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Extra dimensions and atomic transition frequencies*

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New unification theories predict large extra dimensions (LEDs). If that is the case, gravity would be stronger at short ranges than what Newtonian gravity predicts. LEDs could also have effects at atomic level. In this paper we propose a new method to constrain the size of 'gravity-only' LEDs by analysing how these LEDs modify the energy of the atomic transitions 1s–2s and 2s–2p (Lamb shift), in the particular case of the hydrogen and muonium atoms. We estimate these effects by using Bethe's non-relativistic treatment of Lamb shift. In the particular case of three LEDs, which may be a candidate to explain the interaction mechanism of dark matter particles, we have found that current knowledge in atomic spectroscopy could constrain their sizes to less than $10\,\mu$m. Although our contributions do not reach the sensitivity given by SN1987a, they are still slightly better than recent constraints given by Inverse Square Law tests of the Eöt–Wash group at Washington University, which gave $R_3 < 36.6\,\mu$m.

Keywords: large extra dimensions, Lamb shift, atomic spectroscopy
PACC: 0450, 2930, 0155

1. Introduction

The existence of large extra dimensions (LEDs) has received great attention. Recently it has been suggested that the remarkable discrepancy between the Planck scale and electroweak scale, the so-called gauge hierarchy problem in particle physics, could be resolved by introducing appropriate number of spatial extra dimensions with suitable size.

As early as 1990, Antoniadis[1] first suggested TeV-scale LEDs to re-explain supersymmetry breaking. Subsequently others found that LEDs could also be used to lower fundamental physical scales, such as string scale,[2] Grand Unification Theory (GUT) scale,[3] seesaw scale[4] and Peccei–Quinn scale.[5]

In 1998 Arkani-Hamed *et al*[6] proposed a model with LEDs (named as the ADD model by the capital letters of the three authors) to explain why gravity is so weak compared with the other three known interactions. Gravity is so weak because it is reduced by its propagation in extra dimensions. In that case gravity could be stronger than expected at short ranges.

It is important to find empirical tests and constraints on LED gravitation theory and theories of gravity in general for the beneficial interaction between theory and experiment. Ni[7] gave a recent review on this topic. Adelberger *et al*[8] has analysed the empirical constraints on the deviation from inverse square laws. Theories with Ashtekar variables are also of recent interest. Experimental and theoretical tests on these theories together with extra dimension theories are focuses in the literature.[9−11]

In the present paper, we discuss how a modified theory of gravity due to "gravity-only" large extra spatial dimensions, could modify the radiation energies of 1s–2s and 2s–2p transitions of hydrogen and muonium atoms. Our estimates are based on Bethe's non-relativistic approximation for calculating the Lamb shift.[12]

We have established upper limits to the sizes R_3 and R_4 for the cases with three and four LEDs, with $R_3 < 10\,\mu$m and $R_4 < 8.2\,$nm, by considering recent measurements of the hydrogen and muonium transition frequencies. These constraints are shown in Table 1. It is worth mentioning that the ADD theory with three LEDs has been considered as an alternative for the interaction mechanism of dark matter described by Qin *et al.*[13] The ADD model, with three LEDs, can reproduce the inverse-velocity self-interaction cross section of dark matter particles proposed by Firmani *et al*[14] to solve the 'soft core problem' in the simulation of structure formation of galaxies, clusters and cosmological scales.

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Table 1. Upper limits on the compactification radius R_n obtained from our argument and others.

	transition	R_3	R_4	R_5	R_6
hydrogen	1s–2s	13 μm	37 nm	1.0 nm	0.1 nm
	2s–2p	90 μm	200 nm	3.3 nm	0.3 nm
muonium	1s–2s	10 μm	8.2 nm	0.1 nm	6.7×10^{-3} nm
	2s–2p	30 μm	17 nm	0.2 nm	0.01 nm
inverse square law test[15]		36.6 μm	62 μm		
SN1987A[16]		1.14 nm	0.038 nm	0.0048 nm	1.2×10^{-3} nm

In Sections 2 and 3, we review the ADD model and the empirical constraints on the size of LEDs briefly. In Section 4, we calculate the modification to 1s–2s and 2s–2p transition energies of hydrogen and muonium due to extra dimensions. Finally, in Section 5 we summarize the main results obtain in the present paper.

