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Generation of multiple-particle cluster state via cavity QED*

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This paper proposes schemes for generating multiple-photon and multiple-atom cluster states, respectively. The schemes are based on the cavity input–output process and atomic or photonic states measurement, and the successful probabilities approach unity in the ideal case. The numerical simulations show that the produced multiple-particle cluster states have high fidelity even if the Lamb–Dicke condition is not satisfied. Some practical imperfections, such as atomic spontaneous emission and output coupling inefficiency, only decrease the success probability but exert no influence on the fidelity of generated multiple-particle cluster states. From the experimental point of view, smaller operation number and lack of need for individual addressing keeps the schemes easy to implement. These schemes may offer a promising approach to the generation of a large-scale cluster state.

Keywords: cluster state, cavity QED
PACC: 0367, 4250

1. Introduction

The entangled state not only provides possibilities to test quantum nonlocality without inequality,[1] but also plays a key role in quantum-information processing, such as quantum cryptography,[2] quantum teleportation,[3] and quantum secret sharing.[4] Recently, a special type of entangled state, the so-called cluster state,[5] has moved into the centre of interest. It has been shown that cluster states can be regarded as a resource for Greenberger–Horne–Zeilinger (GHZ) states and are more immune to decoherence than GHZ states.[6] A new Bell inequality is maximally violated by four-particle cluster state but not violated by the four-qubit GHZ state.[7] More importantly, cluster states have been shown to constitute a universal resource for quantum computation via local, single-qubit projective measurements and feedforward of their outcomes.[8]

It is well known that optics system and cavity QED system are two of the ideal candidates for quantum communication and quantum computation. For the optics system, much effort has been made towards the preparation of photon cluster states encoded in polarization,[9−12] in which two-photon polarization entangled resources are required. All of them take the spontaneous parametric down-conversion as photon entangled resources. However, the down-conversion process becomes exponentially inefficient for generating larger cluster states due to its intrinsically probabilistic nature. In addition the success probability of 'fusion' mechanisms[11,12] to construct the multiple-photon cluster state exponentially decreases with number increase even if the entangled photons have been prepared. Therefore, only small photon cluster states encoded in polarization have been experimentally implemented.[13−15] We note that Su *et al* have recently presented the first experimental result generating continuous variable quadripartite cluster state of electromagnetic fields.[16] For the cavity QED system, Zou *et al* proposed probabilistic schemes for generating cluster states of four distant trapped atoms in leaky cavities.[17] Dong *et al* proposed a protocol for the generation of cluster states through resonant interactions between two atoms and a single-mode cavity,[18] and so on.[19] However, high-speed realization of cluster states on demand still remains challenge in experiment.

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Very recently, based on the input–output relation of the cavity, Duan and Kimble proposed a new interesting scheme to realize quantum controlled phase flip (CPF) gate between two single-photon pulses.[20] Lin *et al* proposed schemes for robustly generating a multiple-atom graph state and a n-photon GHZ state.[21] Cho *et al* proposed a scheme for preparing an atomic cluster state via performing the CPF gate between every neighbouring qubit.[22] In this paper, we show that with the similar unitary operations the multiple-photon and the multiple-atom cluster states can be prepared. Compared with previous protocol,[9−15] the successful probability for generating a multiple-photon cluster state approaches unity in the ideal case and there is no need of two-photon polarization entangled resources. Contrasted with Ref.[18], our scheme does not require that the atoms are localized in the Lamb–Dicke regime. Some practical imperfections, such as atomic spontaneous emission and output coupling inefficiency, only decrease the success probability but exert no influence on the fidelity of prepared cluster state. Less operation number and no need for individually addressing keep the schemes easy to experimentally implement.

2. The fundamental model and theoretical analysis

The basic component concerned in our proposal is a single three-level atom trapped in a one-sided optical cavity, as shown in Fig.1. The relatively atomic levels and transition are illustrated in Fig.2, where the states $|0\rangle$ and $|1\rangle$ are two different ground-state hyperfine levels while $|e\rangle$ is an excited state, and the transition from $|1\rangle$ to $|e\rangle$ is resonantly coupled to the cavity mode $a_{\rm h}$ with horizontal (h) polarization. According to quantum Langevin equation, we have,[20−27]

$$\dot{a}_{\rm h}(t) = -{\rm i}\left[a_{\rm h}(t), H_{\rm I}\right] - \left({\rm i}\Delta + \frac{\kappa}{2}\right) a_{\rm h}(t) - \sqrt{\kappa}\Xi_{\rm h}^{\rm in}(t), \quad (1)$$

