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Structure equation and Mei conserved quantity for Mei symmetry of Appell equation*

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This paper investigates structure equation and Mei conserved quantity of Mei symmetry of Appell equations for non-Chetaev nonholonomic systems. Appell equations and differential equations of motion for non-Chetaev nonholonomic mechanical systems are established. A new expression of the total derivative of the function with respect to time t along the trajectory of a curve of the system is obtained, the definition and the criterion of Mei symmetry of Appell equations under the infinitesimal transformations of groups are also given. The expressions of the structure equation and the Mei conserved quantity of Mei symmetry in the Appell function are obtained. An example is given to illustrate the application of the results.

Keywords: Appell equation, Mei conserved quantity, Mei symmetry, non-Chetaev nonholonomic system

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1. Introduction

Gibbs found a new method to study the motion of mechanical system in 1899.[1] In the same year, this method is developed by Appell, a famous mathematician in France, and Appell equations are obtained.[2] Now, Appell equations have become one of three important mechanic systems including Lagrange system, Nielsen system and Appell system in the theories of analytical mechanics.[3]

In 1918, Noether, a Germanic woman scientist, revealed the potential relation between the symmetry and the conserved quantity.[4] However, the value of Noether theory was not found until in 1970s in the 20th century. In 1978, Vujanović B first studied the conservation law of a mechanical system according to differential-variational principles.[5] The next year, Lutzky transplanted the Lie method to the mechanical system, and first put forward the concept of Lie symmetry which can keep differential equations of mechanical system invariant,[6] and further gained non-Noether conserved quantity called Lutzky conserved quantity deduced by point symmetry of Lagrange system.[7−9] From then on, the study on both symmetry and conserved quantity is flourished and therefore plentiful and substantial outcomes are achieved.[10−30]

However, the study on both symmetry and conserved quantity in Appell style is very slow. In order to seek the methods of solving Appell equations, first, Mei Feng-Xiang used the theory[10] given by Santilli so that the Appell equation can be expressed as a Lagrange equation; next, he gave a Noether conserved quantity deduced indirectly by using the Noether symmetry according to form invariance, namely Mei symmetry.[31] In the same way, the conserved quantities of variable mass holonomic systems are obtained by Li *et al.*[32] As a result of Noether symmetry and Lie symmetry from form invariance respectively, the conserved quantity of Appell equations for holonomic systems of rotational relativity was achieved by Luo Shao-Kai and he also acquired knowledge of the conserved quantity about general holonomic systems through Lie symmetry.[33,34]

Though Refs.[31–34] provided a new idea to look for Mei conserved quantity of Appell equation, it is still a great pity. That is to say, there is no structure equation and the expression of Mei conserved quantity which can be denoted as Appell function. Therefore, it is not dissolved to the core about the problem of

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Mei conserved quantity of Appell equation.

The study of this paper made up shortages in the Refs.[31–34]. First, Appell equation for nonholonomic systems of non-Chetaev type and the differential equations of motion are established. Second, a new expression of the total derivative of function with respective to t along the curve of motion of the system is given, and the definitions of Mei symmetry, loose Mei symmetry, strict Mei symmetry of Appell equation under the infinitesimal transformations of groups are obtained, and the criterion equations corresponding to three kinds of symmetries are also gained. Third, the structure equation of Mei symmetry and the expression of Mei conserved quantity in the Appell function are achieved. Finally, an example is given to illustrate the application of the results and the conclusions in this paper are discussed.

