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Effects of correlations between the real and imaginary parts of quantum noise on intensity fluctuation for a saturation laser model*

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This paper studies the effects of cross-correlations between the real and imaginary parts of quantum noise on the laser intensity in a saturation laser model. It derives the analytic expressions of the intensity correlation function $C(\tau)$ and the associated relaxation time $T(C)$ in the case of a stable locked phase resulting from the cross-correlation λ_q between the real and imaginary parts of quantum noise. Based on numerical computations it finds that the presence of cross correlations between the real and imaginary parts of quantum noise slow down the decay of intensity fluctuation, i.e., it causes the increase of intensity fluctuation.

Keywords: saturation laser model, correlated noises, correlation function
PACC: 0540, 0250

1. Introduction

The study of laser fluctuations has revealed that the consideration of the additive and multiplicative noises is of importance to understand statistical properties of a laser system deeply.[1−7] Noises that result in fluctuations of a laser system have the same source and can be correlated in certain situation. The presence of correlated noises changes the dynamics of laser system. In 1993, Zhu first introduced correlations between quantum and pump noises to laser system and studied the steady-state fluctuations of a single-mode laser.[5] In 1996, by virtue of the two-dimensional Fokker–Planck equation, Zhou *et al* introduced the correlation between the real and imaginary parts of quantum noise to research into the statistical fluctuations of the cubic model of single-mode laser.[7] In 1997, Long *et al* introduced the phase-locking method into the cubic model of single-mode laser to make the laser field amplitude equation decouple from the laser field phase equation.[8] Based on the phase-locking method, the effects of the correlation between the real and imaginary parts of quantum noise on the stationary statistical properties of the cubic model of single-mode laser are studied by Ke *et al*.[9] Xie and Mei investigated the effects of correlation between the real and imaginary parts of quantum noise on the intensity correlation fluctuation of the cubic laser system.[10] The cubic model of single-mode laser only contains cubic non-linearities in laser system. Chen and Mei studied the effects of correlations between quantum and pump noises on fluctuation of the laser intensity in a saturation laser model.[11] Chen *et al* discussed intensity correlation time of a gain–noise model driven by two coloured noises with coloured cross-correlation.[12] A full account of the saturation effects and correlation effects between the real and imaginary parts of quantum noise on the intensity correlation fluctuation of the laser system should be further considered.

In this paper, we investigate the effects of the correlations between the real and imaginary parts of quantum noise on the laser intensity correlation fluctuation in the saturation laser model of a single-mode laser. In Section 2 the analytic expressions of the intensity correlation function $C(\tau)$ and the associated relaxation time $T(C)$ are derived. In Section 3 $C(\tau)$ and $T(C)$ are discussed by numerical calculations.

2. The intensity correlation function and the associated relaxation

The laser model with a full account of the saturation effects follows the Langevin equation (LE)[1]

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$$\frac{\mathrm{d}E}{\mathrm{d}t} = -KE + \frac{F_1 E}{1 + A|E|^2/F_1} + p(t)E + q(t), \quad (1)$$

where the electric field E is complex and dimensionless, K is the cavity decay rate for the electric field, $F_1 = a_0 + K$ is the gain parameter, a_0 and A are real and stand for net gain and self-saturation coefficients. The random variables $q(t)$ and $p(t)$ are complex and represent the quantum and pump noises. The statistical properties of the noise term are characterized by first and second moments:

$$\langle q_i(t)\rangle = \langle p_i(t)\rangle = 0. \quad (2)$$

$$\langle q_i(t)q_i(t')\rangle = D\delta(t-t'), \quad (3)$$

$$\langle p_i(t)p_i(t')\rangle = Q\delta(t-t'), \quad (4)$$

where $i = \mathrm{R}, \mathrm{I}$ represents the real and imaginary part, D and Q denote strengths of the quantum and the pump noises respectively. Because the real and imaginary components of quantum noise arise from a common origin, they should be correlated each other. Assuming the real and imaginary parts of quantum noise to be Gaussian white noises with correlations, we have

$$\langle q_{\mathrm{R}}(t)q_{\mathrm{I}}(t')\rangle = \lambda_q D\delta(t-t'), \quad (5)$$

$$\langle q_i(t)p_j(t')\rangle = 0, \quad (6)$$

where the correlation coefficient λ_q measures the degree of correlation between $q_{\mathrm{R}}(t)$ and $q_{\mathrm{I}}(t)$, and $-1 \le \lambda_q \le 1$; $\lambda_q \to 0$ means that the quantum noise is a circular complex Gaussian noise, and the real and imaginary parts of the noise are independent of each other; whereas $|\lambda_q| \to 1$ implies that the real and imaginary parts of the noise are (positively or negatively) linearly correlated.