and

$$\Xi_{\rm h}^{\rm out}(t) = \Xi_{\rm h}^{\rm in}(t) + \sqrt{\kappa} a_{\rm h}(t), \quad (2)$$

where the Hamiltonian $H_{\rm I} = g(a_{\rm h}|e\rangle\langle 1| + a_{\rm h}^{+}|1\rangle\langle e|)$ characterizes the interaction between the atom and the cavity mode $a_{\rm h}$, κ is the cavity decay rate, Δ denotes the detuning of the cavity field mode $a_{\rm h}$ from the atomic transition, g represents the coupling rate of the atom to the cavity field, and $\Xi_{\rm h}^{\rm in}(t)$ ($\Xi_{\rm h}^{\rm out}(t)$) depicts the cavity input (output) operator. In the case of the input single-photon pulse duration $T \gg 1/\kappa$, we obtain

$$\Xi_{\rm h}^{\rm out}(t) \approx \frac{{\rm i}\Delta - \kappa/2}{{\rm i}\Delta + \kappa/2}\Xi_{\rm h}^{\rm in}(t). \quad (3)$$

When the atom is in the state $|0\rangle$, the Hamiltonian $H_{\rm I}$ does not operate and results in $\Delta = 0$. So we have $\Xi_{\rm h}^{\rm out}(t) \approx -\Xi_{\rm h}^{\rm in}(t)$. However, when the atom is in the state $|1\rangle$, for the case of $g \gg \left(\frac{1}{T}, \kappa, \gamma\right)$,[28] (here γ is the atomic spontaneous rate), the effective detunings of two dressed cavity modes from the input pulse are $\Delta = \pm g$, respectively, so that we have $\Xi_{\rm h}^{\rm out}(t) \approx \Xi_{\rm h}^{\rm in}(t)$. In practice, the result is still true even if $g \sim \kappa$.[20−26] In a word, after the photon pulse is reflected by the cavity, a CPF operation $U = \exp({\rm i}\pi|0\rangle\langle 0| \otimes |h\rangle\langle h|)$ is achieved between the atom and the photon pulse.

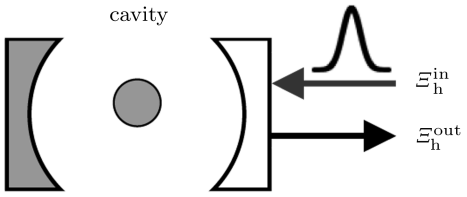


Fig.1. Schematic setup to reflect a single-photon pulse from one-sided cavity with a trapped atom.

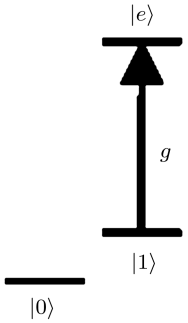


Fig.2. The relevant atomic level structure and transitions.

3. Generation of n-photon cluster state

Next we describe in detail how to prepare an n-photon cluster state. As shown in Fig.3, a one-sided optical cavity traps a three-level atom (the relative

atomic levels and transition are shown in Fig.2) with initial state $|\Phi_0\rangle_{\rm a} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$. The polarization beam splitter (PBS) transmits only the h polarization component and reflects the vertical (v) polarization component. When the jth input photon pulse is reflected by the cavity or mirror M, based on the result above, an operation $U_{{\rm a}j} = \exp(\mathrm{i}\pi\,|0\rangle_{\rm a}\langle 0| \otimes |h\rangle_j\langle h|)$ is applied on the atom and the jth input photon pulse. Then the half-wave plate (HWP) performs a Hadamard operation R_j on the jth single-photon pulse: i.e., $R_j\,|h\rangle_j = \frac{1}{\sqrt{2}}\left(|v\rangle_j - |h\rangle_j\right)$ or $R_j\,|v\rangle_j = \frac{1}{\sqrt{2}}\left(|v\rangle_j + |h\rangle_j\right)$. A Hadamard transformation $R_{\rm a}^{(j)}\,|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)$ or $R_{\rm a}^{(j)}\,|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)$ is performed on the atom via Raman pulses.[29] The net effect of these three subprocesses is that the operations $R_{\rm a}^{(j)}R_jU_{{\rm a}j}$ are accomplished on the jth input photon pulse and the atom. Let n single-photon pulses with initial state $|\Omega_0\rangle = \frac{1}{2^{\frac{n}{2}}}\Pi_{j=1}^n(|v\rangle_j + |h\rangle_j)$ undergo successively the three subprocesses above, the state of the system of n photons and the atom evolves into