2. The ADD model

In the ADD model, the standard model particles live in a D3-brane with a thickness of about $1\,\mathrm{TeV}^{-1}$. The D3-brane is embedded in the $(4+n)$ dimensional spacetime, where n is the number of spatial extra dimensions, which are compactified on a volume $V_n \sim (2\pi R_n)^n$. In this case, gravitons can propagate freely in the bulk. Using Gaussian law in $(4+n)$ dimensional spacetime, the gravitational potential between two point particles, of masses m_1 and m_2, is given by

$$V_{(\mathrm{r})} = \frac{m_1 m_2}{M_{\mathrm{pl}(4+n)}^{n+2}} \times \frac{1}{r^{n+1}}, \quad (r \ll R_n), \tag{1}$$

$$V_{(\mathrm{r})} = \frac{m_1 m_2}{M_{\mathrm{pl}(4+n)}^{n+2}} \times \frac{1}{R_n^n \times r}, \quad (r \gg R_n), \tag{2}$$

where $M_{\mathrm{pl}(4+n)}$ refers to the Planck scale of the $(4+n)$ dimensional theory and R_n is the size of extra dimensions. Compactification on a n-torus has been assumed. The effective 4-dimensional Planck scale can therefore be reinterpreted as $M_{\mathrm{pl}}^2 = M_{\mathrm{pl}(4+n)}^{2+n} \times R_n^n$. To solve the gauge problem we assume that the $(4+n)$ dimensional Planck scale and the electroweak scale coincide, i.e. $M_{\mathrm{pl}(4+n)} \sim m_{\mathrm{ew}} \sim 1\,\mathrm{TeV}$. To recover the observed 4 dimensional Planck scale we should assume the size of extra dimensions, R_n, to be approximately

$$R_n \sim \frac{1}{\pi} 10^{-17+\frac{32}{n}} \left(\frac{1\mathrm{Tev}}{m_{\mathrm{ew}}}\right)^{1+\frac{2}{n}} \mathrm{cm}. \tag{3}$$

The existence of one LED, $R_1 \sim 3 \times 10^{12}\,\mathrm{m}$, is clearly ruled out by tests of Newtonian gravity on solar system scales. The case of two LEDs, giving $R_2 \sim 0.3\,\mathrm{mm}$, is also inconsistent with current short-range inverse-square law tests. Nevertheless, the theory with more than two LEDs, that gives $R_{n>2} \leq 1\,\mathrm{nm}$ is still compatible with the current experimental constraints.

3. Empirical constraints on the size of LEDs

Gravity is intimately connected to the geometrical nature of spacetime. It has already been discussed above that LEDs could modify Newtonian gravity at very short distances, below the millimetre range. Inverse-square law tests can therefore give direct evidence for the existence of these entities. High energy colliders can also explore the effects of LEDs in graviton and virtual graviton production and exchange at TeV energy scale. In addition, LEDs can also have some influences in cosmological and astrophysical processes, for example, the cooling of supernovae explosions. In what follows we will briefly review these constraints.

Inverse square law test The main obstacle for looking at deviations from Newtonian gravity at short ranges is due to the weakness of those forces. In the last 3 decades, great improvements have been made by various groups in the field of weak force measurements. By testing inverse-square law with a low-frequency torsion oscillator,[17,18] the Eöt–Wash group at Washington University recently put constraints on the size of $n = 3$ and $n = 4$ LEDs compactified on a n-torus, to be less than 36.6 μm and 62 μm, respectively, which can be trivially derived by expressing Eq.(18) in Ref.[8] in terms of our Eqs.(1) and (2). We show

them in Table 1.

High energy collider In high energy colliders, two kinds of effects due to LEDs are usually considered: a) direct graviton production[19] and b) virtual graviton exchange.[20] Gravitons can be produced directly in processes such as e^+e^- and p^+p^-, in association with production of photons or jets. The differential cross section of these process depends on the new gravity scale, i.e. $M_{\mathrm{pl}(4+n)}$. Virtual gravitons can appear in many Standard Model processes. Virtual gravitons can be traced by measuring deviations of the differential cross sections from the Standard Model prediction. These phenomena can be searched for at LEP, TEVATRON, HERA and LHC. Several groups have explored these issues and they put strong constraints on the new Planck scale and thus the size of LEDs in ADD model.[19−21]

