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A quantum dense coding implementation in an ion trap*

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This paper reports that a quantum dense coding can be implemented with ions confined in a linear trap and interacting with laser beams. The scheme is insensitive to the interaction between the quantum channel and the environment. The Bell-state measurement is not involved and the probability of success in our scheme is 1.0.

Keywords: ion trap, dense coding, entanglement swapping
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Quantum dense coding, first proposed by Bennett and Wiesner,[1] is one of the quantum information processing protocols[2] that is superior to classical ones. Classically, the capacity of a single bit through the classical channel cannot exceed one bit. However, if we use a pair of entangled qubits between the sender (Alice) and the receiver (Bob), then we can send two bits of classical information from Alice to Bob.[1] For this protocol, Bell-basis states

$$\begin{aligned} |\Psi^+\rangle &= (|00\rangle + |11\rangle)/\sqrt{2}, \\ |\Psi^-\rangle &= (|00\rangle - |11\rangle)/\sqrt{2}, \\ |\Phi^+\rangle &= (|01\rangle + |10\rangle)/\sqrt{2}, \\ |\Phi^-\rangle &= (|01\rangle + |10\rangle)/\sqrt{2}, \end{aligned} \tag{1}$$

are used in dense coding. Bell-basis states are in the Hilbert space of two dimensions, and they are the maximally entangled states. Suppose that Alice and Bob share the maximally entangled state $|\Psi^+\rangle$. Alice then performs one of the four 1-qubit unitary operations given by the identity I or the Pauli matrices $(\sigma^x, \mathrm{i}\sigma^y, \sigma^z)$ on her qubit. Each of four unitary operations maps the Bell state $|\Psi^+\rangle$ to a different member of the four Bell states, i.e., $|\Psi^+\rangle$, $|\Phi^+\rangle$, $|\Phi^-\rangle$ and $|\Psi^-\rangle$, respectively. Then Alice sends his particle back to Bob. Because the four manipulations result in four orthogonal Bell states, four distinguishable messages, i.e., 2 bits of information, can be obtained by Bob via collective measurement on the two particles.

Theoretically, the original dense coding protocol has been generalized by other authors in various directions, such as the case of the continuous variables[3] and multipartite communication.[4] In Ref.[5] the authors generalized the protocol to transmit faithfully $2\log_2 d$ bits of classical information, making use of a maximally entangled state in a d-level quantum system. In Refs.[6] the schemes extended quantum dense coding to the multiqubit entanglement with N pairwise qubits entangled and with maximally entangled N qubits shared between Alice and Bob. Also as in Refs.[7, 8] schemes for quantum dense coding have proposed by using GHZ state and the nonmaximally two-particle entangled state. Two particles quantum dense coding has been experimentally demonstrated by using optical systems and techniques of nuclear magnetic resonance.[9,10] However, the ion trap is also good system for quantum-information processing.[11] Other researchers, Riebe *et al*[12] and Barrett *et al*[13] have implemented quantum teleportation of atomic qubits by using Ca^+ ions in a linear Paul trap and using Be^+ ions in a segmented ion trap, respectively. It shows that the techniques of manipulating and transforming quantum states have made a striking progress in ion trap.

In this article, we propose an alternative implementation of quantum dense coding in an ion trap. Our contribution is twofold. First, we address each ion with a single laser with large detuning from the sideband, the internal state transition is insensitive to the vibrational quantum number n. It is insensitive to the interaction between the quantum channel and the environment. Second, four Bell-state measurements can be exactly discriminated only by one step of detecting ions 1 and 2, i.e., without Bell-state measurement

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(BSM), with probability of success 1. In fact, maximally entangled states are very difficult to prepare and store in practical applications, but it is essentially one for quantum dense coding as a quantum channel. To avoid this problem, an alternative method is adopted to generate entanglement between two distant particles that have never interacted before in our scheme. That is the so-called entanglement swapping.[14]

We consider a collection of two identical two-level ions in a linear trap driven by a different laser field separately. We can neglect all other vibrational modes and concentrate on one collective degree of vibrational excitation of the ions by tuning the lasers sufficiently close to the sidebands. In this case, our system can be described by the following Hamiltonian:[15]

$$H_{\rm int} = \hbar\nu\left(a^+a+\frac{1}{2}\right)+\hbar\omega_{\rm eg}\sum_i\sigma_i^z/2 + \sum_i\frac{\hbar\Omega_i}{2}(\sigma_i^+{\rm e}^{{\rm i}[\eta_i(a+a^+)-\omega_i t]}+{\rm H.c.}), \quad (2)$$

here $a^+(a)$ denotes the creation (annihilation) operator for the centre-of-mass mode of the collective motion of the two ions (frequency ν), $\hbar\omega_{eg}$ is the energy difference between the internal states $|e\rangle$ and $|g\rangle$. $\sigma_i^z=(|e\rangle_i\langle e|-|g\rangle_i\langle g|)$, $\sigma_i^+=|e\rangle_i\langle g|$ and $\sigma_i^-=|g\rangle_i\langle e|$ represent the internal states degrees of freedom for the ith ion, Ω_i is Rabi frequency of the laser field addressing the ith ion, and η_i is Lamb–Dicke parameter. For the laser addressing the first ion, we choose a detuning close to the first blue sideband, i.e., $\omega_1=\omega_{eg}+\delta$, and for the laser addressing the second ion, we choose the detuning to be the negative of the detuning of the first laser, i.e., $\omega_1=\omega_{eg}-\delta$.