$$\begin{aligned}|\Phi\rangle &= \Pi_{j=1}^n R_{\rm a}^{(j)}R_jU_{{\rm a}j}\,|\Omega_0\rangle\otimes|\Phi_0\rangle_{\rm a}\\ &\propto |0\rangle\otimes\Pi_{j=1}^n(|h\rangle_j + |v\rangle_j\,\sigma_z^{j+1}) + |1\rangle\\ &\otimes\Pi_{j=1}^{n-1}(|h\rangle_j + |v\rangle_j\,\sigma_z^{j+1})\otimes(|h\rangle_n - |v\rangle_n),\end{aligned} \quad (4)$$

where $\sigma_z^{j+1} = (|h\rangle_{j+1}\langle h| - |v\rangle_{j+1}\langle v|)$ and $\sigma_z^{n+1} = 1$. Finally the atomic state is measured in the bases $\{|0\rangle, |1\rangle\}$. If the measurement outcome is $|0\rangle$, the state of n-photon is projected into

$$|\Omega\rangle \propto \Pi_{j=1}^n(|v\rangle_j + |h\rangle_j\,\sigma_z^{j+1}), \quad (5)$$

which is the n-photon line cluster state.[5] If the measurement outcome is $|1\rangle$, the state of n-photon is projected into

$$|\Omega\rangle' \propto \Pi_{j=1}^{n-1}(|h\rangle_j + |v\rangle_j\,\sigma_z^{j+1})\otimes(|h\rangle_n - |v\rangle_n), \quad (6)$$

which can be changed into the state $|\Omega\rangle$ by performing local operation σ_z on the nth photon pulse.

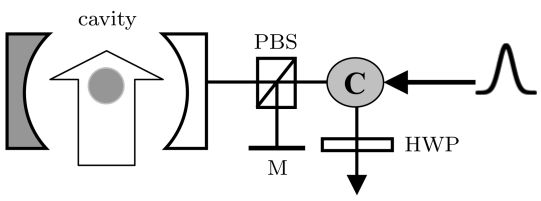


Fig.3. Schematic setup for generation of n-photon cluster state. The wide arrow denotes the classical field. The optical paths from PBS⟶ M and PBS→ Cavity are assumed to be equal.

4. Generation of n-atom cluster state

Now we show how to construct a multiple-atom cluster state. As shown in Fig.4, n one-sided cavities are unidirectionally coupled by circulator (C). n atoms (Fig.2) with initial state $|\phi_0\rangle_{\rm a} = 2^{-\frac{n}{2}}\Pi_{i=1}^n(|0\rangle_i + |1\rangle_i)$ are trapped in n cavities, where the subscript i denotes the atom i trapped in the ith cavity. The HWP$_i$ between the ith cavity and the $(i+1)$th cavity performs a Hadamard operation on the single-photon pulse, i.e., $R_i\,|h\rangle = \frac{1}{\sqrt{2}}(|v\rangle - |h\rangle)$ or $R_i\,|v\rangle = \frac{1}{\sqrt{2}}(|v\rangle + |h\rangle)$. An operation $U_i = \exp(\mathrm{i}\pi\,|0\rangle_i\langle 0|\otimes|h\rangle\langle h|)$ is applied on the atom i and the single-photon pulse after the pulse is reflected by the ith cavity. Let a single-photon pulse with initial state $|\psi_0\rangle = \frac{1}{\sqrt{2}}(|v\rangle + |h\rangle)$ be successively reflected by n cavities and performed n Hadamard operations. The state of the system of n atoms and the single-photon pulse evolves into

$$\begin{aligned}|\Psi\rangle &= \Pi_{i=1}^n R_iU_i\,|\psi_0\rangle\otimes|\phi_0\rangle_{\rm a}\\ &\propto |v\rangle\otimes\Pi_{i=1}^n(|+\rangle_i + |-\rangle_i\,\sigma_z^{i+1})\\ &+ |h\rangle\otimes\Pi_{i=1}^{n-1}(|+\rangle_i + |-\rangle_i\,\sigma_z^{i+1})\\ &\otimes(|+\rangle_n - |-\rangle_n),\end{aligned} \quad (7)$$

