Vol 17 No 1, January 2008
1674-1056/2008/17(01)/0027-07

Chinese Physics B

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Remote preparation of an entangled two-qubit state with three parties*

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(Received 10 April 2007; revised manuscript received 2 August 2007)

We present a scheme for probabilistic remote preparation of an entangled two-qubit state with three parties from a sender to either of two receivers. The quantum channel is composed of a partially entangled two-qubit state and a partially entangled three-qubit state. We calculate the successful total probabilities of the scheme in general and particular cases, respectively. We also calculate total classical communication cost in a general case and two particular cases, respectively.

Keywords: remote state preparation, two-qubit, classical communication cost, three parties
PACC: 0365

1. Introduction

Recently, Lo[1] has presented an interesting new method to transmit a pure known quantum state using a prior shared entanglement and some classical communication when the sender actually knows the transmitted state. This communication protocol is called remote state preparation (RSP). Its main difference from the usual teleportation[2] is that Alice is assumed to know fully the transmitted state to be prepared by Bob, while in the teleportation neither Alice nor Bob has a knowledge of the transmitted state. Pati[3] has found that RSP protocol is more economical than teleportation and requires only one classical bit from Alice to Bob for some special ensembles, but for general states, the RSP protocol requires as much classical communication cost as the quantum teleportation does. Bennett *et al*[4] have studied the tradeoff in cost between the required entanglement and classical communication in RSP. Since then, the researchers have put forward various kinds of theoretical RSP schemes,[5−23] such as low-entanglement RSP,[5] higher-dimension RSP,[6] optimal RSP,[7] RSP without oblivious conditions,[8] generalized RSP,[9] oblivious RSP,[10] faithful RSP,[11] and continuous variable RSP in phase space,[12] etc. Meantime, RSP schemes have been implemented experimentally by using Nuclear magnetic resonance (NMR)[24] and spontaneous parametric down-conversion.[25] So far, there has been no proposal for how to probabilistically and remotely prepare a bipartite entangled state with three parties via the combination of partial EPR pair and tripartite GHZ state, and no report an the calculation of the classical communication cost required in the RSP process with three parties via using this partially entangled quantum channel, either.

In this paper, we propose a protocol for preparing remotely the entangled two-qubit state from a sender to either of two receivers. The quantum channel consists of a partially entangled two-qubit state and a partially entangled three-qubit state. We have calculated the successful total probability and the total classical communication cost of probabilistic RSP scheme in general and particular cases, respectively. It is shown that such a RSP requires only 2.5 bits and 5 bits classical communication cost under the condition of maximally entangled quantum channel for the general quantum states and particular states, respectively, and possesses 50% and 100% in successful probability, respectively.

*Project supported by the National Natural Science Foundation of China (Grant Nos 10404039 and 60674040).
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2. Probabilistic remote preparation of an entangled two-qubit state with three parties for a general case

Suppose that the sender Alice wants to help either of two receivers Bob and Charlie remotely prepare the following entangled two-qubit state

$$|\phi\rangle = \alpha|00\rangle + \beta|11\rangle, \tag{1}$$

where α is real number and β is a complex number with $|\alpha|^2 + |\beta|^2 = 1$. We suppose that Alice knows about α and β completely, but either Bob or Charlie does not know about them at all. We also suppose that the quantum channel is composed of a non-maximally entangled two-qubit state and a non-maximally entangled three-qubit state as follows:

$$|\Psi\rangle_{12} = (a|00\rangle + b|11\rangle)_{12}, \tag{2}$$

$$|\Psi\rangle_{345} = (c|000\rangle + d|111\rangle)_{345}, \tag{3}$$

where the real coefficients a and b satisfy $|a|^2+|b|^2 = 1$ and $|b| \leq |a|$; the real coefficients c and d satisfy $|c|^2 + |d|^2 = 1$ and $|d| \leq |c|$. Particles (1, 3) are in the possession of Alice, but particle 2 only belongs to one of receivers, Bob or Charlie, while particles (4, 5) belong to the receivers Bob and Charlie, respectively. Figure 1 shows the schematic diagram for RSP of the entangled two-qubit state with three parties.

