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Complete synchronization of uncertain chaotic dynamical network via a simple adaptive control*

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In this paper, based on the invariance principle of differential equations, we propose a simple adaptive control method to synchronize the network with coupling of the general form. Comparing with other control approaches, this scheme only depends on each node's state output. So we need not to know the concrete network structure and the solutions of the isolate nodes of the network in advance. In order to demonstrate the effectiveness of the method, a special example is provided and numerical simulations are performed. The numerical results show that our control scheme is very effective and robust against the weak noise.

Keywords: complete synchronization, adaptive control, dynamical network
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1. Introduction

Synchronization of oscillations has been known to scientists since the first observation by Huygens in a pendulum clock[1] as a common phenomenon that takes place in natural systems and artificial oscillators. In the classical sense, synchronization means the adjustment or entrainment of the frequency of periodic oscillators by means of the weak interaction.[2] In recent years, inspired by the work of Pecora and Carroll,[3,4] the notion of synchronization has been extended to the coupled chaotic systems. Due to the intrinsic character of chaotic system, the chaotic synchronization is believed to have a great potential in a large number of application areas, e.g. secure communications,[5,6] biological systems,[7] chemical reaction[8] and many physical systems.[9] Because of its importance, chaotic synchronization has been studied extensively by many researchers in recent years, and various modern control methods have been proposed to synchronize chaotic systems, such as back-stepping design,[10] nonlinear control,[11] adaptive control,[12−14] active control[15] and adaptive active control.[16]

In fact, many natural and artificial systems, such as neural systems, food webs and social systems, can be described by network. Traditionally, the study of network is concentrated on networks with completely regular topological structures, such as chains, grids and lattices. Erdös and Rényi[17] proposed a completely random network, which was on the opposite of the completely regular network. However, many real complex networks are neither completely regular nor completely random. Recently, Watts and Strogatz (WS)[18] introduced the so-called small-network to describe the network between the completely regular network and completely random network. But the WS small world is also homogeneous in nature. Barabási and Albert[19] showed that most of large-scale networks are scale-free. The scale-free networks are inhomogeneous in nature. Their work makes study of the network enter upon a new phase.

The recent researches of networks have provided a fresh and increasingly challenging framework for the study of synchronization, and have attracted much attention of researchers in different areas. The local stability of the synchronization manifold was studied via a linearization method, named the 'master stability function' method.[20] Sufficient conditions for synchronization of an array of linear couple systems were discussed.[21,22] In 2006,[23] a new method was proposed to the global stability of the synchronization. A connection graph stability method was developed

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based on graph theory.[24] From these researches, we know that the synchronization conditions of the network are rigorous, so it is difficult for the real networks to be synchronized. But sometimes we want all the nodes of the network to be synchronized. Some effective controllers were designed which can synchronize the complex network with chaotic nodes without knowing the concrete network structure.[25−27] But these controllers were designed using a solution of the node system $x' = f(x)$. So we must know a solution of the node system in advance if we use the above control scheme. However, sometimes we can not obtain a solution of the node system, for example, the network may not have an isolated node, and we can not acquire the accurate value of parameters of the node system. In this case, the control schemes of Refs.[25–27] are not valid. So we need a more effective control scheme to deal with this problem. Motivated by the above discussions, in this paper we design a simple controller to deal with the problem. Comparing with the methods of Refs.[25–27], our method only depends on the state outputs of the network. So we need not know the concrete network structure and the solution of the node system $x' = f(x)$ in advance to synchronize the dynamical network.

The rest of this paper is organized as follows. In Section 2, based on the invariance principle of differential equations, a simple adaptive feedback controller is designed to synchronize all the nodes of the network. In Section 3, we provide an example to validate the control scheme. Some conclusions are drawn in Section 4.

2. An adaptive synchronization scheme

Consider a dynamical network consisting of m identical nodes with the general form of coupling, and each node is an n-dimensional chaotic dynamical oscillator. The state equation of this network can be written as

$$\dot{x}^i = f(x^i, t) + g^i(x^1, x^2, \ldots, x^m), \quad i = 1, 2, \ldots, m \tag{1}$$

where $x^i = (x_1^i, x_2^i, \ldots, x_n^i)^{\mathrm{T}} \in R^n$ is a state vector of node i, $f(x,t) = (f_1, f_2, \ldots, f_n) : R^n \times R^+ \to R^n$ is a smooth nonlinear vector function describing the dynamics of each node. $g^i(x^1, x^2, \ldots, x^m) : R^{m\times n} \to R^n \ \ (i = 1, 2, \ldots, m)$ are smooth coupling functions. For functions $f(x,t)$ and $g^i(x^1, x^2, \ldots, x^m)$ $(i = 1, 2, \ldots, m)$, we make the following assumptions.