2. Appell equation for nonholonomic systems of non-Chetaev type

Suppose that the nonholonomic system of non-Chetaev's type consists of N particles with masses m_i and its position vector $\boldsymbol{r}_i$, and let the position of the system be determined by n generalized coordinates $q_s(s = 1, 2, \ldots, n)$, it is subjected to the g ideal nonholonomic constraints of non-Chetaev type

$$f_\beta(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) = 0, \quad (\beta = 1, 2, \ldots, g), \tag{1}$$

and if the restriction for which constraint Eq.(1) imposes on virtual displacements is

$$f_{\beta s}\delta q_s = 0, \quad (\beta = 1, 2, \ldots, g), \tag{2}$$

in which and in the following text, we use the Einstein summation convention. Generally speaking, there is no relation between $f_{\beta s}$ and $\partial f_\beta/\partial \dot{q}_s$. Especially if they are equivalent, the constraints of non-Chetaev type will be ones of Chetaev type. In this way, the energy of acceleration for the system is expressed as

$$S = S(t, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) = \frac{1}{2} m_i \ddot{\boldsymbol{r}}_i^2, \tag{3}$$

and Appell equations are written in the form

$$\begin{aligned} \frac{\partial S}{\partial \ddot{q}_s} &= Q_s + \lambda_\beta f_{\beta s} = Q'_s + Q''_s + \lambda_\beta f_{\beta s} \\ &= Q'_s + Q''_s + \Gamma_s, \quad (s = 1, 2, \ldots, n), \end{aligned} \tag{4}$$

where

$$\Gamma_s = \Gamma_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) = \lambda_\beta f_{\beta s}, \quad (s = 1, 2, \ldots, n), \tag{5}$$

$$Q_s = Q'_s + Q''_s, \quad (s = 1, 2, \ldots, n), \tag{6}$$

λ_β is a constraint multiplicator corresponding to the β constraint, $\Gamma_s, Q'_s, Q''_s, Q_s = Q_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$ are generalized constrained force, generalized potential force, generalized non-potential force and generalized force respectively corresponding to the s generalized coordinate q_s. Before integrating the differential equations of motion of system, we can figure out $\lambda_\beta = \lambda_\beta(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$ by Eqs.(1) and (4). Let

$$\Lambda_s = \Lambda_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) = Q'_s + Q''_s + \Gamma_s, \quad (s = 1, 2, \ldots, n), \tag{7}$$

where Λ_s is called generalized composition of forces. So Eq.(4) becomes

$$\frac{\partial S}{\partial \ddot{q}_s} = \Lambda_s, \quad (s = 1, 2, \ldots, n), \tag{8}$$

which is intituled Appell equation of holonomic system opposite to nonholonomic system of non-Chetaev type expressed as Eqs.(1) and (4). If initial condition of system satisfies Eq.(1), solution of Eq.(8) will give the orbit of motion of system above. By virtue of Eq.(8), we can solve all generalized accelerations that are the differential equations of motion of system. Namely

$$\ddot{q}_s = \alpha_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}), \quad (s = 1, 2, \ldots, n). \tag{9}$$

3. Mei symmetry of Appell equation

Introduce the infinitesimal transformations of time and generalized coordinates as

$$\begin{aligned} &t^* = t + \Delta t, \quad q_s^*(t^*) = q_s(t) + \Delta q_s, \\ &\qquad (s = 1, 2, \ldots, n), \end{aligned} \tag{10}$$

or

$$\begin{aligned} &t^* = t + \varepsilon \xi_0(t, \boldsymbol{q}, \dot{\boldsymbol{q}}), \\ &q_s^*(t^*) = q_s(t) + \varepsilon \xi_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}), \\ &\qquad (s = 1, 2, \ldots, n), \end{aligned} \tag{11}$$

where ε is an infinitesimal parameter and ξ_0, ξ_s are infinitesimal generators. Similarly, we introduce vector of infinitesimal generators again

$$X^{(0)} = \xi_0 \frac{\partial}{\partial t} + \xi_s \frac{\partial}{\partial q_s}, \tag{12}$$

as well as its expansions

$$\tilde{X}^{(1)} = X^{(0)} + \left(\frac{\bar{\mathrm{d}}\xi_s}{\mathrm{d}t} - \dot{q}_s\frac{\bar{\mathrm{d}}\xi_0}{\mathrm{d}t}\right)\frac{\partial}{\partial\dot{q}_s},$$
$$\tilde{X}^{(2)} = \tilde{X}^{(1)} + \left[\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\left(\frac{\bar{\mathrm{d}}\xi_s}{\mathrm{d}t} - \dot{q}_s\frac{\bar{\mathrm{d}}\xi_0}{\mathrm{d}t}\right) - \ddot{q}_s\frac{\bar{\mathrm{d}}\xi_0}{\mathrm{d}t}\right]\frac{\partial}{\partial\ddot{q}_s}, \quad (13)$$

where the total derivative with respect to t along the curve of a function determined by the motion of system can be written as follows:

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \dot{q}_s\frac{\partial}{\partial q_s} + \alpha_s\frac{\partial}{\partial\dot{q}_s} + \dot{\alpha}_s\frac{\partial}{\partial\ddot{q}_s}. \quad (14)$$

From Eq.(11), then we obtain

$$\frac{\mathrm{d}q_s^*}{\mathrm{d}t^*} = \frac{\mathrm{d}q_s + \varepsilon\mathrm{d}\xi_s}{\mathrm{d}t + \varepsilon\mathrm{d}\xi_0} = \dot{q}_s + \varepsilon(\dot{\xi}_s - \dot{q}_s\dot{\xi}_0) + O(\varepsilon^2),$$
$$\frac{\mathrm{d}^2q_s^*}{\mathrm{d}t^{*2}} = \ddot{q}_s + \varepsilon[(\dot{\xi}_s - \dot{q}_s\dot{\xi}_0)^\bullet - \ddot{q}_s\dot{\xi}_0] + O(\varepsilon^2). \quad (15)$$

Let dynamic functions S, Λ_s, f_β of system be S^*, Λ_s^*, f_β^* under the infinitesimal transformations Eq.(11). According to Eq.(8), taking the Taylor expansion of S^*, Λ_s^*, f_β^* with respect to $(t, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$, one can easily obtain

$$\begin{aligned} S^* =& S\left(t^*, \boldsymbol{q}^*, \frac{\mathrm{d}\boldsymbol{q}^*}{\mathrm{d}t^*}, \frac{\mathrm{d}^2\boldsymbol{q}^*}{\mathrm{d}t^{*2}}\right) = S(t, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) \\ &+ \varepsilon\left\{\frac{\partial S}{\partial t}\xi_0 + \frac{\partial S}{\partial q_s}\xi_s + \frac{\partial S}{\partial\dot{q}_s}(\dot{\xi}_s - \dot{q}_s\dot{\xi}_0)\right. \\ &\left. + \frac{\partial S}{\partial\ddot{q}_s}[(\dot{\xi}_s - \dot{q}_s\dot{\xi}_0)^\bullet - \ddot{q}_s\dot{\xi}_0]\right\} + O(\varepsilon^2), \end{aligned}$$

that is

$$S^* = S(t, \boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) + \varepsilon\tilde{X}^{(2)}(S) + O(\varepsilon^2),] \quad (16)$$

$$\begin{aligned} \Lambda_s^* &= \Lambda_s^*\left(t^*, \boldsymbol{q}^*, \frac{\mathrm{d}\boldsymbol{q}^*}{\mathrm{d}t^*}\right) \\ &= \Lambda_s(t, \boldsymbol{q}, \dot{\boldsymbol{q}}) + \varepsilon\tilde{X}^{(1)}(\Lambda_s) + O(\varepsilon^2), \\ &\quad (s = 1, 2, \ldots, n), \end{aligned} \quad (17)$$

$$\begin{aligned} f_\beta^* &= f_\beta\left(t^*, \boldsymbol{q}^*, \frac{\mathrm{d}\boldsymbol{q}^*}{\mathrm{d}t^*}\right) \\ &= f_\beta\left(t, \boldsymbol{q}, \frac{\mathrm{d}\boldsymbol{q}}{\mathrm{d}t}\right) + \varepsilon\tilde{X}^{(1)}(f_\beta) + O(\varepsilon^2), \\ &\quad (\beta = 1, 2, \ldots, g). \end{aligned} \quad (18)$$

Definition 1 If Appell equation (8) keeps its form invariant when dynamic functions S, Λ_s are substituted by the functions S^*, Λ_s^* under the infinitesimal transformations Eq.(11), namely

$$\frac{\partial S^*}{\partial\ddot{q}_s} = \Lambda_s^*, \qquad (s = 1, 2, \ldots, n), \quad (19)$$

then the invariance is called Mei symmetry of Eq.(8) for the holonomic system of non-Chetaev type.