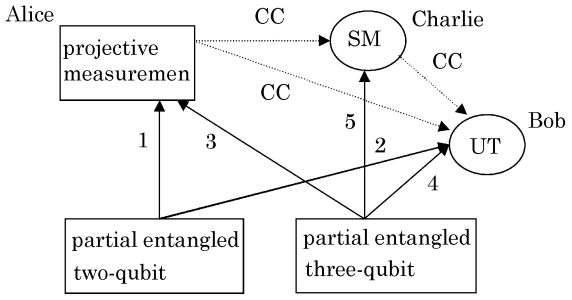


Fig.1. A sketch of the RSP of an entangled two-qubit state with three parties. A partially entangled two-qubit state (particles 1 and 2) and a partially entangled three-qubit state (particles 3, 4 and 5) are used as a quantum channel. Particles (1, 3) are in the possession of Alice, but particle 2 only belongs to one of receivers, Bob, while particles (4, 5) belong to the receivers Bob and Charlie, respectively. "SM" stands for separate measurement, which is on particle 5 and must be carried out by Charlie, whereas "UT" stands for unitary transformation, which Bob must perform in order to retrieve the original state, and CC stands for classical communication.

In order to help Bob or Charlie remotely prepare the entangled two-qubit state of Eq.(1), Alice must perform a projective measurement on particles (1, 3). The measurement basis chosen by Alice is a set of mutually orthogonal basis vectors $\{|\varphi\rangle, |\varphi_\perp\rangle, |\psi\rangle, |\psi_\perp\rangle\}$, which are related to the computation basis vectors $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ by the following relations:

$$|\varphi\rangle_{13} = (\alpha|00\rangle + \beta|11\rangle)_{13}, \tag{4a}$$

$$|\varphi_\perp\rangle_{13} = (\beta^*|00\rangle - \alpha|11\rangle)_{13}, \tag{4b}$$

$$|\psi\rangle_{13} = (\alpha|01\rangle + \beta|10\rangle)_{13}, \tag{4c}$$

$$|\psi_\perp\rangle_{13} = (\beta^*|01\rangle - \alpha|10\rangle)_{13}. \tag{4d}$$

Therefore, without loss of generality, we have

$$\begin{aligned} &|\Psi\rangle_{12} \otimes |\Psi\rangle_{345} \\ =&|\varphi\rangle_{13}(ac\alpha|000\rangle + bd\beta^*|111\rangle)_{245} \\ &+ |\varphi_\perp\rangle_{13}(ac\beta|000\rangle - bd\alpha|111\rangle)_{245} \\ &+ |\psi\rangle_{13}(ad\alpha|011\rangle + bc\beta^*|100\rangle)_{245} \\ &+ |\psi_\perp\rangle_{13}(ad\beta|011\rangle - bc\alpha|100\rangle)_{245}. \end{aligned} \tag{5}$$

Then Alice measures particles (1, 3). For example, if her von Neumann measurement result is $|\varphi_\perp\rangle_{13}$, the state of particles (2, 4, 5), as shown by Eq.(5), will collapse into

$$(ac\beta|000\rangle - bd\alpha|111\rangle)_{245}. \tag{6}$$

Now, we see how the original state described in Eq.(1) can be prepared remotely from the state in Eq.(6). Firstly, Alice informs the receivers Bob and Charlie of her measurement outcomes via a classical communication. If Bob wants to receive the original state of Eq.(1), particle 2 belongs to the receiver Bob. Bob can ask Charlie to perform the following Hadamard operation on particle 5:

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \tag{7a}$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle), \tag{7b}$$

then in basis $\{|0\rangle, |1\rangle\}$ we have

$$\begin{aligned} &\frac{1}{\sqrt{2}}(ac\beta|00\rangle - bd\alpha|11\rangle)_{24}|0\rangle_5 \\ &+ \frac{1}{\sqrt{2}}(ac\beta|00\rangle + bd\alpha|11\rangle)_{24}|1\rangle_5. \end{aligned} \tag{8}$$