Let $\Omega \in R_{n\times m}$ be a bounded close set which contains the whole attractor of Eq.(1). For any $x = \left(x^{1^{\mathrm{T}}}, x^{2^{\mathrm{T}}}, \ldots, x^{m^{\mathrm{T}}}\right)^{\mathrm{T}}$, $y = \left(y^{1^{\mathrm{T}}}, y^{2^{\mathrm{T}}}, \ldots, y^{m^{\mathrm{T}}}\right)^{\mathrm{T}} \in \Omega$, there exists positive constants l_1, l_2 satisfying:

$$|f_k(x^i, t) - f_k(y^i, t)| \le l_1 \max\{|x_j^i - y_j^i|, \quad j = 1, 2, \ldots, n\} \quad i = 1, 2, \ldots, m, k = 1, 2, \ldots, n, \tag{2}$$

$$|g_k^i(x^1, x^2, \ldots, x^m) - g_k^i(y^1, y^2, \ldots, y^m)| \le l_2 \max\{|x_j^h - y_j^h|, \quad j = 1, 2, \ldots, n, \quad h = 1, 2, \ldots, m\},$$
$$k = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, m. \tag{3}$$

Moreover, for any $x^1 = x^2 = \cdots = x^m$, we assume

$$g^i(x^1, x^2, \ldots, x^m) = 0, \quad (i = 1, 2, \ldots, m). \tag{4}$$

Remark 1 The conditions (2) and (3) are every loose. If $f(x,t)$ is independent of time t, the conditions (2) and (3) hold as long as $\dfrac{\partial f_i}{\partial x_j}(i, j = 1, 2, \ldots, n)$ and $\dfrac{\partial g^i}{\partial x_j^k}(i, k = 1, 2, \ldots m, \ j = 1, 2, \ldots, n)$ are bounded. If $f(x,t)$ depends on time t, condition (2) is also very general. For example, it is not difficult to find a kind of chaotic system with $f(x,t) = A(t)h(x)+b(t)$ satisfying condition (2), where $A(t) \in R^n \times R^n$ is a bounded matrix function, $b(t) \in R^n$ is a bounded vector function, $h(x) \in R^n$ is vector function satisfying the condition (2). The condition (4) ensures that the solution of a single isolate node is also a solution of the synchronization coupled network.

In order to synchronize all the nodes of the network, we add the controller $u^i = (u_1, u_2, \ldots, u_n)^{\mathrm{T}}$ to each node of the network

$$\dot{x}^i = f(x^i, t) + g^i(x^1, x^2, \ldots, x^m) + u^i, \quad i = 1, 2, \ldots, m. \tag{5}$$

Unlike the scheme in Refs.[25,26], instead of $s(t)$ which is a solution of the isolated node of the network, we use $X(t) = \dfrac{1}{m}\sum_{i=1}^{m} x^i$ to design the controller. It is obvious that $X(t)$ satisfies the equation

$$\dot{X}(t) = \frac{1}{m}\sum_{i=1}^{m}[f(x^i, t) + g^i(x^1, x^2, \ldots, x^m) + u^i]. \tag{6}$$

We define the distance variables

$$\delta x^i = x^i - X(t) \quad i = 1, 2, \ldots, m, \tag{7}$$

where $\delta x^i = (\delta x_1^i, \delta x_2^i, \ldots, \delta x_n^i)^{\mathrm{T}} \in R^n$ describes the deviation of state vector of node i from $X(t)$. Obviously, the variables $\delta x^i (i = 1, 2, \ldots, m)$ satisfy the following equation:

$$\delta\dot{x}^i = f(x^i, t) + g^i(x^1, x^2, \ldots, x^m) + u^i - G(x), \quad i = 1, 2, \ldots, m, \tag{8}$$

where $G(x) = \dfrac{1}{m}\sum_{i=1}^{m}[f(x^i, t) + g^i(x^1, x^2, \ldots, x^m) + u^i]$. Here, we should note that $\delta x^i (i = 1, 2, \ldots, m)$ satisfy the condition