Definition 2 If Appell equation (8) and constraint equation (1) keep their forms invariant when dynamic functions S, Λ_s are substituted by the functions S^*, Λ_s^* under the infinitesimal transformations Eq.(11), namely

$$f_\beta^* = f_\beta\left(t^*, \boldsymbol{q}^*, \frac{\mathrm{d}\boldsymbol{q}^*}{\mathrm{d}t^*}\right) = 0, \quad (\beta = 1, 2, \ldots, g), \quad (20)$$

and Eq.(8) both come into existence, then the invariance is called loosely Mei symmetry of Eq.(8) for holonomic system of non-Chetaev type.

It can be easily proved that restriction equation (2) for which constraint equation (1) imposes on virtual displacements can be rewritten in the form

$$f_{\beta s}(\xi_s - \dot{q}_s\xi_0) = 0,$$
$$(\beta = 1, 2, \ldots, g; \quad s = 1, 2, \ldots, n), \quad (21)$$

which is called additional restriction equation.

Definition 3 If Appell equation (8) and constraint equation (1) keep their forms invariant and infinitesimal generators ξ_0, ξ_s satisfy Eq.(21) when dynamic functions S, Λ_s are substituted by the functions S^*, Λ_s^* under the infinitesimal transformations Eq.(11), then the invariance is called strictly Mei symmetry of Eq.(8) for holonomic system of non-Chetaev type.

4. Criterion of Mei symmetry

Substituting Eqs.(17) and (18) into Eq.(20) and neglecting terms that are $O(\varepsilon^2)$ or higher, by using Eq.(8), we obtain

$$\frac{\partial}{\partial\ddot{q}_s}[\tilde{X}^{(2)}(S)] - \tilde{X}^{(1)}(\Lambda_s) = 0, \quad (22)$$

here Eq.(22) is called discriminant equation of Mei symmetry of Appell equation. From Eq.(19), we can see that the invariance of Eq.(1) under the infinitesimal transformations Eq.(11) can come down to the following constraint equation

$$\tilde{X}^{(1)}[f_\beta(t, \boldsymbol{q}, \dot{\boldsymbol{q}})] = 0. \quad (23)$$

Hence, we have

Criterion 1 For Appell equation (8), if infinitesimal generators ξ_0, ξ_s make Eq.(22) come into existence, then the invariance of Eq.(8) under the infinitesimal transformations Eq.(11) is called Mei symmetry of Eq.(8).

Criterion 2 For Appell equation (8), if infinitesimal generators ξ_0, ξ_s make Eqs.(22–23) come into existence, then the invariance of Eq.(8) under the infinitesimal transformations Eq.(11) is called loosely Mei symmetry of Eq.(8).

Criterion 3 For Appell equation (8), if infinitesimal generators ξ_0, ξ_s make Eqs.(21–23) come into existence, then the invariance of Eq.(8) under the infinitesimal transformations Eq.(11) is called strictly Mei symmetry of Eq.(8).