Charlie measures particle 5. If Charlie obtains the result of $|0\rangle_5$, then the state of particles (2, 4) will become

$$\frac{1}{\sqrt{2}}(ac\beta|00\rangle - bd\alpha|11\rangle)_{24}. \tag{9}$$

Otherwise, if the result is $|1\rangle_5$, the state of particles (2, 4) will be projected to

$$\frac{1}{\sqrt{2}}(ac\beta|00\rangle + bd\alpha|11\rangle)_{24}. \tag{10}$$

After his measuring, Charlie informs Bob of his measurement outcomes via a classical communication. Secondly, Bob needs to establish a correspondence such that coefficients α and β can correspond to $|00\rangle_{24}$ and $|11\rangle_{24}$, respectively. According to Alice's and Charlie's measurement results, Bob carries out the following unitary transformation U_1 on particles (2, 4) of Eq.(9):

$$U_1 = (|1\rangle\langle 0| - |0\rangle\langle 1|)_2 \otimes (|1\rangle\langle 0| + |0\rangle\langle 1|)_4. \tag{11}$$

That is, unitary transformation U_1 in Eq.(11) will transform the state described in Eq.(9) into

$$|\Psi\rangle_{24} = \frac{1}{\sqrt{2}}(bd\alpha|00\rangle + ac\beta|11\rangle)_{24}. \tag{12}$$

Then, Bob introduces an auxiliary two-level particle A with initial state $|0\rangle_A$ and performs another unitary transformation U_2 on particles 2, 4 and A under basis $\{|00\rangle_{24}|0\rangle_A,\ |11\rangle_{24}|0\rangle_A,\ |00\rangle_{24}|1\rangle_A,\ |11\rangle_{24}|1\rangle_A\}$. Unitary transformation U_2 may take the form of the following 4×4 matrices:

$$U_2 = \begin{pmatrix} A_1 & A_2 \\ A_2 & -A_1 \end{pmatrix}, \tag{13}$$

where A_i (i=1, 2), a kind of 2×2 matrices, may be written as

$$A_1 = \mathrm{diag}(a_1,\ a_2), \tag{14a}$$

$$A_2 = \mathrm{diag}(\sqrt{1-a_1^2},\ \sqrt{1-a_2^2}) \tag{14b}$$

with $a_i (i = 1,\ 2\ )$ ($|a_i| \leq 1$) depending on the state of particles 2 and 4. Namely, after Bob has introduced an auxiliary particle A with its initial state $|0\rangle_A$, the state described in Eq.(12) will become

$$|\Psi\rangle_{24} \otimes |0\rangle_A = \frac{1}{\sqrt{2}}(bd\alpha|00\rangle + ac\beta|11\rangle)_{24} \otimes |0\rangle_A, \tag{15}$$

if we choose

$$(a_1, a_2) = \left(1,\ \frac{bd}{ac}\right). \tag{16}$$

Unitary transformation U_2 will transform the state in Eq.(15) into

$$\frac{1}{\sqrt{2}}bd(\alpha|00\rangle + \beta|11\rangle)_{24} \otimes |0\rangle_A + \frac{1}{\sqrt{2}}\sqrt{(ac)^2 - (bd)^2}\beta|11\rangle_{24} \otimes |1\rangle_A. \tag{17}$$

Finally, Bob measures the state of auxiliary particle A. If his measurement result is $|1\rangle_A$, the remote preparation of the original state fails. If the result $|0\rangle_A$is measured, the remote preparation of an entangled two-qubit state is successfully realized and the successful probability is $\dfrac{(bd)^2}{2}$.

Similarly, from Eq.(5), if Alice's measurement result on particles (1, 3) is $|\psi_\perp\rangle_{13}$, the state of particles 2, 4 and 5 will become

$$(ad\beta|011\rangle - bc\alpha|100\rangle)_{245}. \tag{18}$$

After Charlie has performed the Hadamard operations in Eq.(7) on particle 5 in basis $\{|0\rangle, |1\rangle\}$ and measures them, for states shown in Eq.(18), Bob will obtain the same successful probability of RSP from the state of Eq.(18) through the above steps.