$$\sum_{i=1}^{m} \delta x^i = 0. \tag{9}$$

By introducing the variables $\delta x^i (i = 1, 2, \ldots, m)$, the problem of synchronization in network (1) can be transformed into a problem of how to realize the asymptotical stabilization of system (8). So our aim is to design a controller $u^i (i = 1, 2, \ldots, m)$ to make the system (8) asymptotically stable at the origin. Inspired by Refs.[27,28], we introduce the following simple adaptive-feedback controller u^i:

$$u_k^i = \varepsilon_k^i \left(x_k^i - \frac{1}{m}\sum_{j=1}^{m} x_k^j \right), \quad i = 1, 2, \ldots, m, \quad k = 1, 2, \ldots, n, \tag{10}$$

where $\varepsilon^i = (\varepsilon_1^i, \varepsilon_2^i, \ldots, \varepsilon_n^i)^{\mathrm{T}} \in R^n (i = 1, 2, \ldots, m)$ are variable feedback strengths. Instead of linear feedback, the feedback strengths $\varepsilon^i (i = 1, 2, \ldots, m)$ will be adapted according to the following algorithm:

$$\dot{\varepsilon}_k^i = -\gamma_k^i \left(x_k^i - \frac{1}{m}\sum_{j=1}^{m} x_k^j \right)^2, \quad i = 1, 2, \ldots, m, \quad k = 1, 2, \ldots, n, \tag{11}$$

where $\gamma_k^i (i = 1, 2, \ldots, m, \quad k = 1, 2, \ldots, n)$ are arbitrary positive constants.

For the $n \times m$ dimensional equation (8), we introduce the following non-negative function

$$V(\delta x^i) = \frac{1}{2}\sum_{i=1}^{m} \delta x^{i^{\mathrm{T}}} \delta x^i + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{1}{\gamma_j^i}(\varepsilon_j^i + L)^2, \tag{12}$$

where L is an arbitrary positive constant. By differentiating the function V along the system (8), we obtain

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^{m} \delta x^{i^{\mathrm{T}}} \delta\dot{x}^i + \sum_{i=1}^{m}\sum_{j=1}^{n} \frac{1}{\gamma_j^i}(\varepsilon_j^i + L)\dot{\varepsilon}_j^i \\
&= \sum_{i=1}^{m} \delta x^{i^{\mathrm{T}}} \left[f(x^i, t) + g^i(x^1, x^2, \ldots, x^m) + u^i - G(x) \right] \\
&\quad - \sum_{i=1}^{m}\sum_{j=1}^{n} \varepsilon_j^i \left(x_k^i - \frac{1}{m}\sum_{j=1}^{m} x_k^j \right)^2 - \sum_{i=1}^{m}\sum_{j=1}^{n} L \left(x_k^i - \frac{1}{m}\sum_{j=1}^{m} x_k^j \right)^2 \\
&= \sum_{i=1}^{m} \delta x^{i^{\mathrm{T}}} [f(x^i, t) - f(X(t), t) + g^i(x^1, x^2, \ldots, x^m) - g^i(X(t), X(t), \ldots, X(t))] \\
&\quad + \sum_{i=1}^{m} \delta x^{i^{\mathrm{T}}} [f(X(t), t) - G(x)] - \sum_{i=1}^{m}\sum_{j=1}^{n} L \left(x_k^i - \frac{1}{m}\sum_{j=1}^{m} x_k^j \right)^2 \\
&\leq \sum_{i=1}^{m}\sum_{j=1}^{n} (nl_1 + m^2 nl_2)\delta {x_j^i}^2 - \sum_{i=1}^{m}\sum_{j=1}^{n} L\delta {x_j^i}^2,
\end{aligned}$$

where we have assumed $x^i \in \Omega$, and used the conditions (2), (3), (4) and (9). Obviously, if L is large enough, then $\dot{V} \leq 0$, and $\dot{V} = 0$ if and only if $\delta x_j^i = 0$. Namely, the set $E = \{(\delta x^i, \varepsilon^i) | \delta x^i = 0, \varepsilon^i = \varepsilon_0^i, i = 1, 2, \ldots, m\}$ is the largest invariant set contained in $\dot{V} = 0$ for system (8). Then according to the well-known LaSalle invariant principle of differential equation, starting with arbitrary initial values of system (3), the orbit will converge asymptotically to the set E, i.e., $\delta x^i \to 0$ and $\varepsilon^i \to \varepsilon_0^i$, as $t \to \infty$.