The Rabi frequency Ω can be determined in the second order perturbation theory,[15,16]

$$\widetilde{\Omega}=-\frac{(\Omega\eta)^2}{2(\nu-\delta)}, \quad (3)$$

where $\delta=\omega_1-\omega_{eg}$ is the detuning of the laser addressing the first ion. The evolution of two ions system for different initial states will be described by

$$|gg\rangle\rightarrow\cos\left(\frac{\widetilde{\Omega}t}{2}\right)|gg\rangle+{\rm i}\sin\left(\frac{\widetilde{\Omega}t}{2}\right)|ee\rangle,$$
$$|ee\rangle\rightarrow\cos\left(\frac{\widetilde{\Omega}t}{2}\right)|ee\rangle+{\rm i}\sin\left(\frac{\widetilde{\Omega}t}{2}\right)|gg\rangle,$$
$$|ge\rangle\rightarrow\cos\left(\frac{\widetilde{\Omega}t}{2}\right)|ge\rangle-{\rm i}\sin\left(\frac{\widetilde{\Omega}t}{2}\right)|eg\rangle,$$
$$|eg\rangle\rightarrow\cos\left(\frac{\widetilde{\Omega}t}{2}\right)|eg\rangle-{\rm i}\sin\left(\frac{\widetilde{\Omega}t}{2}\right)|ge\rangle. \quad (4)$$

Firstly, we must prepare a maximally entangled state of two-ion system as quantum channel for realizing the quantum dense coding. Suppose that three users Alice, Bob and Tom are spatially separated. Tom is in the middle between Alice and Bob. They share ions 1, 2, 3 and 4 that have been previously prepared in the following entangled states:

$$|\Psi\rangle_{12}=a|ge\rangle_{12}+b|eg\rangle_{12}, \quad (5)$$
$$|\Psi\rangle_{34}=c|ge\rangle_{34}+d|eg\rangle_{34}, \quad (6)$$

where $|g\rangle_i$ and $|e\rangle_i$ are the ground and excited states of the ith ion, a, b, c and d are coefficients. Ions 2 and 3 belong to Tom, ions 1 and 4 belong to Alice (the sender) and Bob (the receiver), respectively. By means of entanglement swapping,[17,18] the ions 1 and 4 will be in maximally entangled state:

$$|\Psi\rangle_{14}=ab(|gg\rangle_{14}+{\rm i}|ee\rangle_{14}), \quad (7)$$

or

$$|\Psi\rangle_{14}=cd(|ee\rangle_{14}+{\rm i}|gg\rangle_{14}), \quad (8)$$

i.e., Alice and Bob share a maximally entangled state (Bell state). We can assume that the quantum state of the ions 1 and 4 is

$$|\Psi\rangle_{14}=\frac{1}{\sqrt{2}}(|gg\rangle_{14}+{\rm i}|ee\rangle_{14}). \quad (9)$$

To realize the quantum dense coding, Alice performs on her ion 1 one of the four unitary operations $I,\sigma^x,{\rm i}\sigma^y$ and σ^z, where I is the identity operator and σ^i are three Pauli operators. Each of four unitary operations maps the Bell state $|\Phi\rangle_{14}$ to a different member of the four Bell states as follows:

$$|\Psi^+\rangle_{14}=\frac{1}{\sqrt{2}}(|gg\rangle_{14}+{\rm i}|ee\rangle_{14}), \quad (10)$$
$$|\Phi^+\rangle_{14}=\frac{1}{\sqrt{2}}(|eg\rangle_{14}+{\rm i}|ge\rangle_{14}), \quad (11)$$
$$|\Phi^-\rangle_{14}=\frac{1}{\sqrt{2}}(|eg\rangle_{14}-{\rm i}|ge\rangle_{14}), \quad (12)$$
$$|\Psi^-\rangle_{14}=\frac{1}{\sqrt{2}}(|gg\rangle_{14}-{\rm i}|ee\rangle_{14}). \quad (13)$$

Then Alice sends her ion 1 to Bob. In this case Bob possesses two ions 1 and 4. Now what Bob wants to do is to know how to discriminate the four states. Because all four Bell states form orthogonal bases, Bob can discriminate them in the joint BSM to obtain two bits of classical information. However, the joint