Based on Eq.(5), we find that if Alice's measurement outcomes on the particles 1 and 3 are $|\varphi\rangle_{13}$ and $|\psi\rangle_{13}$, respectively, the state of particles (2, 4, 5) described by Eq.(5) will become $(ac\alpha|000\rangle + bd\beta^*|111\rangle)_{245}$ and $(ad\alpha|011\rangle + bc\beta^*|100\rangle)_{245}$, respectively. Since Bob and Charlie have no knowledge of these states, they cannot unitarily convert them into the original state $|\phi\rangle$, thus the remote state preparation fails.

From the above analysis, after Charlie has performed the Hadamard operations in Eq.(7) on particle 5 in basis $\{|0\rangle, |1\rangle\}$ and measures them, for states shown in Eq.(5), Bob can obtain 4 kinds of states in all. It can be easily proven that the 4 kinds of states have the same results for Bob, therefore the total probability of successful RSP is $2(bd)^2$. Table 1 shows all possible unitary transformations U_1 and coefficients a_1 and a_2 for unitary transformations U_2 performed by Bob on the states of particles (2, 4), where symbol $\pm^5$ in the angular brackets with a subscript 4 corresponds to the measurement results $|0\rangle_5$ and $|1\rangle_5$ on particle 5 after Charlie has performed the Hadamard operation on particle 5.

Table 1. All unitary transformations U_1 and coefficients a_1 and a_2 for unitary transformations U_2 performed by Bob.

Alice's von Neumann measurement results	U_1	a_1	a_2
$\|\varphi_\perp\rangle_{13}$ in Eq.(5)	$(\|1\rangle\langle 0\| - \|0\rangle\langle 1\|)_2 \otimes (\|1\rangle\langle 0\| \pm^5 \|0\rangle\langle 1\|)_4$	1	$\frac{bd}{ac}$
$\|\psi_\perp\rangle_{13}$ in Eq.(5)	$(\|1\rangle\langle 0\| - \|0\rangle\langle 1\|)_2 \otimes (\|0\rangle\langle 0\| \pm^5 \|1\rangle\langle 1\|)_4$	$\frac{d}{c}$	$\frac{b}{a}$

If $|a| = |b| = 1/\sqrt{2}$, $|c| = |d| = 1/\sqrt{2}$, i.e. the quantum channel consists of an EPR pair and a maximally entangled tripartite GHZ state, the total probability of successful RSP is 50%. Clearly, in this case, the unitary transformation made by Bob is only the unitary transformation U_1 operated on the particles (2, 4) only, in which the auxiliary particle A and unitary transformation U_2 do not need introducing. Therefore, the probabilistic RSP of a entangled two-qubit state with three parties becomes the usual standard RSP with three parties via a maximally entangled channel.

3. Probabilistic remote preparation of a two-qubit state with three parties for two particular cases

For states shown in Eq.(1), however, if α and β are real numbers, and if the Alice's measurement outcomes are $|\varphi\rangle_{13}$ or $|\psi\rangle_{13}$, Bob can prepare remotely the original two-qubit state $|\phi\rangle$ via a unitary operation and certain classical communication. All possible unitary transformations U_1 and the coefficients a_1 and a_2 for unitary transformations U_2 performed by Bob on the states of particles (2, 4) are given in Table 2,

Table 2. All unitary transformations U_1 and coefficients a_1 and a_2 for unitary transformations U_2 performed by Bob.