3. Application of the proposed method

In this section, an illustrative example is given to show the effectiveness of the proposed control scheme, and numerical simulations are performed to validate the proposed method. As we know, mixing of the useful signal with noise takes place during transfer of the information through communications channel. So noise is considered in the following example, and the numerical simulations show that our adaptive control scheme is robust against the effect of weak noise.

In the numerical simulation, in order to measure the synchronization, we define the following quantities

$$\text{error}_j(t) = \frac{1}{m} \sum_{i=1}^{m} |\delta x_j^i| \ j = 1, 2, \ldots, n. \tag{13}$$

It is obvious that if $\text{error}_j(t)$ converge to zero as $t \to \infty$, all δx_j^i converge to zero with time increasing.

Example In this example, we use the unified chaotic system[29,30] to describe the dynamics of the network (1), and set the coupling function

$$g_k^i = \sin^2 \left(\frac{1}{m} \sum_{j=1}^{m} x_k^j - x_k^i \right). \tag{14}$$

So the state equations of the entire network are give by

$$\begin{aligned}
\dot{x}_1^i &= -(25\alpha + 10)(x_1^i - x_2^i) + \sin^2 \left(\frac{1}{m} \sum_{j=1}^{m} x_1^j - x_1^i \right) + u_1^i, \\
\dot{x}_2^i &= -x_1^i x_3^i + (28 - 35\alpha) x_1^i - (29\alpha - 1) x_2^i + \sin^2 \left(\frac{1}{m} \sum_{j=1}^{m} x_2^j - x_2^i \right) + u_2^i, \quad i = 1, 2, \ldots, m, \\
\dot{x}_3^i &= x_1^i x_2^i - \frac{\alpha + 8}{3} x_3^i + \sin^2 \left(\frac{1}{m} \sum_{j=1}^{m} x_3^j - x_3^i \right) + u_3^i,
\end{aligned} \tag{15}$$

where $u^i (i = 1, 2 \ldots, 50)$ are adaptive feedback controllers with the update laws (10) and (11). α is an positive real constant in the range $[0, 1]$. According to Refs.[29,30], in the absence of coupling, the single oscillator belongs to the general Lorenz system for $0 \leq \alpha < 0.8$, belongs to the general Lü system for $\alpha = 0.8$, and belongs to the general Chen system for $0.8 < \alpha \leq 1$. So in this paper, we set $\alpha = 0.3$ and $\alpha = 0.9$, respectively, and set $\gamma_k^i = 0.01$ $(k = 1, 2, 3)$.

In the numerical simulation, we use fourth-order Runge–Kutta method to solve the differential equations (5) and (11) with a time step size of 0.01, and set the initial conditions $x_k^i = 2\sin(10i)$ and $\varepsilon_k^i = 0$ $(i = 1, 2, \ldots, 50, \ k = 1, 2, 3)$. The simulation results are shown in Fig.1(a)–1(c) $(\alpha = 0.3)$ and Fig.2(a)–2(c) $(\alpha = 0.9)$. In these figures, one can see that error_j (t) $(j = 1, 2, \ldots, n)$ converge to zero quickly as time increases.

To test the robustness of the scheme against noise, we add independent uniformly distributed random noise in the range $[0, 1]$ with amplitude of 0.1 to each node of the network. The corresponding numerical simulations are shown in Fig.3(a)–3(c) $(\alpha = 0.3)$ and Fig.4 (a)–4(c) $(\alpha = 0.9)$. The simulation results show that the error_j (t) $(j = 1, 2, \ldots, n)$ eventually approach to zero with negligibly small fluctuations.