BSM is very complex for experimental realization,[13] and thus is a bottle-neck. To overcome this difficulty, Bob confine ions 1 and 4 simultaneously into another ion trap described by Eq.(2). For an interaction time t, the state of the ions 1 and 4 evolves according to Eq.(4) and can be expressed as

$$|\Psi^+\rangle'_{14} = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\widetilde{\Omega}t}{2}\right)|gg\rangle_{14} + \mathrm{i}\sin\left(\frac{\widetilde{\Omega}t}{2}\right)|ee\rangle + \mathrm{i}\cos\left(\frac{\widetilde{\Omega}t}{2}\right)|ee\rangle_{14} - \sin\left(\frac{\widetilde{\Omega}t}{2}\right)|gg\rangle\right), \tag{14}$$

$$|\Phi^+\rangle'_{14} = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\widetilde{\Omega}t}{2}\right)|eg\rangle_{14} - \mathrm{i}\sin\left(\frac{\widetilde{\Omega}t}{2}\right)|ge\rangle + \mathrm{i}\cos\left(\frac{\widetilde{\Omega}t}{2}\right)|ge\rangle_{14} + \sin\left(\frac{\widetilde{\Omega}t}{2}\right)|eg\rangle\right), \tag{15}$$

$$|\Phi^-\rangle'_{14} = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\widetilde{\Omega}t}{2}\right)|gg\rangle_{14} - \mathrm{i}\sin\left(\frac{\widetilde{\Omega}t}{2}\right)|ee\rangle - \mathrm{i}\cos\left(\frac{\widetilde{\Omega}t}{2}\right)|ee\rangle_{14} - \sin\left(\frac{\widetilde{\Omega}t}{2}\right)|gg\rangle\right), \tag{16}$$

$$|\Psi^-\rangle'_{14} = \frac{1}{\sqrt{2}}\left(\cos\left(\frac{\widetilde{\Omega}t}{2}\right)|gg\rangle_{14} + \mathrm{i}\sin\left(\frac{\widetilde{\Omega}t}{2}\right)|ee\rangle - \mathrm{i}\cos\left(\frac{\widetilde{\Omega}t}{2}\right)|ee\rangle_{14} + \sin\left(\frac{\widetilde{\Omega}t}{2}\right)|gg\rangle\right). \tag{17}$$

We can choose $\widetilde{\Omega}t = \pi/2$. Then the states in Eqs.(14–17) will become

$$|\Psi^+\rangle_{14} \rightarrow \mathrm{i}|ee\rangle_{14}, \tag{18}$$

$$|\Phi^+\rangle_{14} \rightarrow |eg\rangle_{14}, \tag{19}$$

$$|\Phi^-\rangle_{14} \rightarrow -\mathrm{i}|ge\rangle_{14}, \tag{20}$$

$$|\Psi^-\rangle_{14} \rightarrow |gg\rangle_{14}. \tag{21}$$

Therefore, the joint BSM can be achieved by detecting ions 1 and 4 separately. With the outcome of measurement on Bob's two ions, Bob can distinguish Alice's operation on ion 1. So he receives two of classical information from Alice. That is, the dense coding is achieved.

It is necessary to give a brief discussion on the experimental matters. On the one hand, for the present experimental technology, we can generate the desired laser pulse with sufficiently narrow linewidth for the quantum manipulations of the vibrational sidebands. For example, the linewidth (1 kHz) of laser beams (at 729 nm) used in Ref.[19] to drive the trapped ion $^{40}Ca^+$ is very much smaller, corresponding to a resolution of better than $\triangle\nu/\nu = 2.5 \times 10^{-12}$. This linewidth is also very much smaller than frequency of the applied trap (138 kHz). Thus, the vibrational sidebands can be well resolved. On the other hand, we choose the detuning which is very large from the sideband, the heating of vibrational mode has been eliminated during the interaction between trapped ions and the lasers. We only need to consider the radiative time of ions. For the Rydberg ions with principal numbers 50 and 51, the radiative time is $T_\mathrm{r} = 3 \times 10^{-2}$ s. Because the interaction between trapped ions and the laser field is on the order of 10^{-3} s, which is much shorter than the atomic radiative time T_r. In addition, when Tom detects the state of the ions, he must discriminate the ions. Fortunately, Riebe *et al*[12] have proposed this problem. So our scheme is realizable.

In this article, we have proposed a potentially practical scheme to realize quantum dense coding . There exist still advantages in our scheme, first the internal state transition is insensitive to the vibrational quantum number n, and it may be applied even with ions which exchange vibrational energy with a surrounding reservoir, i.e., it is insensitive to the interaction between the quantum channel and the environment. Second, our scheme does not involve the joint BSM, four states codified by Alice can be completely discriminated only via the ion-field interaction in a trap, and the successful probability can reach 1.0. So our scheme has potential value of applications in practice.

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