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Total teleportation of zero- and one-photon entangled states in running waves*

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Inspired by a recent paper [2002 *J. Opt.* B **4** 316] we present an alternative scheme to teleport an entanglement of zero- and one-photon states of a running-wave field. The scheme employs only linear optical elements plus single-photon sources and detectors.

Keywords: total teleportation; entangled states; linear optical elements
PACC: 0367, 0365, 4250

Quantum entanglement plays an important role in various fields of quantum information, such as quantum computation,[1−3] quantum teleportation,[4] dense coding,[5] quantum communication,[6−9] etc. Quantum teleportation was experimentally reported by Bouwmeester *et al*,[10] Boschi *et al*[11] and Lombardi *et al*,[12] all of them used entangled pairs of photon states created from parametric down-conversion. Also, other experiments showing teleportation were reported by Furusawa *et al*[13] and Takei *et al*,[14] both used continuous variables; Fattal *et al*[15] used a quantum dot single-photon source; Riebe *et al*[16] and Barrett *et al*[17] used states of trapped ions. An increasing number of protocols have been suggested to implement the process in different systems, such as running waves[18−22] and cavity QED.[23−25]

Single particle entanglement is useful as a resource for fundamentals in quantum mechanics and applications in quantum teleportation. The remarkable ingredient in quantum teleportation is the transfer of information between non-interacting systems by using a quantum channel. To implement teleportation between two distinct points, usually named Alice and Bob, one should i) prepare a state to be teleported; ii) prepare a nonlocal quantum channel; iii) make a Bell measurement (Alice); iv) communicate (to Bob) the measured result by a classical channel. Among these steps, the main experimental challenge consists in the so-called Bell state measurement,[4] performed on the Bell operator basis for the particle (or field), whose state is to be teleported, plus its partner composing the quantum channel. The same procedure is employed when the issue is concerned with the teleportation of entangled states, with major interest in quantum information processing.[26−28]

Teleportation of entangled states does not mean necessarily the total teleportation of the state of a system.[20] In Ref.[20] the authors presented a scheme for partial quantum teleportation based on single-photon entangled states, by using optical devices. The total teleportation of a single-particle entangled state was proposed by Lee,[21] whose scheme consisted of a parametric down-conversion source emitting a pair of entangled photon states, and a dual Bell measurement system by using two beam splitters and two pairs of detectors. The feature characterizing the total teleportation is the fact that the entirely entangled state is transferred directly from one place (sender, Alice) to another (receiver, Bob).

A recent paper by Serra *et al*[22] has shown how to teleport an entanglement of zero- and one-photon states in a running-wave field. In this case the scheme basically consists of symmetric (50/50) beam splitters (BS), nonlinear (Kerr) media, and photodetectors. The main difficulty arises from the necessity of using nonlinear optical elements to prepare the nonlocal channel as well as to perform the Bell state measurement. Actually, despite the promising proposals in literature,[29−31] the perfect discrimination of Bell-states through nonlinear effects in running waves, re-

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mains a challenge in practice. Here we present an alternative scheme to teleport an entanglement of zero- and one-photon states in running-wave fields. In a way different from that in Ref.[22], we employ only linear optical elements, which makes our scheme feasible with the current technology. With respect to Ref.[20], instead of the partial teleportation, the distinct feature of our scheme lies in the fact that our proposal is concerned with the total teleportation, with no direct bonding between Alice and Bob. As depicted in Fig.1, our scheme employs a single BS with unknown transmittance and reflectance coefficients, necessary to prepare the state to be teleported, four additional symmetric BS, three single-photon sources, and four photodetectors. Schemes to teleport entangled photon states by using the linear optical elements were proposed recently by Cao and Song[32] and Song and Zhang.[33] Different from the present proposal, the quantum channel in Ref.[33] was composed of a tripartite (GHZ) states and in Ref.[32] of a general class of W states involving photon number states, which, as far as we know, suffers from experimental demonstrations yet. Also, their schemes employ additional auxiliary states to complete the teleportation process of entangled photon states, i.e. polarization[33] and photon number states,[32] thus requiring additional quantum resources. Experimentally, the teleportation of entanglements of photon states was recently reported by Lombardi *et al* in Ref.[12], where a source of parametric down conversion was used to prepare the nonlocal channel, requiring a sample of coincidence measurement that combined zero- and one-photon states at pairs of detectors.

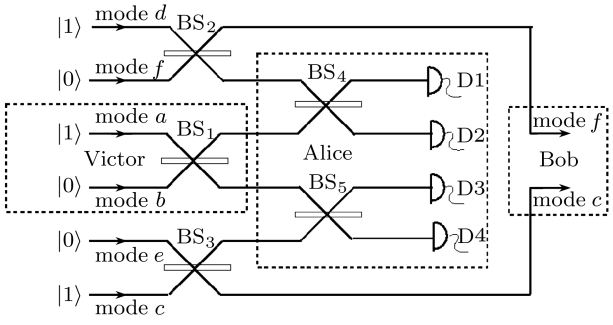


Fig.1. Scheme of the experimental setup required for the total teleportation of an entanglement of zero- and one-photon states.