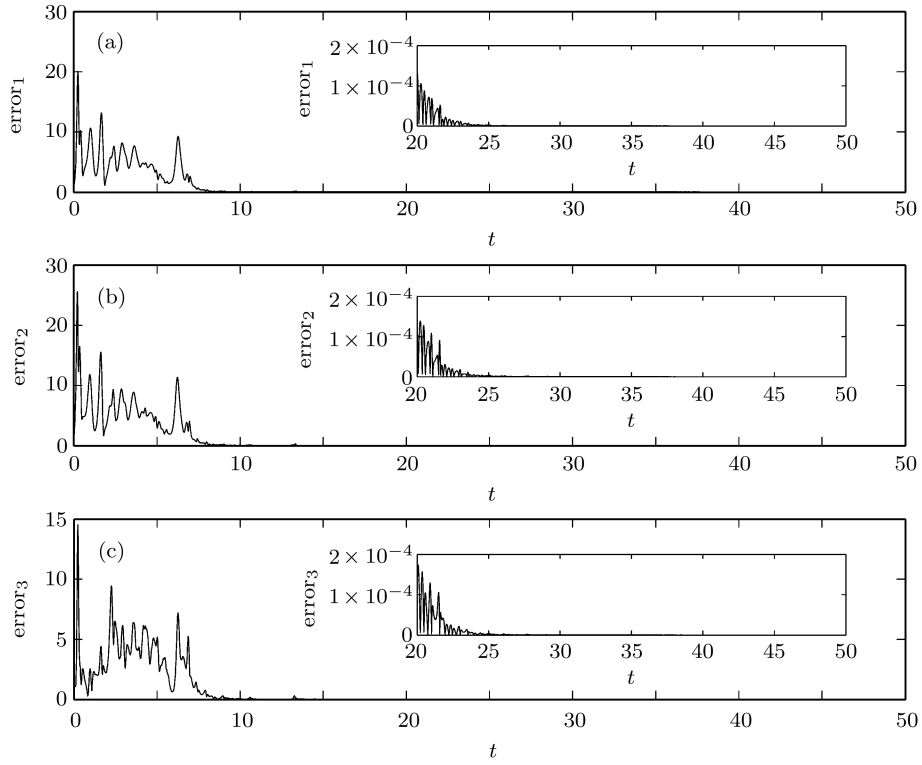


Fig.1. (a)–(c) The temporal evolution of $\mathrm{error}_i(t)$ $(i = 1, 2, 3)$ for $\alpha = 0.3$.

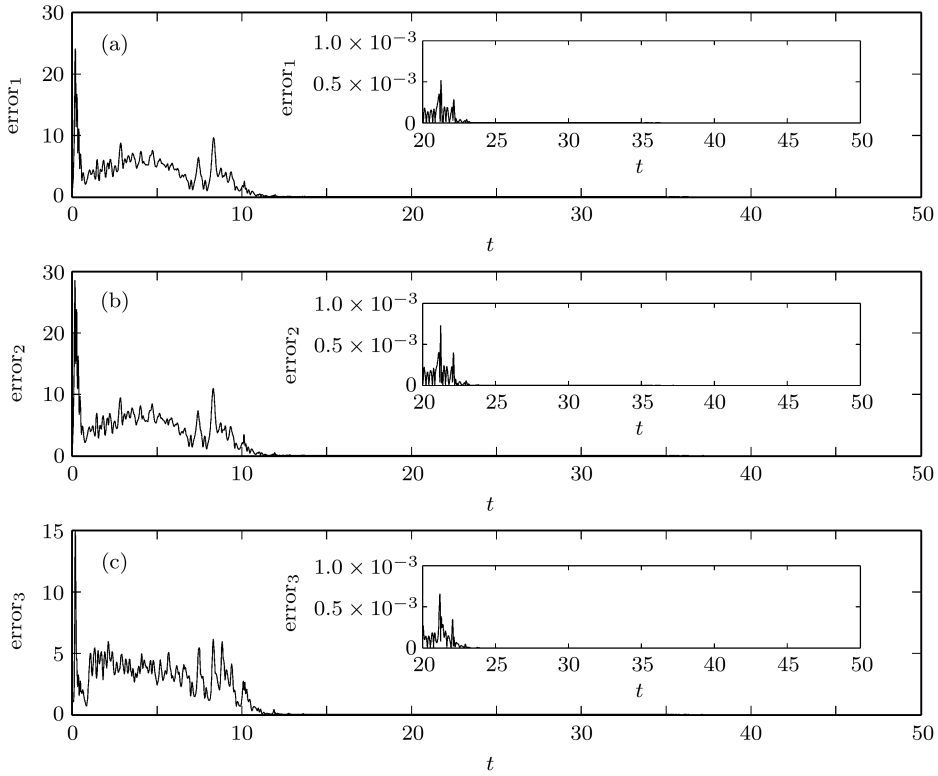


Fig.2. (a)–(c) The temporal evolution of $\mathrm{error}_i(t)$ $(i = 1, 2, 3)$ for $\alpha = 0.9$.