Alice's von Neumann measurement results	U_1	a_1	a_2
$\|\varphi_\perp\rangle_{13}$ in Eq.(5)	$(\|1\rangle\langle 0\| - \|0\rangle\langle 1\|)_2 \otimes (\|1\rangle\langle 0\| \pm^5 \|0\rangle\langle 1\|)_4$	1	$\frac{bd}{ac}$
$\|\psi_\perp\rangle_{13}$ in Eq.(5)	$(\|1\rangle\langle 0\| - \|0\rangle\langle 1\|)_2 \otimes (\|0\rangle\langle 0\| \pm^5 \|1\rangle\langle 1\|)_4$	$\frac{d}{c}$	$\frac{b}{a}$
$\|\varphi\rangle_{13}$ in Eq.(5)	$I_2 \otimes (\|0\rangle\langle 0\| \pm^5 \|1\rangle\langle 1\|)_4$	$\frac{bd}{ac}$	1
$\|\psi\rangle_{13}$ in Eq.(5)	$I_2 \otimes (\|1\rangle\langle 0\| \pm^5 \|0\rangle\langle 1\|)_4$	$\frac{b}{a}$	$\frac{d}{c}$

where symbol $\pm^5$ in the angular brackets with a subscript 4 corresponds to the measurement results $|0\rangle_5$ and $|1\rangle_5$ after Charlie has performed the Hadamard operation on particle 5, and I_2 is the identity matrix. Thus, for a particular case in which α and β are real numbers, the total probability of successful RSP becomes $4(bd)^2$.

As for the particular case in which the two-qubit state are

$$|\phi\rangle = (1/\sqrt{2})(|00\rangle + \mathrm{e}^{\mathrm{i}\phi_0}|11\rangle), \tag{19}$$

namely the state chosen from the equatorial line on Bloch sphere, we may realize successfully its remote preparation. In this case, the measurement basis chosen by Alice will become a set of mutually orthogonal basis vectors $\{|\varphi\rangle, |\varphi_\perp\rangle, |\psi\rangle, |\psi_\perp\rangle\}$ as follows:

$$|\varphi\rangle_{13} = \frac{1}{\sqrt{2}}|00\rangle_{13} + \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\phi_0}|11\rangle_{13}, \tag{20a}$$

$$|\varphi_\perp\rangle_{13} = \frac{1}{\sqrt{2}}|00\rangle_{13} - \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\phi_0}|11\rangle_{13}, \tag{20b}$$

$$|\psi\rangle_{13} = \frac{1}{\sqrt{2}}|01\rangle_{13} + \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\phi_0}|10\rangle_{13}, \tag{20c}$$

$$|\psi_\perp\rangle_{13} = \frac{1}{\sqrt{2}}|01\rangle_{13} - \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\phi_0}|10\rangle_{13}. \tag{20d}$$

For the quantum channel given by Eqs.(2) and (3), we can obtain

$$\begin{aligned} &|\,\Psi\rangle_{12} \otimes |\,\Psi\rangle_{345} \\ &= |\varphi\rangle_{13}\left(\frac{1}{\sqrt{2}}ac|000\rangle + \frac{1}{\sqrt{2}}bd\mathrm{e}^{\mathrm{i}\phi_0}|111\rangle\right)_{245} \end{aligned}$$

$$+|\varphi_\perp\rangle_{13}\left(\frac{1}{\sqrt{2}}ac|000\rangle-\frac{1}{\sqrt{2}}bd\mathrm{e}^{\mathrm{i}\phi_0}|111\rangle\right)_{245}$$
$$+|\psi\rangle_{13}\left(\frac{1}{\sqrt{2}}ad|011\rangle+\frac{1}{\sqrt{2}}bc\mathrm{e}^{\mathrm{i}\phi_0}|100\rangle\right)_{245}$$
$$+|\psi_\perp\rangle_{13}\left(\frac{1}{\sqrt{2}}ad|011\rangle-\frac{1}{\sqrt{2}}bc\mathrm{e}^{\mathrm{i}\phi_0}|100\rangle\right)_{245}. \quad (21)$$

Similarly, if Bob wants to receive the original entangled two-qubit state shown in Eq.(19), particle 2 belongs to the receiver Bob. By using the above mentioned method, after Charlie has performed the Hadamard operations in Eq.(7) on particle 5 in basis $\{|0\rangle, |1\rangle\}$ and measured them, for states shown in Eq.(21), Bob can obtain 8 kinds of states in all. It can easily be proven that the 8 kinds of states have the same results for Bob, thus the total probability of successful RSP is $4(bd)^2$. Table 3 presents all possible unitary transformations U_1 and coefficients a_1 and a_2 for unitary transformations U_2 performed by Bob on the states of particles (2, 4), where symbol $\pm^5$ in the angular brackets with a subscript 4 corresponds to the measurement results $|0\rangle_5$ and $|1\rangle_5$ on particle 5 after Charlie has performed the Hadamard operation on particle 5.