As shown in Fig.1, Victor prepares in modes a and b the state to be teleported. Then, Alice receives from Victor field-modes a and b in an unknown entangled state. The nonlocal channel shared by Alice and Bob is composed of field-modes c, d, e, and f. While field-modes c and d are given to Alice, the other field-modes e and f are given to Bob. Then, after she has made a measurement on particles a, b, c, and d, Alice sends two bits of classical information to Bob for communicating her result. According to the present protocol, depending on the measurements by Alice, Bob is required to perform only one of three possible unitary operations on modes e and f, in order to complete the teleportation. As Bob cannot discriminate among the four Bell states, the success probability of our teleportation scheme is limited to 25%. On the other hand, in the ideal case, i.e. ignoring decoherence effects such as those stemming from photo-absorption, the fidelity is 100%.

The BS appearing in our scheme are represented by the unitary operator, $\hat{U}_{\rm BS} = \exp\left[{\rm i}\theta\left(\widehat{a}^{\dagger}_{\rm in}\widehat{b}_{\rm in} + \widehat{a}_{\rm in}\widehat{b}^{\dagger}_{\rm in}\right)\right]$, leading to the following relationships between the input and output operators, $\widehat{a}$ and $\widehat{b}$:

$$\widehat{a}_{\rm out} = T\widehat{a}_{\rm in} + R\widehat{b}_{\rm in}, \tag{1}$$

$$\widehat{b}_{\rm out} = T\widehat{b}_{\rm in} + R\widehat{a}_{\rm in}, \tag{2}$$

where $T = \cos(\theta)$ and $R = {\rm i}\sin(\theta)$ are the BS transmission and reflection coefficients, satisfying $|T|^2 + |R|^2 = 1$. For a 50/50 symmetric BS, $\theta = \pi/4$, and $T = |R| = 1/\sqrt{2}$. These coefficients, and thus the operators, depend on the field frequency, and here a monochromatic source is assumed. While Victor uses the $\rm BS_1$ of unknown coefficients T and R to prepare a state to be teleported

$$|\psi\rangle_{\rm in} = C_0\,|1\rangle_a\,|0\rangle_b + C_1\,|0\rangle_a\,|1\rangle_b\,, \tag{3}$$

the non-local channel is prepared in $\rm BS_2$ and $\rm BS_3$. After exiting from the $\rm BS_2$ and $\rm BS_3$, the modes d, f and c, e are in the entangled state $|\varphi\rangle_1 = 1/\sqrt{2}(|1\rangle_d\,|0\rangle_f + {\rm i}\,|0\rangle_d\,|1\rangle_f)$ and $|\varphi\rangle_2 = 1/\sqrt{2}\,(|1\rangle_c\,|0\rangle_e + {\rm i}\,|0\rangle_c\,|1\rangle_e)$, respectively. Note that the nonlocal channel formed by modes d, f, c, and e can be written as a state product $|\varphi\rangle_1 \otimes |\varphi\rangle_2$ for the entangled modes d, f and c, e separately. The nonlocality is exploited from state $|\varphi\rangle_1$ by sending particle f to Bob and particle d to Alice and, from state $|\varphi\rangle_2$, by sending particle e to Bob and particle c to Alice. The particles e and f, initially uncorrelated, become entangled after the joint measurement, thus swapping the entanglement with particles c and d, with the coefficients given by the state prepared in the modes a, b.

The whole state after the first three BS results in the form $|\phi\rangle_1 = |\psi\rangle_{\rm in} \otimes |\varphi\rangle_1 \otimes |\varphi\rangle_2$, leading to

$$
\begin{aligned}
|\phi\rangle_1 = \Big\{ & \frac{C_0}{2}\Big[|1\rangle_a|0\rangle_b|1\rangle_c|1\rangle_d|0\rangle_e|0\rangle_f + \mathrm{i}\,|1\rangle_a|0\rangle_b|0\rangle_c|1\rangle_d|1\rangle_e|0\rangle_f + \mathrm{i}\,|1\rangle_a|0\rangle_b|1\rangle_c|0\rangle_d|0\rangle_e|1\rangle_f \\
& - |1\rangle_a|0\rangle_b|0\rangle_c|0\rangle_d|1\rangle_e|1\rangle_f\Big] + \frac{C_1}{2}\Big[|0\rangle_a|1\rangle_b|1\rangle_c|1\rangle_d|0\rangle_e|0\rangle_f + \mathrm{i}\,|0\rangle_a|1\rangle_b|0\rangle_c|1\rangle_d|1\rangle_e|0\rangle_f \\
& + \mathrm{i}\,|0\rangle_a|1\rangle_b|1\rangle_c|0\rangle_d|0\rangle_e|1\rangle_f - |0\rangle_a|1\rangle_b|0\rangle_c|0\rangle_d|1\rangle_e|1\rangle_f\Big]\Big\}. \qquad (4)
\end{aligned}
$$