Cosmology and astrophysics Cosmological and astrophysical constraints on LEDs arise from exploration of the production of Kaluza–Klein gravitons coming from Standard Model particles. Overproduction of gravitons would increase the density of our universe significantly or even close it. This can give strong constraints on the gravitational scale $M_{\mathrm{pl}(4+n)}$ or equivalently R_n.[22] Nevertheless, this scenario strongly depends on the so-called 'normalcy temperature' T_0, temperature at which graviton production effectively starts. In this case the constraints could be relaxed by setting larger values of T_0.[23] The gravitons produced in supernova would also take energy away and, in this way reduce the observable neutrino emission. By considering the production of Kaluza–Klein gravitons and dilatons via bremsstrahlung processes from the nucleon-nucleon system in SN1987a, Hannestad and Raffelt[16] put stringent constraints on toroidally compactified 'gravity-only' LEDs. This scenario depends strongly on the nature of Kaluza–Klein gravitons. These constraints are shown in Table 1.

4. Modification of transition energies of hydrogen and muonium due to LEDs

In this section we estimate the energy shift in the 1s–2s and 2s–2p transitions of hydrogen and muonium, when a modified law of gravity on atomic scales is considered. We use Bethe's non-relativistic method for calculating Lamb shift given in 1947.[12] A more precise relativistic calculation needs a complete quantum gravity theory which has not been known well yet. After introducing Bethe's method, we report the detailed calculation of our estimation and put constraints on the size of LEDs.

4.1. Bethe's non-relativistic method

In 1947 Bethe applied Kramer's mass renormalization technique to calculate the shift of hydrogen atom energy levels due to the self-energy of electron. A Feynman's diagram describing the self-energy term of electron is shown in Fig.1 in which I stands for intermediate state. Bethe showed that the 2s–2p level shift was approximately 1040 MHz, which agreed well with observations in a few percent accuracy.

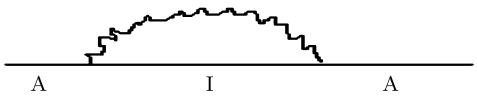


Fig.1. Feynman's diagram of the self-energy term of electron in state A.

From the classical theory of radiation, where the electron is treated as non-relativistic and the electromagnetic field is quantized, the energy shift of electron in state A due to self-energy contributions, is given by[24]

$$\Delta E_{\mathrm{A}} = \frac{e^2}{6\pi^2\varepsilon_0\hbar m_{\mathrm{e}}^2 c^3}\sum_{\mathrm{I}}\int \mathrm{d}E_\gamma \frac{E_\gamma \left|(\boldsymbol{p})_{\mathrm{IA}}\right|^2}{E_{\mathrm{A}} - E_{\mathrm{I}} - E_\gamma}, \quad (4)$$

where m_{e} and $\boldsymbol{p}$ is the mass and momentum of electron, respectively, and E_γ is the energy of emitted and/or absorbed photon, as shown in Fig.1. The sum is taken all over the intermediate states I. The result is obviously linearly divergent. The shift of the kinetic energy in the case of a free electron with momentum $\boldsymbol{p}$ is given by

$$\Delta E_{\mathrm{free}} = -\frac{e^2\boldsymbol{p}^2}{6\pi^2\varepsilon_0\hbar m_{\mathrm{e}}^2 c^3}\int_0^{E_\gamma^{\max}} \mathrm{d}E_\gamma \equiv C\cdot\boldsymbol{p}^2, \quad (5)$$

where $E_\gamma^{\max}$ is the cut-off energy of the interacting photon. In a non-relativistic approximation this energy cut-off is set to be of the electron rest mass, i.e. $m_{\mathrm{e}}c^2$.

The energy term defined by Eq.(5) cannot be "observed" separately from the electron kinetic energy in

state A. Therefore the main idea of mass renormalization will then consist of subtracting Eq.(5) from Eq.(4). The observable energy shift is then given by

$$\Delta E_{\rm A}^{(\rm obs)} = -\frac{e^2}{6\pi^2\varepsilon_0\hbar m_{\rm e}^2c^3}\ln\frac{E_\gamma^{\max}}{< E_{\rm I}-E_{\rm A} >_{\rm Av}} \times\sum_{\rm I}|(\boldsymbol{p})_{\rm IA}|^2\,(E_{\rm I}-E_{\rm A}). \quad (6)$$