Transforming Eq.(1) into the LE about the intensity $I(E = r\mathrm{e}^{\mathrm{i}\phi},\ I = r^2)$ and applying the phase-locking method,[8] we obtain the laser intensity LE under a stable locked phase,

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -2KI + \frac{2F_1 I}{1 + AI/F_1} + P(1 - |\lambda_q|) + 2IP_{\mathrm{R}}(t) + 2I^{1/2}\tilde{\varepsilon}_{\mathrm{r}}(t), \quad (7)$$

where

$$\langle \tilde{\varepsilon}_{\mathrm{r}}(t)\rangle = 0,$$

$$\langle \tilde{\varepsilon}_{\mathrm{r}}(t)\tilde{\varepsilon}_{\mathrm{r}}(t')\rangle = D(1 + |\lambda_q|)\delta(t-t'), \quad (8)$$

$$\langle p_{\mathrm{R}}(t)\rangle = 0, \langle p_{\mathrm{R}}(t)p_{\mathrm{R}}(t')\rangle = Q\delta(t-t'). \quad (9)$$

The Fokker–Planck equation for the probability density function $P(I,t)$ corresponding to Eq.(7) with Eqs.(8) and (9) is given by[13]

$$\partial_t P(I,t) = L_{\mathrm{FP}} P(I,t), \quad (10)$$

where

$$L_{\mathrm{FP}} = -\partial_I f(I) + \partial_I^2 G(I), \quad (11)$$

here $G(x)$ and $f(x)$ are given by

$$f(I) = 2(Q-K)I + \frac{2F_1 I}{1 + AI/F_1} + 2D, \quad (12)$$

$$G(I) = 2[QI + D(1 + |\lambda_q|)]I. \quad (13)$$

From Eq.(10) the stationary probability density is obtained

$$P_{\mathrm{st}}(I) = N\frac{\left[QI + \dfrac{Q}{A}F_1\right]^{\beta_1}}{I^{\beta_2}/[QI + D(1+|\lambda_q|)]^{\beta_3}}, \quad (14)$$

where

$$\beta_1 = \frac{F_1^2}{AD(1+|\lambda_q|) - QF_1}, \quad (15)$$

$$\beta_2 = \frac{|\lambda_q|}{1+|\lambda_q|}, \quad (16)$$

and

$$\beta_3 = \frac{K}{Q} + \frac{1}{1+|\lambda_q|} + \beta_1. \quad (17)$$

N in Eq.(14) is the normalization constant. For a laser system the normalized intensity correlation function in a stationary state is defined by

$$C(\tau) = \lim_{t\to\infty}\frac{\langle \delta I(t+\tau)\delta I(t)\rangle}{\langle (\delta I)^2\rangle} = \frac{\langle \delta I(t+\tau)\delta I(t)\rangle_{\mathrm{st}}}{\langle (\delta I)^2\rangle_{\mathrm{st}}}, \quad (18)$$

where $\delta I(t) = I(t) - \langle I^2\rangle_{\mathrm{st}}$. The associated relaxation time is

$$T_C = \int_0^\infty C(\tau)\mathrm{d}\tau. \quad (19)$$

By using the projection operator method, the zeroth approximation for the relaxation time is given by[14]

$$T_C^0 = \gamma_0^{-1} = \frac{< (\delta I)^2 >_{\mathrm{st}}}{< G(I) >_{\mathrm{st}}}, \quad (20)$$

and the first-order approximation for the relaxation time is given by[14]

$$T_C = \left[\gamma_0 + \frac{\eta_1}{\gamma_1}\right]^{-1}, \quad (21)$$

where

$$\eta_1 = \frac{< G(I)f'(I) >_{\mathrm{st}}}{< (\delta I)^2 >_{\mathrm{st}}} + \gamma_0^2, \quad (22)$$

and

$$\gamma_1 = \frac{< G(I)[f'(I)]^2 >_{\rm st}}{\eta_1 < (\delta I)^2 >_{\rm st}} + \frac{\gamma_0^3}{\eta_1} - 2\gamma_0. \quad (23)$$

Performing converse Laplace transformation, we obtain the correlation function of the laser intensity[15]

$$C(\tau) = \alpha \exp(-\pi_- \tau) + (1-\alpha)\exp(-\pi_+ \tau), \quad (24)$$

where

$$\pi_\pm = \frac{\gamma_0 + \gamma_1}{2} \pm \frac{1}{2}\sqrt{(\gamma_1 - \gamma_0)^2 - 4\eta_1}$$

and

$$\alpha = (\gamma_1 - \pi_-)/(\pi_+ - \pi_-). \quad (25)$$

The parameter η_1 is the memory strength of the first-order approximation. When we set $\eta_1 = 0$, namely the effects of memory kernel is completely neglected, the intensity correlation function is given by

$$C^0(\tau) = \exp(-\gamma_0 \tau) = \exp(-\tau/T_C^0). \quad (26)$$

Equation (26) can also be obtained by virtue of the Stratonovich-like ansatz.

3. Discussion and conclusions

To see the effects of λ_q on the intensity fluctuation of the laser system with the cross correlations between the real and imaginary parts, we have plotted the curves of T_C and $C(s)$ in Figs.1–4 by numerically calculating the expression of the relaxation time Eq.(21) and the correlation function Eq.(24).