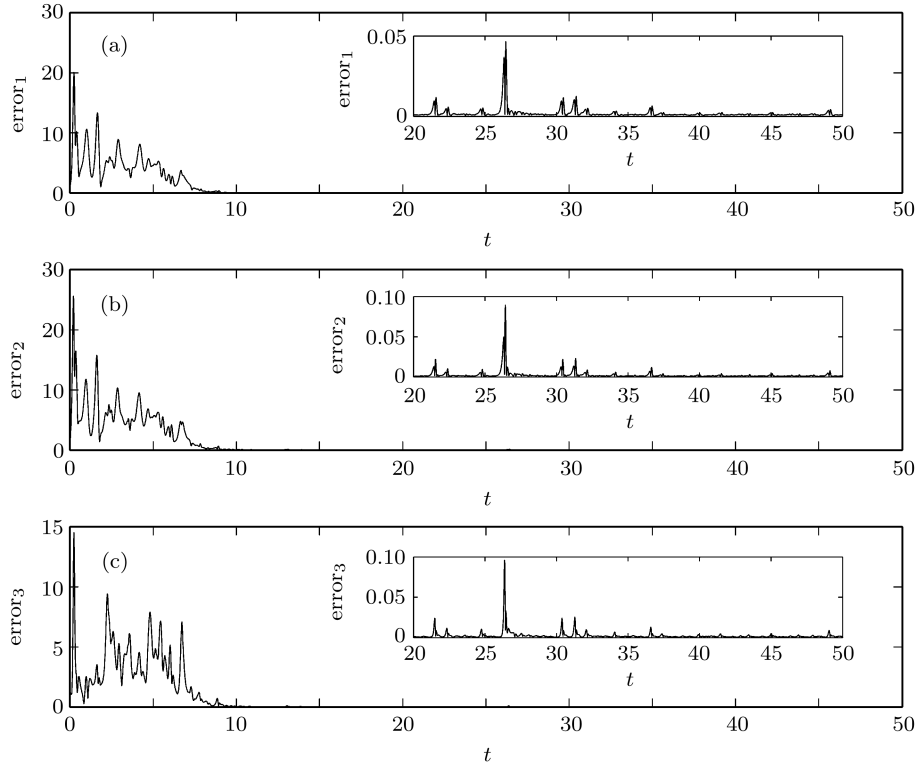


Fig.3. (a)–(c) show the effect on $\mathrm{error}_i(t)$ $(i = 1, 2, 3)$ in Fig.1(a)–1(c) when uniformly distributed noises in range [0,1] of the strength 0.1 are added to each node.