5. Structure equation and Mei conserved quantity

Proposition If the infinitesimal generators ξ_0, ξ_s and the gauge function $G_{\rm M} = G_{\rm M}(t, \boldsymbol{q}, \dot{\boldsymbol{q}})$ of Mei symmetry, loosely Mei symmetry, strictly Mei symmetry of Eq.(8) satisfy

$$\tilde{X}^{(2)}(S)\frac{\bar{\rm d}\xi_0}{{\rm d}t} + \tilde{X}^{(1)}[\tilde{X}^{(2)}(S)] + \xi_s - \dot{q}_s\xi_0)\tilde{E}_s[\tilde{X}^{(2)}(S)] + \xi_0[\tilde{X}^{(1)}(\Lambda_s)]\frac{\bar{\rm d}\alpha_s}{{\rm d}t} + \frac{\bar{\rm d}G_{\rm M}}{{\rm d}t} = 0, \tag{24}$$

then the system possesses Mei conserved quantity deduced by these symmetries as follows:

$$I_{\rm M} = \xi_0\tilde{X}^{(2)}(S) + \frac{\partial\tilde{X}^{(2)}(S)}{\partial\dot{q}_s}(\xi_s - \dot{q}_s\xi_0) + G_{\rm M} = \text{const.} \tag{25}$$

Proof:

$$\begin{aligned}\frac{\bar{\rm d}I_{\rm M}}{{\rm d}t} &= \left[\frac{\partial\tilde{X}^{(2)}(S)}{\partial t} + \dot{q}_s\frac{\partial\tilde{X}^{(2)}(S)}{\partial q_s} + a_s\frac{\partial\tilde{X}^{(2)}(S)}{\partial\dot{q}_s} + \frac{\partial\tilde{X}^{(2)}(S)}{\partial\ddot{q}_s}\frac{\bar{\rm d}\alpha_s}{{\rm d}t}\right]\xi_0 + \tilde{X}^{(2)}(S)\frac{\bar{\rm d}\xi_0}{{\rm d}t} \\ &\quad + \left[\frac{\bar{\rm d}}{{\rm d}t}\frac{\partial\tilde{X}^{(2)}(S)}{\partial\dot{q}_s}\right](\xi_s - \dot{q}_s\xi_0) + \frac{\partial\tilde{X}^{(2)}(S)}{\partial\dot{q}_s}\left(\frac{\bar{\rm d}\xi_s}{{\rm d}t} - a_s\xi_0 - \dot{q}_s\frac{\bar{\rm d}\xi_0}{{\rm d}t}\right) + \frac{\bar{\rm d}G_{\rm M}}{{\rm d}t}.\end{aligned} \tag{26}$$

Note that

$$\tilde{X}^{(1)}[\tilde{X}^{(2)}(S)] = \xi_0\frac{\partial\tilde{X}^{(2)}(S)}{\partial t} + \xi_s\frac{\partial\tilde{X}^{(2)}(S)}{\partial q_s} + \left(\frac{\bar{\rm d}\xi_s}{{\rm d}t} - \dot{q}_s\frac{\bar{\rm d}\xi_0}{{\rm d}t}\right)\frac{\partial\tilde{X}^{(2)}(S)}{\partial\dot{q}_s},$$

therefore, Eq.(26) becomes

$$\begin{aligned}\frac{\bar{\rm d}I_{\rm M}}{{\rm d}t} &= \tilde{X}^{(1)}[\tilde{X}^{(2)}(S)] - \xi_s\frac{\partial\tilde{X}^{(2)}(S)}{\partial q_s} + \xi_0\dot{q}_s\frac{\partial\tilde{X}^{(2)}(S)}{\partial q_s} + \xi_0\frac{\partial\tilde{X}^{(2)}(S)}{\partial\ddot{q}_s}\frac{\bar{\rm d}\alpha_s}{{\rm d}t} + \tilde{X}^{(2)}(S)\frac{\bar{\rm d}\xi_0}{{\rm d}t} \\ &\quad + \left[\frac{\bar{\rm d}}{{\rm d}t}\frac{\partial\tilde{X}^{(2)}(S)}{\partial\dot{q}_s}\right](\xi_s - \dot{q}_s\xi_0) + \frac{\bar{\rm d}G_{\rm M}}{{\rm d}t} \\ &= \tilde{X}^{(1)}[\tilde{X}^{(2)}(S)] - (\xi_s - \dot{q}_s\xi_0)\frac{\partial\tilde{X}^{(2)}(S)}{\partial q_s} + \xi_0\frac{\partial\tilde{X}^{(2)}(S)}{\partial\ddot{q}_s}\frac{\bar{\rm d}\alpha_s}{{\rm d}t} + \tilde{X}^{(2)}(S)\frac{\bar{\rm d}\xi_0}{{\rm d}t} \\ &\quad + \left[\frac{\bar{\rm d}}{{\rm d}t}\frac{\partial\tilde{X}^{(2)}(S)}{\partial\dot{q}_s}\right](\xi_s - \dot{q}_s\xi_0) + \frac{\bar{\rm d}G_{\rm M}}{{\rm d}t}.\end{aligned}$$

