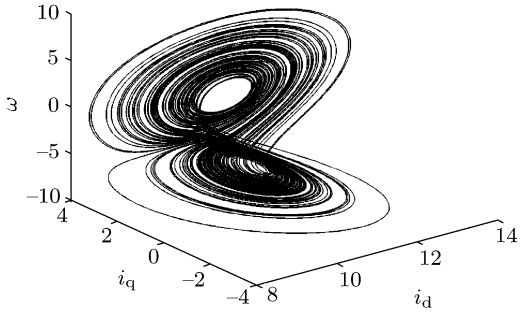


Fig.1. Typical chaotic attractor in SynRM with systemic parameters A=1.58, B=0.18 and C=10.

3. Basic conception of passivity of nonlinear system and passive control method

Consider a nonlinear affine system

$$\begin{aligned}\dot{x} &= f(x) + g(x)u,\\ y &= h(x),\end{aligned} \tag{3}$$

where the state variable $x \in \Re^n$; the external input $u \in \Re^m$; output $y \in \Re^m$; both f and m column of g are smooth vector fields; h is smooth mapping. Without loss of generality, we suppose that the vector field f has at least one equilibrium point. The notion of passivity can be captured as follows:

Definition 1[7] System (3) is said to be passive if there exists a real constant β, such that for $\forall t \geq 0$ the following inequality holds:

$$\int_0^t u^\mathrm{T} y(\tau)\mathrm{d}\tau \geq \beta, \tag{4}$$

or if there exist a constant ρ equal to or greater than zero, i.e. $\rho \geq 0$, and a real constant β, such that

$$\int_0^t u^\mathrm{T} y(\tau)\mathrm{d}\tau + \beta \geq \int_0^t \rho y^\mathrm{T}(\tau) y(\tau)\mathrm{d}\tau. \tag{5}$$

The physical meaning of passive system is that the energy of the nonlinear system can be increased only through the supply from an external source. In other words, a passive system cannot store more energy than that supplied externally. The passive system is a naturally stable system.[7]

Let $z = \vartheta(x)$, then system (3) will be changed into the following generalized form:

$$\begin{aligned} \dot{z} &= f_0(z) + p(z, y)y, \\ \dot{y} &= b(z, y) + a(z, y)u, \end{aligned} \tag{6}$$

where $a(z, y)$ is nonsingular for any (z, y). With the conception of passivity of nonlinear system, a well-known lemma is introduced as follows.

Lemma 1[8] If system (3) is a minimum phase system, system (6) will be equivalent to a passive system and asymptotically stabilized at an equilibrium point by using a state feedback in the form of

$$u = \alpha(x) + \beta(x)v. \tag{7}$$

Lemma 1 implies that the non-inherently-passive nonlinear systems can be equivalent to passive systems by designing system controller and then stabilized at the equilibrium point, which is essential for the proof of main result in this paper.

4. Passive adaptive control of chaos in synchronous reluctance motor

A SynRM is a practical engineering plant, in which the feasibility of control must be taken into account, namely, the operational variable must be accessible and the control objective should be realizable. It is well known that one usually controls the angle speed to a desired value when the conventional drive system suffers from destabilization. Without loss of generality, we choose the angle speed of the SynRM as our manipulated variable and control object, which is desirable for the real application. Then the controlled system can be expressed as

$$\begin{aligned} \frac{\mathrm{d}i_\mathrm{d}}{\mathrm{d}t} &= -Bi_\mathrm{d} + i_\mathrm{q}\omega, \\ \frac{\mathrm{d}i_\mathrm{q}}{\mathrm{d}t} &= -i_\mathrm{q} - i_\mathrm{d}\omega + C\omega, \\ \frac{\mathrm{d}\omega}{\mathrm{d}t} &= i_\mathrm{d}i_\mathrm{q} - A\omega + u, \\ y &= \omega, \end{aligned} \tag{8}$$

where u is the control law, and y is the output signal. With the knowledge above, the main results of our proposal are described in the following.

Theorem 1 The controlled SynRM system (8) is a minimum phase system.