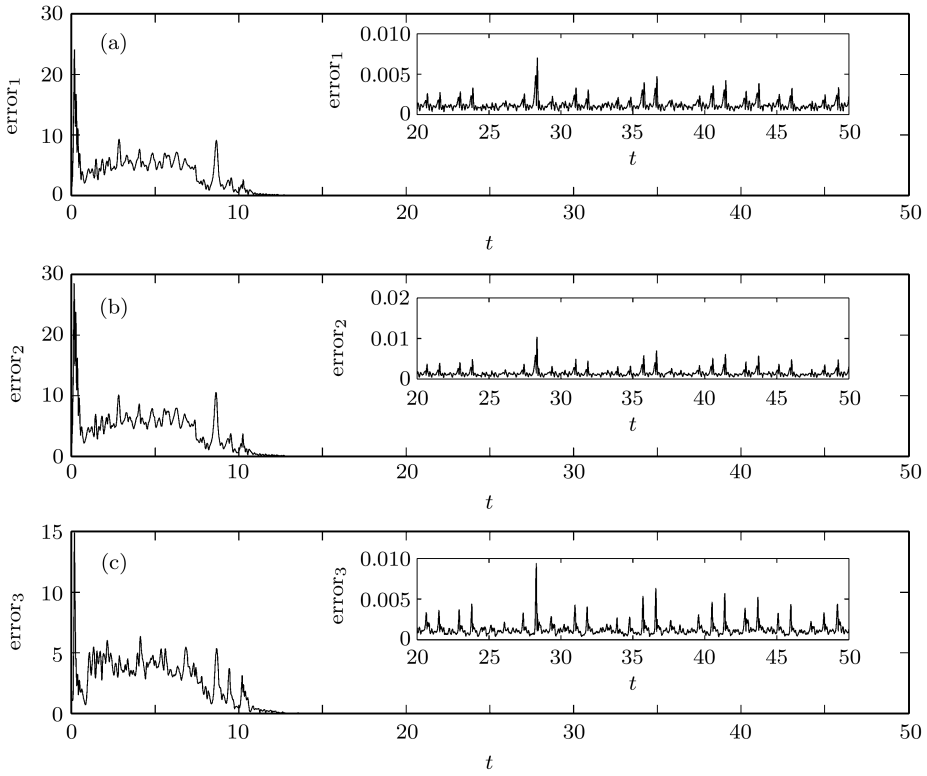


Fig.4. (a)–(c) show the effect on $\mathrm{error}_i(t)$ $(i = 1, 2, 3)$ in Fig.2(a)–2(c) when uniformly distributed noises in range [0,1] of the strength 0.1 are added to each node.

4. Conclusions

In this paper, based on the principle of differential equations, we propose a simple adaptive control scheme to synchronize all the nodes of network. Comparing with other control methods, instead of solution of isolated node of the network, we design a controller only depending on the state output of each node. So in our method, we need not know the concrete structure of the isolated node of the network and the solution of isolated node of the network in advance. In order to demonstrate the effectiveness of our method, an illustrative example is considered, and numerical simulations are performed. The numerical simulation results show that our method is every effective and robust against the weak noise. Due to the effectiveness and robustness of this method, we believe that this method has practical applications in the future.

References

[1] Huygens C 1669 *Phil. Trans. R. Soc. London* **4** 937
[2] Blekman I I 1971 *Synchronization of Dynamical Systems* (Moscow: Nauka)
[3] Pecora L M and Carroll T L 1990 *Phys. Rev. Let.* **64** 821
[4] Pecora L M and Carroll T L 1991 *Phys. Rev.* A **44** 2374
[5] Li C D, Liao F and Wong K 2004 *Physica* D **194** 187
[6] Zhang J S and Xiao X C 2001 *Acta Phys. Sin.* **50** 2121 (in Chinese)
[7] Wang X J and Rinzel J 1992 *Neural Comput.* **4** 84
[8] Kuramoto Y 1984 *Chemical Oscillations, Waves and Turbulence* (Berlin: Springer)
[9] Chua L O, Itah M, Kosarev L and Eckert K 1993 *J. Circ. Syst. Comput.* **3** 93
[10] Zhang H, Ma X K, Yang Y and Xu C D 2005 *Chin. Phys.* **14** 86
[11] Yue L J and Shen K 2005 *Chin. Phys.* **14** 1760
[12] Chen Z S, Sun K H and Zhang T S 2005 *Acta Phys. Sin.* **54** 2580 (in Chinese)
[13] Li Z and Han C Z 2002 *Chin. Phys.* **11** 9
[14] Chen S H, Zhao L M and Liu J 2002 *Chin. Phys.* **11** 543
[15] Li Z and Han C Z 2001 *Acta. Phys. Sin.* **50** 847 (in Chinese)
[16] Li G H 2005 *Chin. Phys.* **14** 472
[17] Erdös P and Rényi A 1959 *Publ. Math. Inst. Hung. Acad. Sci.* **5** 17
[18] Watts D J and Strogatz 1998 *Nature* **393** 440
[19] Barabási A L and Albert R 1999 *Scienc* **286** 509
[20] Pecora L M and Carroll T L 1998 *Phys. Rev. Lett.* **80** 2109
[21] Wu C W and Chua L O 1995 *IEEE Trans. Circuits Sys.* **42** 430
[22] Wu C W and Chua L O 1994 *Int. J. Bifur. Chaos* **4** 979
[23] Lu W L and Chen T P 2006 *Physica* D **213** 214
[24] Belykh V N, Belykh I V and Hasler M 2004 *Physica* D **195** 159
[25] Li Z and Chen G 2004 *Phys. Lett.* A **326** 333
[26] Lü J H and Chen G R 2005 *IEEE Trans. Automatic Control* **50** 841
[27] Zhou J and Lu J A and Lü J H 2006 *IEEE Trans. Automatic Control* **51** 652
[28] Huang D B 2005 *Phys. Rev.* E **71** 037203
[29] Lü J H, Chen G R, Cheng D Z and Celikovsky S 2002 *Int. J. Bifur. Chaos* **12** 2917
[30] Lü J H and Chen G R 2002 *Int. J. Bifur. Chaos* **12** 659