where $|\pm\rangle = (|0\rangle \pm |1\rangle)$, $\sigma_z^{i+1} = (|+\rangle_{i+1}\langle +| - |-\rangle_{i+1}\langle -|)$ and $\sigma_z^{n+1} = 1$. Then the single-photon pulse passes through a PBS, and make a click on the single-photon detectors D_v or D_h. If the detector D_v is triggered, the atomic state is collided into

$$|\phi\rangle \propto \Pi_{i=1}^n(|+\rangle_i + |-\rangle_i\,\sigma_z^{i+1}), \quad (8)$$

which is the n-atom line cluster state.[5] If the detector D_h is triggered, the atomic state is collided into

$$|\phi\rangle' \propto \Pi_{i=1}^{n-1}(|+\rangle_i + |-\rangle_i\,\sigma_z^{i+1})\otimes(|+\rangle_n - |-\rangle_n), \quad (9)$$

which can be changed into the state $|\phi\rangle$ by performing local operation σ_z on the nth atom. If the qubit is encoded with the bases $\{|0\rangle, |1\rangle\}$, the corresponding cluster state can be achieved by applying a filter operation $\prod\limits_{i=1,2,3,\cdots,n}|0\rangle_i\langle +| + |1\rangle_i\langle -|$ on each atom via Raman pulses.[29] We note that Cho and Lee have proposed a scheme for robust generation of atomic cluster states via performing controlled-phase operations between every neighbouring qubit,[22] in which to obtain n-atom cluster state need n-1 times measurement on photonic state at least. Here we need only once measurement on photonic state. The decrease of operation number is very important on account of current experimental technology.

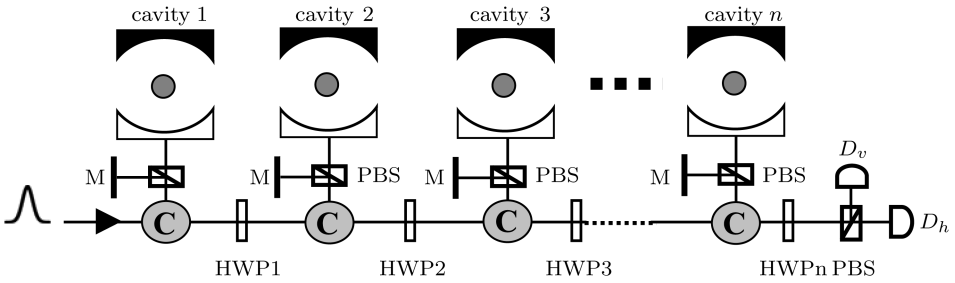


Fig.4. Schematic setup for generation of n-atom cluster state. $D_{h(v)}$ denotes the single-photon detector.

5. Numerical simulations and brief discussion

In the following, we quantify the quality of the four-particle cluster state through the numerical simulation method. Assuming that the input single-photon pulse is a Gaussian pulse $f(t) \propto \exp\left[-\left(t-\frac{T}{2}\right)^2 \Big/ \left(\frac{T}{5}\right)^2\right]$ where T is the pulse duration, we take the practical parameter in Ref.[30], i.e. $g_0 = 4\kappa$, $(\kappa, \gamma)/2\pi = (8.2, 5.2)\,\mathrm{MHz}$. Numerical simulation results show that the fidelity of the four-particle cluster state is very high when $T \gg 1/\kappa$. Figure 5 shows the fidelity F associated with different pulse duration T. In our estimations, the fidelity F is up to 0.9979 for $T = 5\,\mu\mathrm{s}$. The simulation also shows that the variation of the fidelity is smaller than 1×10^{-6} when the coupling rate g varying from g_0 to $g_0/2$, which means that the schemes are insensitive to randomness in the coupling rates caused by fluctuations of atomic position. The other dominant noise arises from photon loss due to atomic spontaneous emission, which leads to a vacuum-state output when the input is a single-photon pulse and yields a the leakage error. Figure 6 shows that the probability P_e of the leakage error for generation of four-particle cluster state is well simulated by empirical formula $P_\mathrm{e} \approx 1/(1+79.2g^2/\gamma\kappa)$. In the scheme for generating multiple-photon cluster state, if each photon pulse is registered through quantum nondemolition measurement, the photon loss due to atomic spontaneous emission and output coupling inefficiency has no influence on the fidelity of generated cluster state. Similarly, in the scheme for generating a multiple-atom cluster state the photon loss only affects the successful probability but has no influence on fidelity of the state since single-photon detection will never occur when the photon is lost.