The BS_4 entangles modes b and d, yielding the output state in the form

$$
\begin{aligned}
|\phi\rangle_2 = & \frac{C_0}{2}\Big(\frac{1}{\sqrt{2}}|1\rangle_a|0\rangle_b|1\rangle_c|1\rangle_d|0\rangle_e|0\rangle_f + \frac{\mathrm{i}}{\sqrt{2}}|1\rangle_a|1\rangle_b|1\rangle_c|0\rangle_d|0\rangle_e|0\rangle_f + \frac{\mathrm{i}}{\sqrt{2}}|1\rangle_a|0\rangle_b|0\rangle_c|1\rangle_d|1\rangle_e|0\rangle_f \\
& - \frac{1}{\sqrt{2}}|1\rangle_a|1\rangle_b|0\rangle_c|0\rangle_d|1\rangle_e|0\rangle_f + \mathrm{i}\,|1\rangle_a|0\rangle_b|1\rangle_c|0\rangle_d|0\rangle_e|1\rangle_f - |1\rangle_a|0\rangle_b|0\rangle_c|0\rangle_d|1\rangle_e|1\rangle_f\Big) \\
& + \frac{C_1}{2}\Big(\frac{\mathrm{i}}{\sqrt{2}}|0\rangle_a|2\rangle_b|1\rangle_c|0\rangle_d|0\rangle_e|0\rangle_f + \frac{\mathrm{i}}{\sqrt{2}}|0\rangle_a|0\rangle_b|1\rangle_c|2\rangle_d|0\rangle_e|0\rangle_f - \frac{1}{\sqrt{2}}|0\rangle_a|2\rangle_b|0\rangle_c|0\rangle_d|1\rangle_e|0\rangle_f \\
& - \frac{1}{\sqrt{2}}|0\rangle_a|0\rangle_b|0\rangle_c|2\rangle_d|1\rangle_e|0\rangle_f + \frac{\mathrm{i}}{\sqrt{2}}|0\rangle_a|1\rangle_b|1\rangle_c|0\rangle_d|0\rangle_e|1\rangle_f - \frac{1}{\sqrt{2}}|0\rangle_a|0\rangle_b|1\rangle_c|1\rangle_d|0\rangle_e|1\rangle_f \\
& - \frac{1}{\sqrt{2}}|0\rangle_a|1\rangle_b|0\rangle_c|0\rangle_d|1\rangle_e|1\rangle_f - \frac{\mathrm{i}}{\sqrt{2}}|0\rangle_a|0\rangle_b|0\rangle_c|1\rangle_d|1\rangle_e|1\rangle_f\Big). \qquad (5)
\end{aligned}
$$

The same is done by the BS_5 on modes a and c, and the whole output state results in