Table 3. All unitary transformations U_1 and coefficients a_1 and a_2 for unitary transformations U_2 performed by Bob, where I_2 is the identity matrix.

Alice's von Neumann measurement results	U_1	a_1	a_2
$\|\varphi\rangle_{13}$ in Eq.(21)	$I_2\otimes(\|0\rangle\langle 0\| \pm^5 \|1\rangle\langle 1\|)_4$	$\frac{bd}{ac}$	1
$\|\varphi_\perp\rangle_{13}$ in Eq.(21)	$I_2\otimes(\|0\rangle\langle 0\| - (\pm^5)\|1\rangle\langle 1\|)_4$	$\frac{bd}{ac}$	1
$\|\psi\rangle_{13}$ in Eq.(21)	$I_2\otimes(\|1\rangle\langle 0\| \pm^5 \|0\rangle\langle 1\|)_4$	$\frac{b}{a}$	$\frac{d}{c}$
$\|\psi_\perp\rangle_{13}$ in Eq.(21)	$I_2\otimes(-\|1\rangle\langle 0\| \pm^5 \|0\rangle\langle 1\|)_4$	$\frac{b}{a}$	$\frac{d}{c}$

Thus for the case in which α and β are real numbers or state $|\phi\rangle = (1/\sqrt{2})(|00\rangle + \mathrm{e}^{\mathrm{i}\theta}|11\rangle)$, the total probability of successful RSP for a maximally entangled channel equals 100%.

4. Classical communication cost

The classical message plays an important role in the RSP process. How many bits of classical information are needed in the RSP process with three parties by a partially entangled quantum channel?

For convenience, we divide this classical information into two transmitted processes. One is that the classical information is sent from the sender Alice to the receiver Bob, in which Alice performs a projective measurement on particles (1, 3) and informs him of her measurement result. The other is that the classical information is sent from Alice to Charlie and then from Charlie to the receiver Bob. That is, it includes the classical information in which Alice performs a projective measurement on particles (1, 3) and informs him of her measurement results, and one in which Charlie makes the Hadamard operation on particle 5, then measures particle 5, and informs Bob of his measurement outcome，as well as one in which Bob introduces an auxiliary two-level particle A with initial state $|0\rangle_A$ and measures the auxiliary particle A.

We first consider the classical information from the sender Alice to the receiver Bob. After projective measurement on particles (1, 3), Alice can obtain four outcomes $|\varphi\rangle_{13}$, $|\varphi_\perp\rangle_{13}$, $|\psi\rangle_{13}$ and $|\psi_\perp\rangle_{13}$ with the following probabilities:

$$p_1 = a^2c^2\alpha^2 + b^2d^2|\beta|^2, \quad (22a)$$
$$p_2 = b^2d^2\alpha^2 + a^2c^2|\beta|^2, \quad (22b)$$
$$p_3 = a^2d^2\alpha^2 + b^2c^2|\beta|^2, \quad (22c)$$
$$p_4 = b^2c^2\alpha^2 + a^2d^2|\beta|^2, \quad (22d)$$

respectively. Without loss of generality, the classical information in this process from the sender Alice to receiver Bob requires

$$\begin{aligned} S_{AB} &= -p_2\log_2 p_2 - p_4\log_2 p_4 \\ &= -(b^2d^2\alpha^2 + a^2c^2|\beta|^2)\mathrm{lb}(b^2d^2\alpha^2 + a^2c^2|\beta|^2) \\ &\quad -(b^2c^2\alpha^2 + a^2d^2|\beta|^2) \\ &\quad \times \log_2(b^2c^2\alpha^2 + a^2d^2|\beta|^2) \text{ bits}, \end{aligned} \quad (23)$$

while the classical information in this process from the sender Alice to Charlie and from Charlie to the receiver Bob needs

$$S_{ACB} = 2(bd)^2\mathrm{lb}\left[\frac{2}{(bd)^2}\right] \text{ bits}. \quad (24)$$