The average excitation energy $< E_{\rm I} - -E_{\rm A} >_{\rm Av}$ over all intermediate states has been defined and calculated numerically in Refs.[12] and [24]. The sum term can also be easily calculated, giving

$$\sum_I|(\boldsymbol{p})_{\rm IA}|^2\,(E_{\rm I}-E_{\rm A}) = -\frac{\hbar^2}{2}\int {\rm d}^3\boldsymbol{x}\,|\Psi_{\rm A}|^2\,\nabla^2V(\boldsymbol{x}), \quad (7)$$

where $V(x)$ is the potential part of the unperturbed Hamiltonian and $\Psi_{\rm A}$ is wave function of electron in state A. Taking all of the discussions above into account, the observable energy shift can be written as

$$\Delta E_{\rm A}^{(\rm obs)} = -\frac{e^2\hbar}{12\pi^2\varepsilon_0 m_{\rm e}^2c^3}\ln\frac{E_\gamma^{\max}}{< E_{\rm I}-E_{\rm A} >_{\rm Av}} \times\int {\rm d}^3\boldsymbol{x}\,|\Psi_{\rm A}|^2\,\nabla^2V(\boldsymbol{x}). \quad (8)$$

Taking the electron's wavefunction of hydrogen as $\Psi_{\rm A}$ and $V(\boldsymbol{x})$ the Coulomb potential, one obtains $\Delta\nu_{\rm 2s-2p}^{\rm obs} = 1040\,{\rm MHz}$. This result is in excellent agreement (2%) with the experimental value of 1057.85 MHz.

As a conclusion we can estimate the leading order of the energy shift caused by modified gravity using this non-relativistic approximation, as we will do in what follows.

4.2. Energy shift due to a modified Newtonian law of gravity

In this section we estimate the energy shift for the 1s–2s and 2s–2p transitions in the particular case of hydrogen and muonium atoms due to a modified gravitational law at short distances. As mentioned above, the gravitational interaction could be stronger as expected at atomic or subatomic scale due to the presence of LEDs. Because this effect is still much weaker than those due to the electromagnetic interaction, the wavefunction of the electron is unaffected in first order approximation. We can also treat gravity as a small perturbation to the Coulomb potential. Inserting the gravitational potential Eqs.(1) and (2) into Eq.(8), we can write the shift of energy levels caused by modified gravity as

$$\Delta E_{\rm A}^{(\rm obs)} = -\frac{e^2\hbar}{12\pi^2\varepsilon_0 m_{\rm e}^2c^3}\ln\frac{E_\gamma^{\max}}{< E_{\rm I}-E_{\rm A} >_{\rm Av}} \times\int {\rm d}^3\boldsymbol{x}\,|\Psi_{\rm A}|^2\,\nabla^2V_{\rm g}(\boldsymbol{x}). \quad (9)$$

Because of the finite size of the proton we cut off the integration volume at the proton radius, in the hydrogen atom case, that is $r_{\rm c} = r_{\rm p} = 0.875\,{\rm fm}$, where $r_{\rm c}$ is the cut-off distance. In the case of muonium, the electron's wavefunction determined by the electromagnetic interaction, should be valid until, approximately, $10^{-17}\,{\rm m}$. At this distance the corrections for weak interaction become important. We should also remember that the gravitational potential due to LEDs, in the ADD model, should switch off theoretically at distances below the electroweak distance scale $\sim 10^{-18}\,{\rm m}$ $(1\,{\rm TeV}^{-1})$. Taking these into account, we can safely take $r_{\rm c} = 10^{-17}\,{\rm m}$ as the cut-off distance when performing the integration in Eq.(9).

The average excitation energy in the logarithmic factor in Eq.(9), for the 2s state, was numerically calculated by Bethe *et al.*[15] They found that the major contribution (about 97 percent) arose from the continuous spectrum, while the discrete part only contributed less than 3 percent. Bethe and colleagues reported a value of 16.646 ± 0.007Ry, where Ry(13.6 eV) is the ionization energy of the ground state of hydrogen. This value is much smaller than the electron's rest mass (0.511 MeV) and does not contribute appreciably to the logarithmic factor. In this way, we have taken the average excitation energy for the 1s state to be approximately the value numerically calculated in Ref.[15] for the 2s state, that is $< E_{\rm I} - E_{\rm 1s} >_{\rm Av} \sim < E_{\rm I} - E_{\rm 2s} >_{\rm Av}$. This will result in an uncertainty of less than 20 percent in our constraints.