$$
\begin{aligned}
|\phi\rangle_3 = & \frac{C_0}{2}\Big(\frac{\mathrm{i}}{2}|2\rangle_a|0\rangle_b|0\rangle_c|1\rangle_d|0\rangle_e|0\rangle_f + \frac{\mathrm{i}}{2}|0\rangle_a|0\rangle_b|2\rangle_c|1\rangle_d|0\rangle_e|0\rangle_f - \frac{1}{2}|2\rangle_a|1\rangle_b|0\rangle_c|0\rangle_d|0\rangle_e|0\rangle_f \\
& - \frac{1}{2}|0\rangle_a|1\rangle_b|2\rangle_c|0\rangle_d|0\rangle_e|0\rangle_f + \frac{\mathrm{i}}{2}|1\rangle_a|0\rangle_b|0\rangle_c|1\rangle_d|1\rangle_e|0\rangle_f - \frac{1}{2}|0\rangle_a|0\rangle_b|1\rangle_c|1\rangle_d|1\rangle_e|0\rangle_f \\
& - \frac{1}{2}|1\rangle_a|1\rangle_b|0\rangle_c|0\rangle_d|1\rangle_e|0\rangle_f - \frac{\mathrm{i}}{2}|0\rangle_a|1\rangle_b|1\rangle_c|0\rangle_d|1\rangle_e|0\rangle_f - \frac{1}{\sqrt{2}}|2\rangle_a|0\rangle_b|0\rangle_c|0\rangle_d|0\rangle_e|1\rangle_f \\
& - \frac{1}{\sqrt{2}}|0\rangle_a|0\rangle_b|2\rangle_c|0\rangle_d|0\rangle_e|1\rangle_f - \frac{1}{\sqrt{2}}|1\rangle_a|0\rangle_b|0\rangle_c|0\rangle_d|1\rangle_e|1\rangle_f - \frac{\mathrm{i}}{\sqrt{2}}|0\rangle_a|0\rangle_b|1\rangle_c|0\rangle_d|1\rangle_e|1\rangle_f\Big) \\
& + \frac{C_1}{2}\Big(\frac{\mathrm{i}}{2}|0\rangle_a|2\rangle_b|1\rangle_c|0\rangle_d|0\rangle_e|0\rangle_f - \frac{1}{2}|1\rangle_a|2\rangle_b|0\rangle_c|0\rangle_d|0\rangle_e|0\rangle_f + \frac{\mathrm{i}}{2}|0\rangle_a|0\rangle_b|1\rangle_c|2\rangle_d|0\rangle_e|1\rangle_f \\
& - \frac{1}{2}|1\rangle_a|0\rangle_b|0\rangle_c|2\rangle_d|0\rangle_e|0\rangle_f - \frac{1}{\sqrt{2}}|0\rangle_a|2\rangle_b|0\rangle_c|0\rangle_d|1\rangle_e|0\rangle_f - \frac{1}{\sqrt{2}}|0\rangle_a|0\rangle_b|0\rangle_c|2\rangle_d|1\rangle_e|0\rangle_f \\
& + \frac{\mathrm{i}}{2}|0\rangle_a|1\rangle_b|1\rangle_c|0\rangle_d|0\rangle_e|1\rangle_f - \frac{1}{2}|1\rangle_a|1\rangle_b|0\rangle_c|0\rangle_d|0\rangle_e|1\rangle_f - \frac{1}{2}|0\rangle_a|0\rangle_b|1\rangle_c|1\rangle_d|0\rangle_e|1\rangle_f \\
& - \frac{\mathrm{i}}{2}|1\rangle_a|0\rangle_b|0\rangle_c|1\rangle_d|0\rangle_e|1\rangle_f - \frac{1}{\sqrt{2}}|0\rangle_a|1\rangle_b|0\rangle_c|0\rangle_d|1\rangle_e|1\rangle_f - \frac{\mathrm{i}}{\sqrt{2}}|0\rangle_a|0\rangle_b|0\rangle_c|1\rangle_d|1\rangle_e|1\rangle_f\Big). \qquad (6)
\end{aligned}
$$

Thus, after the photon detection in modes a, b, c, and d we have the following alternatives:

i) If Alice detects the photon states $|0\rangle_a|0\rangle_b|1\rangle_c|1\rangle_d$ or $|1\rangle_a|1\rangle_b|0\rangle_c|0\rangle_d$, Bob receives the teleported state,

$$|\psi\rangle_{\rm out} = C_0\,|1\rangle_e|0\rangle_f + C_1\,|0\rangle_e|1\rangle_f\,. \qquad (7)$$

Then, Alice sends Bob a message informing that she has received the wanted state $|\psi\rangle_{\rm in}$. The success probability for this case is $P_1 = P_{0011} + P_{1100} = 1/8$, where P_{abcd} represents the detection probability of 'a' photons in mode a; 'b' photons in mode b; etc.

ii) If Alice detects the photon states $|1\rangle_a|0\rangle_b|0\rangle_c|1\rangle_d$ or $|0\rangle_a|1\rangle_b|1\rangle_c|0\rangle_d$, then the state received by Bob is

$$|\psi\rangle_{\rm out} = C_0\,|1\rangle_e|0\rangle_f - C_1\,|0\rangle_e|1\rangle_f\,, \qquad (8)$$

which no longer coincides with the state $|\psi\rangle_{\rm in}$ to be teleported. Then Alice sends Bob a message informing him that she has to apply a phase shift π in order to change the relative phase of the state $|0\rangle_e|1\rangle_f$ with respect to the state $|1\rangle_e|0\rangle_f$, thus recovering the initial state $|\psi\rangle_{\rm in}$. The success probability is again $P_2 = P_{1001} + P_{0110} = 1/8$.

iii) If Alice detects any result other than those in (i) and (ii), she sends Bob a message informing him to neglect the state in his hands.

Note that expression (6) exhibits terms with two photons. Therefore, the detectors should be able to discriminate between the detected one and two photons in the same mode, a point presenting difficulties with detectors yet available. Proposals circumventing this difficulty can be found in Ref.[34].

In conclusion, we present a proposal to teleport an entanglement of zero- and one-photon states with fidelity 100% and success probability 25%. Our proposal simplifies that in Refs.[22, 32, 33] by neglecting the use of nonlinear optical elements and the GHZ-state (or W-state) as a quantum channel. As in Refs.[32, 33] we have disregarded any loss of information during the whole process of teleportation by considering the BS and photodetectors to be ideal.

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