Proof Suppose $z_1 = i_\mathrm{d}$ and $z_2 = i_\mathrm{q}$ and consider the case where $y = \omega$, controlled system (8) can be rewritten in the normal form of Eq.(6) as

$$\begin{aligned} \dot{z_1} &= -Bz_1 + z_2 y, \\ \dot{z_2} &= -z_2 - z_1 y + Cy, \\ \dot{y} &= z_1 z_2 - Ay + u. \end{aligned}$$

We rewrite Eq.(6) as follows:

$$\begin{aligned} \dot{z} &= f_0(z) + p(z, y)y, \\ \dot{y} &= b(z, y) + a(z, y)u, \end{aligned}$$

where

$$\begin{aligned} z &= [z_1, z_2]^\mathrm{T}, \\ f_0(z) &= [-Bz_1, -z_2]^\mathrm{T}, \\ p(z, y) &= [z_2, -z_1 + C]^\mathrm{T}, \\ b(z, y) &= z_1 z_2 - Ay, \\ a(z, y) &= 1. \end{aligned} \tag{9}$$

Choose a storage function candidate to be

$$\begin{aligned} V =& W(z) + \frac{1}{2}y^2 + \frac{1}{2k_1}(\hat{A} - A)^2 \\ &+ \frac{1}{2k_2}(\hat{B} - B)^2 + \frac{1}{2k_3}(\hat{C} - C)^2, \end{aligned} \tag{10}$$

where $W(z) = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2$, which is Lyapunov function with $W(0) = 0$; $\hat{A}$, $\hat{B}$ and $\hat{C}$, which can be constants or time-varied functions, are the estimated values of undeterministic parameters A, B and C, respectively; k_1, k_2 and k_3 are arbitrary positive scalars.

The zero dynamics of system (6) describes the internal dynamics which is consistent with external constraint $y = 0$, i.e.

$$\dot{z} = f_0(z). \tag{11}$$

Considering Eq.(11) and parameter B to be a positive constant, we have

$$\begin{aligned} \frac{\mathrm{d}W(z)}{\mathrm{d}t} =& \frac{\partial W(z)}{\partial z} f_0(z) \\ =& [z_1, z_2][-Bz_1, -z_2]^\mathrm{T} \le 0. \end{aligned} \tag{12}$$

Since $W(z)$ is the Lyapunov function of $f_0(z)$, the zero dynamics of the controlled SynRM system is Lyapunov stable, namely, the controlled SynRM system is of minimum phase. QED.

Theorem 2 For controlled SynRM system (8), if there exists a the loop control as follows:

$$\begin{aligned} u &= (\hat{A}-\alpha)y + v - z_1 z_2 - \hat{C} z_2, \\ \dot{\hat{A}} &= -k_1 y^2, \\ \dot{\hat{B}} &= 0, \\ \dot{\hat{C}} &= k_3 z_2 y, \end{aligned} \tag{13}$$

the controlled SynRM system can be equivalent to a passive system and globally asymptotically stabilized at the set equilibrium point, moreover, the control properties can avoid the influence of undeterministic systemic parameters.

In Eq.(13) v is the external input signal that is connected to the reference input; α is the positive constant which can be selected arbitrarily; $\dot{\hat{A}}$, $\dot{\hat{B}}$ and $\dot{\hat{C}}$ are adaptation algorithms. The arbitrary positive scalars k_1 and k_3 can adjust the performance of adaptation algorithm.

Proof Taking the time derivative of V along the trajectory of controlled system (8), one can obtain

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}t} =& \frac{\partial W(z)}{\partial z}\dot{z} + y\dot{y} + \frac{1}{k_1}(\hat{A}-A)\dot{\hat{A}} \\ &+ \frac{1}{k_2}(\hat{B}-B)\dot{\hat{B}} + \frac{1}{k_3}(\hat{C}-C)\dot{\hat{C}} \\ =& \frac{\partial W(z)}{\partial z} f_0(z) + \frac{\partial W(z)}{\partial z} p(z,y)y \\ &+ [b(z,y) + a(z,y)u]y \\ &+ \frac{1}{k_1}(\hat{A}-A)\dot{\hat{A}} + \frac{1}{k_2}(\hat{B}-B)\dot{\hat{B}} \\ &+ \frac{1}{k_3}(\hat{C}-C)\dot{\hat{C}}. \end{aligned} \tag{14}$$

According to Theorem 1, $\dfrac{\partial W(z)}{\partial z} f_0(z) \le 0$, hence Eq.(14) becomes