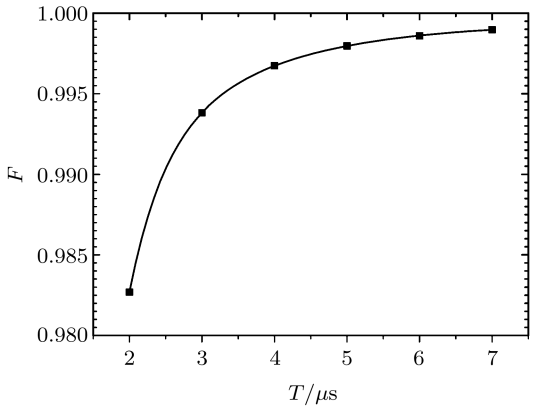


Fig.5. The fidelity of the four-particle cluster state for different pulse duration T, assuming $g_0 = 4\kappa$ and $(\kappa, \gamma)/2\pi = (8.2, 5.2)$MHz.

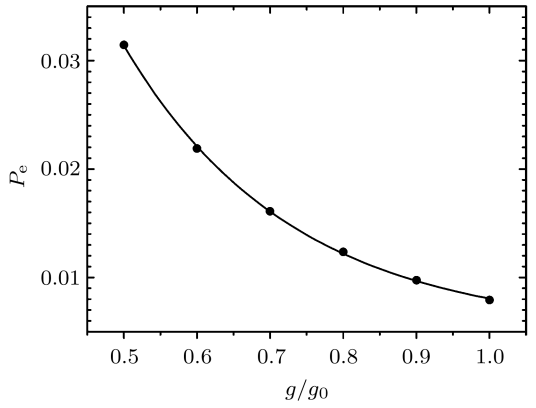


Fig.6. The probability P_e of photon loss as a function of g/g_0. The dots denote the results of numerical simulation, and the solid curve describes the empirical formula $P_\mathrm{e} \approx 1/(1+79.2g^2/\gamma\kappa)$. Here, we have taken $(\kappa, \gamma)/2\pi = (8.2, 5.2)$MHz, $g_0 = 4\kappa$, and $T = 5\,\mu\mathrm{s}$.

Next we briefly address the experiment feasibility of the proposed schemes. We consider a Cs atom trapped in a one-sided Fabry–Perot cavity.[30] The

states $|0\rangle$ and $|1\rangle$ correspond to $|F=3,m=3\rangle$ and $|F=4,m=4\rangle$ of $6S_{1/2}$, respectively, while $|e\rangle$ corresponds to $|F=5,m=5\rangle$ of $6P_{3/2}$. The transition from $|1\rangle$ to $|e\rangle$ is resonantly coupled to a single TEM_{00} mode of the cavity. The Cs hyperfine splitting between the $F=3$ and $F=4$ manifolds is $\delta \sim 9.2\,\mathrm{GHz} \gg g_0$, thus the state $|0\rangle$ is not affected by the TEM_{00} mode of the cavity. For photonic qubits are initially assumed to encode in h and v polarizations in above schemes, we need a QWP inserted between the PBSs and cavities in Figs.3 and 4 to perform the transformation between h polarized photons and right-circularly (R) polarized photons. Trapping time for single Cs atom is about several milliseconds,[31] which is much larger than the duration of the single-photon pulse $T \sim 5\mu$s. Of course, our schemes require a perfect single photon source. Remarkably, there have been significant experimental advances in the realization of the single-photon source using a variety of physical systems including parametric down conversion,[32] single quantum dots,[33] and high-Q cavities.[34] Thus our schemes may be implemented in the near future.

6. Conclusion

In summary, we propose schemes for generating multiple-photon and multiple-atom cluster states, respectively. The numerical simulations show that the produced multiple-particle cluster state has high fidelity even if the atoms are not localized in the Lamb–Dicke regime. Some practical imperfections, such as atomic spontaneous emission and output coupling inefficiency, only decrease the success probability but exert no influence on the fidelity of prepared cluster state. The successful probability of our protocol approaches unity in the ideal case. In addition, the decrease of operation number and lack of need for individual addressing keeps the schemes easy to implement from the experimental point of view. Hence, our schemes may offer a promising approach to generation of large-scale cluster state.

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