Therefore, without loss of generality, the total classical communication for this reliable probabilistic remote preparation of an entangled two-qubit state with three parties require

$$\begin{aligned} S =& S_{AB} + S_{ACB} \\ =& -(b^2d^2\alpha^2 + a^2c^2|\beta|^2)\mathrm{lb}(b^2d^2\alpha^2 + a^2c^2|\beta|^2) \\ & -(b^2c^2\alpha^2 + a^2d^2|\beta|^2)\mathrm{lb}(b^2c^2\alpha^2 + a^2d^2|\beta|^2) \\ & -2(bd)^2\mathrm{lb}\left[\frac{(bd)^2}{2}\right] \quad \text{bits.} \end{aligned} \tag{25}$$

If the quantum channel consists of an EPR pair and a maximally entangled tripartite GHZ state, the classical information will consume 2.5 bits in total.

However, for two particular cases in which α and β are real numbers or state $|\phi\rangle = (1/\sqrt{2})(|00\rangle + \mathrm{e}^{\mathrm{i}\theta}|11\rangle)$, the total classical communication cost required in this RSP scheme with three parties will be

$$S = -\sum_{i=1}^{4} p_i \log_2 p_i - 4(bd)^2\mathrm{lb}\left[\frac{(bd)^2}{2}\right] \quad \text{bits.} \tag{26}$$

where p_i (i=1, 2, 3, 4) are determined by Eq.(22). For the maximally entangled channel, the classical information will require 5 bits in total.

5. Conclusion

In summary, we present a scheme for the probabilistic RSP of an entangled two-qubit state with three parties. A partially entangled two-qubit state and a partially entangled three-qubit state are considered as a quantum channel. We have calculated the total successful probability of the probabilistic RSP scheme in a general case and two particular cases, respectively. We have also obtained the total classical communication cost of probabilistic RSP scheme with three parties in a general quantum state and two particular states, respectively. It is shown that the total successful probability of such a RSP scheme for a general state is only 50% and total classical communication will consume 2.5 bits for a maximally entangled quantum channel. While the remote preparation of the two-qubit state for two particular states has a successful probability of 100% and requires 5 bits of classical communication cost for a maximally entangled channel. We must point out that this scheme can be directly generalized to the remote preparation of an entangled k-qubit state from a sender to either of two receivers by using a non-maximally k particle entangled state and a non-maximally tripartite GHZ state as a quantum channel. Similarly, this scheme can be directly generalized to the remote preparation of the two-qubit state from a sender to any of $(k-1)$ receivers by a non-maximally partial EPR pair and a non-maximally k particle entangled state. Comparing with the previous RSP schemes,[1,3−25] the present one has the following advantages. First, our protocol has investigated the probabilistic RSP of a two-qubit state with three parties from a sender to either of two receivers, while the previous RSP schemes considered only the RSP with two parties from a sender to a receiver. Second, the quantum channels are different in form. In our present scheme, a partially entangled two-qubit state and a partially entangled three-qubit state are used as a quantum channel, whereas the quantum channel in the previous schemes was composed of either the EPR pairs or GHZ state. Third, we have calculated total classical communication of probabilistic RSP scheme with three parties, which not only involves the transmitted communication from a sender to the receivers, but also contains one between two receivers, while the earlier RSP schemes just considered the classical communication cost for a maximally quantum channel, which is related to the transmitted communication from a sender to a receiver only. Nowadays, three-photon entanglement has been observed and used to verify quantum nonlocality,[26,27] and a number of feasible protocols for generating GHZ states have been proposed.[28,29] Thus we believe that the present scheme may be realized in the near future, and useful not only in understanding the essence of the classical communication in a multi-party RSP process, but also in expanding the application field of classical information science.

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