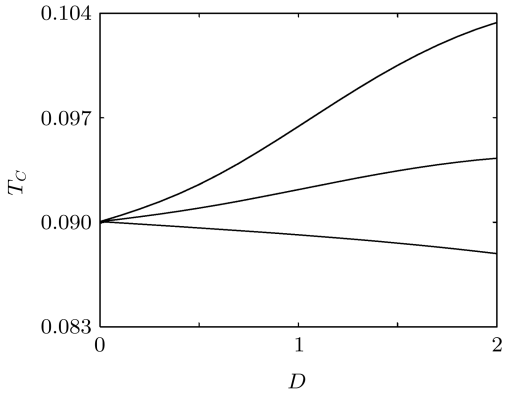


Fig.1. Relaxation time T_C as a function of D with different values of λ_q. The curves from the top to bottom correspond to $\lambda_q = 1, 0.5$, and 0 respectively, with $a_0 = 6$, $A = 1$ and $Q = 0.8$.

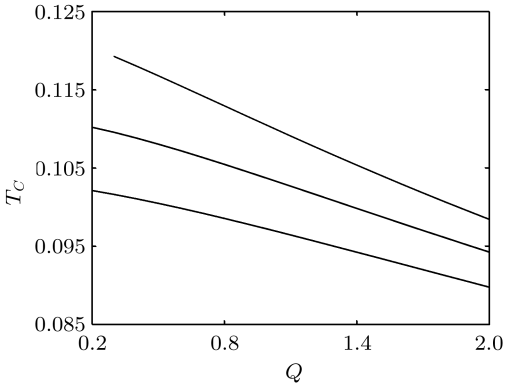


Fig.2. Relaxation time T_C as a function of Q with different values of λ_q for $a_0 = 6$, $A = 1$ and $D = 1.8$. The curves from the top to the bottom correspond to $\lambda_q = 1, 0.5$, and 0 respectively.

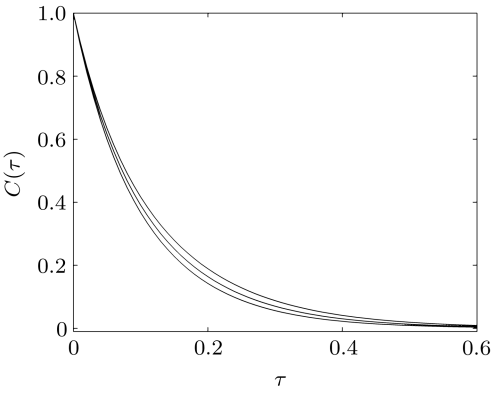


Fig.3. Intensity correlation function $C(\tau)$ as a function of decay time τ with different values of λ_q for $Q = 0.3$, $A = 1$, $a_0 = 6$ and $D = 1.8$. The curves from the top to the bottom correspond to $\lambda_q = 1, 0.5$, and 0, respectively.

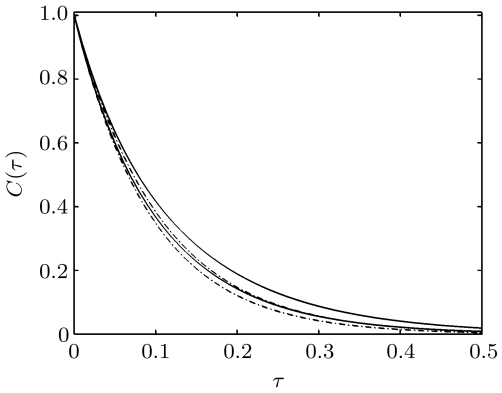


Fig.4. Intensity correlation function $C(\tau)$ and $C^0(\tau)$ as a function of decay time τ with different λ_q for $Q = 0.3$, $A = 1$, $a_0 = 6$ and $D = 1.8$. The solid curves correspond to $C(\tau)$. The dotted curves correspond to $C^0(\tau)$. Here λ_q takes 1 and 0 from the top to the bottom.

It is clear that T_C increases with the increase of D when $\lambda_q \neq 0$, however, T_C decreases with the increase of D when $\lambda_q = 0$.

In Fig.2, T_C always decreases with the increase of Q whether $\lambda_q = 0$ or $\lambda_q \neq 0$. It is also shown that the larger $|\lambda_q|$ the larger T_C is. Therefore, the cross correlations between the real and imaginary parts of quantum noise slow down the decay of intensity fluctuation when the laser is operated above the threshold.

The intensity correlation function $C(\tau)$ decays with time τ, as might be expected (see Fig.3). Nevertheless the effects of correlations between the real and imaginary parts of quantum noise cause decay rate of $C(\tau)$ with τ to become slow. The large $|\lambda_q|$ is, the slow the intensity fluctuation decays.

Finally, it should be pointed out that when the influence of memory is completely not taken into account, the relaxation time of intensity correlation drops (i.e. $T_C^0 < T_C$) since $T_C = T_C^0/(1-\epsilon)$, and $0 < \epsilon < 1$. Thus the decay rate of intensity correlation function $C^0(\tau)$ decreases, which is reflected in Fig.4.

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