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}t} \le& \frac{\partial W(z)}{\partial z} p(z,y)y + [b(z,y) + a(z,y)u]y \\ &+ \frac{1}{k_1}(\hat{A}-A)\dot{\hat{A}} + \frac{1}{k_2}(\hat{B}-B)\dot{\hat{B}} + \frac{1}{k_3}(\hat{C}-C)\dot{\hat{C}} \\ \le& -Ay^2 + Cz_2 y + z_1 z_2 y + uy + \frac{1}{k_1}(\hat{A}-A)\dot{\hat{A}} \\ &+ \frac{1}{k_2}(\hat{B}-B)\dot{\hat{B}} + \frac{1}{k_3}(\hat{C}-C)\dot{\hat{C}}. \end{aligned} \tag{15}$$

Substituting Eq.(13) into inequality (15), we have

$$\frac{\mathrm{d}V}{\mathrm{d}t} \le vy - \alpha y^2. \tag{16}$$

Assuming the initial state of system to be V_0 and taking integration of both sides of inequality (16) with respect to time t, we obtain

$$V - V_0 \le \int_0^t v(\tau)y(\tau)\mathrm{d}\tau - \int_0^t \alpha y^2(\tau)\mathrm{d}\tau. \tag{17}$$

As $V \ge 0$, let $\beta = V_0$, then we will have

$$\begin{aligned} \int_0^t v(\tau)y(\tau)\mathrm{d}\tau + \beta &\ge \int_0^t \alpha y^2(\tau)\mathrm{d}\tau + V \\ &\ge \int_0^t \alpha y^2(\tau)\mathrm{d}\tau, \end{aligned}$$

which satisfies Definition 1. In terms of Lemma 1, system (8) is rendered passive, and it is asymptotically stabilized at an equilibrium point by using the state feedback control of Eq.(13). On the other hand, since the adaptation algorithm is employed to estimate the unknown parameters, the control properties can avoid the influence of uncertainties. QED.

It is noted that the designed control law is simple and straight-forward, which does not need to use any information of system except the system output and state variables. Therefore, the designed controller is feasible.

5. Numerical simulation and analysis

In this section, the validity and robustness of the proposed controller are simulated. Suppose that the actual system parameters are taken to be the values the same as those in Section 2, i.e. $A = 1.58$, $B = 0.18$ and $C = 10$, with which one of the systematic nontrivial equilibrium points is $X_{\mathrm{eq}} = (i_{\mathrm{d}}^*, i_{\mathrm{q}}^*, \omega^*) = (9.8446, 0.52479, 3.3766)$. Let $\dot{i}_{\mathrm{d}} = \dot{i}_{\mathrm{q}} = \dot{\omega} = 0$ and substitute X_{eq} into Eq.(8), then we will have $v = \alpha\omega^* + Ci_{\mathrm{q}}^*$. The SynRM experiences chaotic oscillations before the controller is performed. To show how effectively the proposed controller can control the chaos in SynRM, we let the controller u come into play at 100s. Figures 2 and 3 show the performance of controller and the curves of undeterministic parameter estimated when the control parameters are set as α=0.5, v=6.9373, k_1=2 and k_3=5, respectively. Figures 4 and 5 show the curves of undeterministic parameter estimated and the performance of controller when the control parameters are set as α=5, v=22.1320, k_1=6 and k_3=12, respectively. Note that the undeterministic parameter B does not influence the final control law, so its parameter estimate simulation is ignored here. From Figs.2 and 5, in which the fore 50s transient is discarded, one can find that it takes only a very

short time for the system to be quickly stabilized at the equilibrium point $X_{\rm eq}$. From Figs.3 and 4, where only 100s time response is shown in order to see the curve clearly, it can be seen that the controlled system distinguishes unknown system parameters on-line in a short time from different control parameters. In other words, the designed controller can avoid the influence of undeterministic parameters.