Then by performing the integrations in Eq.(9) we obtain the energy shifts by modified gravity due to LEDs. We show them in Table 2, in which $\rm B_{\it n}$ is defined as $(R_n/\rho_{\rm B})^n$ and $\rho_{\rm B} = 5.29\,{\rm nm}$ is the Bohr radius of hydrogen. H and M stand for hydrogen and muonium, respectively.

Table 2. The energy shift of hydrogen and muonium due to modified gravity.

	state A	$n=3$	$n=4$	$n=5$	$n=6$
H	1s	1.3×10^{-29} B_3 eV	1.0×10^{-24} B_4 eV	7.4×10^{-20} B_5 eV	5.1×10^{-15} B_6 eV
	2s	1.7×10^{-30} B_3 eV	1.3×10^{-25} B_4 eV	9.3×10^{-21} B_5 eV	6.4×10^{-16} B_6 eV
M	1s	6.8×10^{-24} B_3 eV	8×10^{-17} B_4 eV	1×10^{-9} B_5 eV	1.1×10^{-2} B_6 eV
	2s	0.85×10^{-24} B_3 eV	1×10^{-17} B_4 eV	1.3×10^{-10} B_5 eV	1.4×10^{-3} B_6 eV

4.3. Constraints on the size of LEDs from hydrogen atom spectroscopy

Measurements of hydrogen transition frequencies have been improved during the last two decades. By using a laser-cooled caesium atom clock and with several other improvements, Niering *et al*[25] have achieved the most accurate measurement of hydrogen 1s–2s transition frequency. They reported a value of 2466 061 413 187.103 (46) kHz (1.9×10^{-14}), which is commonly used in the current adjustment for fundamental physical constants.[26] For the 2s–2p level splitting, the Lamb shift of hydrogen, a result of 1057851.4 (1.9) kHz was reported by Pal'chikov *et al*[27] in 1985, which is the best precision result up to date. The theoretical calculation is limited by the mean square charge radius of proton.

With these precise experimental data in hand, we can put stringent constraints on LEDs. We have shown these constraints in Table 1, together with the constraints from inverse-square law tests and astrophysical phenomena (SN1987a).

4.4. Constraints on the size of LEDs from muonium atom spectroscopy

The muonium atom, consisting of two simplest leptons, a positive muon and an electron, was first discovered by Hughes in 1960.[28] Muonium spectroscopy has received a great impulse since then. The Doppler-free excitation of 1s–2s muonium transition was achieved at KEK[29] and independently at RAL.[30] The latest experiments at RAL give $\Delta\nu_{1s-2s}(\mathrm{exp}) = 2455528941.0(9.8)$ MHz.[31] While the accuracy of the theoretical value, $\Delta\nu_{1s-2s}(\mathrm{theo}) = 2455528935.4(1.4)$ MHz, is limited mainly by the uncertainty of the muon's rest mass.

Contrary to the highly precise value for 1s–2s transition frequency, the measured value for 2s–2p (Lamb shift) transition frequency is not so accurate. The experiments at LAMPF[32] and at TRIUMPF[33] give 1042(+21)(-23) MHz and 1070(+12)(-15), respectively. Constraints on the size of LEDs estimated from these measurements are also shown in Table 1.

There are several plans for muon research in the future, such as the J-PARC facility in Japan and EURISOL facility in Europe.[34] These plans aim at more accurate values for transition frequencies of muonium, and in this way our constraints will be improved.

5. Conclusion

LEDs have received great attention because of their value in solving the gauge hierarchy problem and lowering other fundamental physical scales. We have estimated the effects of a modified gravitational interaction due to LEDs in the atomic transition of hydrogen and muonium atoms. We have established stringent constraints on the compactification radius of the LEDs based on ADD theoretical model. In particular, for the special cases of three and four spatial LEDs compactified on a torus, our constraints, $R_3 < 10\,\mu$m and $R_4 < 8.2$ nm, are slightly better than the recent constraints, $R_3 < 36\,\mu$m and $R_4 < 62\,\mu$m, established with a torsion balance experiment to test the inverse-square law of gravity at Washington.[8,18] Of course our constraints and the constraints obtained from inverse-square law tests are both not as stringent as those from astrophysics and cosmology which depend on the nature of quantum theory of gravity. The subsequent relativistic consideration and the numerical calculation of average excitation energy are needed.

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