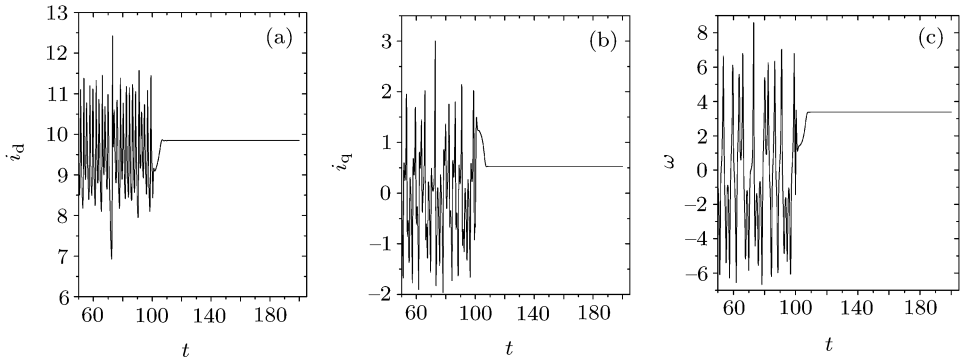


Fig.2. Control chaotic SynRM to an equilibrium point $X_{\rm eq}$=(9.8446, 0.52479, 3.3766) with α=0.5, v=6.9373, k_1=2 and k_3=5. Subfigure (a) is for the dynamic response curves of $i_{\rm d}$; (b) for the dynamic response curves of $i_{\rm q}$; (c) for the dynamic response curves of ω.

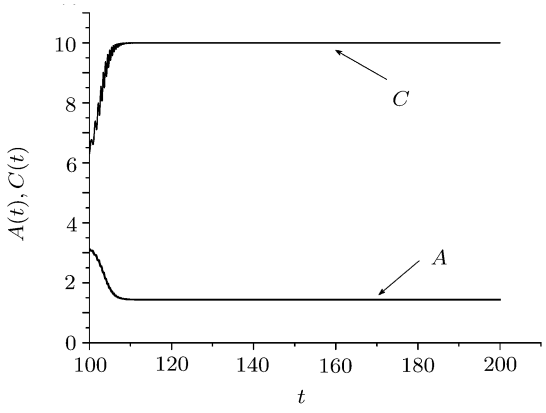


Fig.3. Curve of the parameter estimated with α=0.5, v=6.9373, k_1=2 and k_3=5.

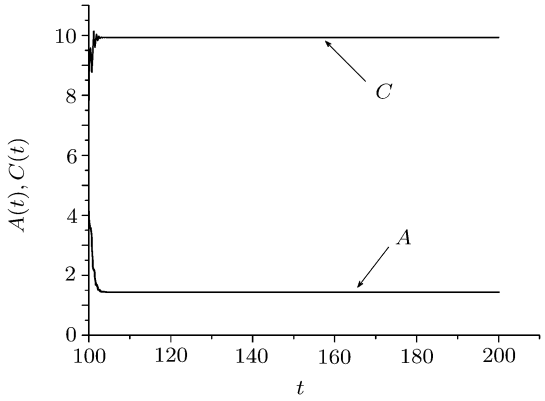


Fig.4. Curve of the parameter estimated with α=5, v=22.1320, k_1=6 and k_3=12.

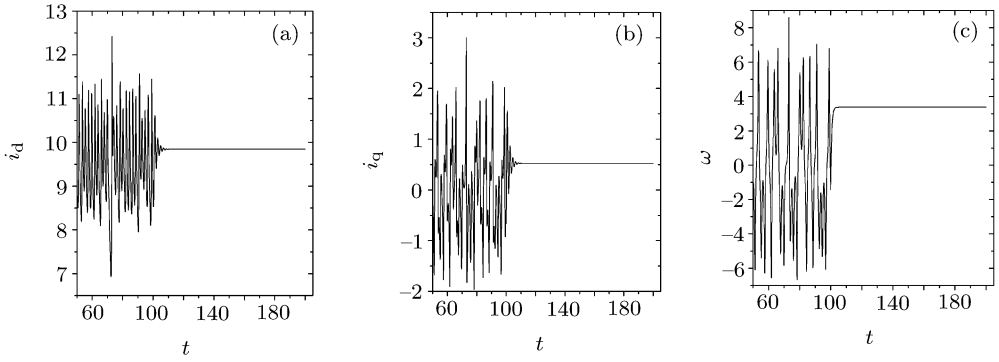


Fig.5. Control chaotic SynRM to an equilibrium point $X_{\rm eq}$=(9.8446, 0.52479, 3.3766) with α=5, v=22.1320, k_1=6 and k_3=12. Subfigure (a) is for the dynamic response curves of $i_{\rm d}$; (b) for the dynamic response curves of $i_{\rm q}$; (c) for the dynamic response curves of ω.

6. Conclusion

In this paper, we have discussed the design and application of passive adaptive control of chaos in SynRM with constant but unknown parameters. Since the feasibility has been considered throughout the design process, the proposed controller is more applicable to the practical systems. Simulation results indicate that the proposed control law is very effective and robust against the uncertainties in system parameters. The present study is significant because it is helpful to maintain the secure operation of industrial servo drive system and it is a good try at the application of the chaos control method to a special plant.

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