Taking into account discriminant equation (22) and structure equation (24), we obtain

$$\begin{aligned}\frac{\bar{\rm d}I_{\rm M}}{{\rm d}t} &= \tilde{X}^{(1)}[\tilde{X}^{(2)}(S)] + (\xi_s - \dot{q}_s\xi_0)\tilde{E}_s[\tilde{X}^{(2)}(S)] + \xi_0\frac{\partial\tilde{X}^{(2)}(S)}{\partial\ddot{q}_s}\frac{\bar{\rm d}\alpha_s}{{\rm d}t} + \tilde{X}^{(2)}(S)\frac{\bar{\rm d}\xi_0}{{\rm d}t} + \frac{\bar{\rm d}G_{\rm M}}{{\rm d}t} \\ &= \xi_0\frac{\bar{\rm d}\alpha_s}{{\rm d}t}\left[\frac{\partial\tilde{X}^{(2)}(S)}{\partial\ddot{q}_s} - \tilde{X}^{(1)}(\Lambda_s)\right] = 0.\end{aligned} \tag{27}$$

6. Example

As an example, we study Mei symmetry and Mei conserved quantity of Appell system, of which the energy of acceleration, constraint equation and restriction condition imposed on virtual displacements are respectively

$$S = \frac{1}{2}(\ddot{q}_1^2 + \ddot{q}_2^2), \tag{28}$$

$$f = \dot{q}_2 - t\dot{q}_1 = 0, \tag{29}$$

$$\delta q_1 - \delta q_2 = 0, \tag{30}$$

potential generalized force and non-potential generalized force are

$$Q'_1 = Q'_2 = 0, \quad Q''_1 = -1, \quad Q''_2 = 0. \tag{31}$$

Comparing expansion of Eq.(2) with Eq.(30), we can easily have

$$f_{11} = 1, \quad f_{12} = -1. \tag{32}$$

Considering Eqs.(5, 7, 8, 31, 32), we obtain

$$\ddot{q}_1 + 1 = \Gamma_1 = \lambda_1, \tag{33}$$

$$\ddot{q}_2 = \Gamma_2 = -\lambda_1. \tag{34}$$

Clearly, from Eq.(29), we can easily obtain

$$\ddot{q}_2 - \dot{q}_1 - t\ddot{q}_1 = 0. \tag{35}$$

Similarly, considering Eqs.(7, 33, 34, 35), we have

$$\lambda = \frac{t - \dot{q}_1}{1 + t}, \tag{36}$$

$$\Lambda_1 = \Gamma_1 = \frac{t - \dot{q}_1}{1 + t}, \tag{37}$$

$$\Lambda_2 = \Gamma_2 = -\frac{t - \dot{q}_1}{1 + t}, \tag{38}$$

$$\ddot{q}_1 = -\frac{1 + \dot{q}_1}{1 + t}, \tag{39}$$

$$\ddot{q}_2 = \frac{\dot{q}_1 - t}{1 + t}. \tag{40}$$

Using Eqs.(39) and (40), we achieve

$$\ddot{q}_1 + \ddot{q}_2 + 1 = 0. \tag{41}$$

By Eq.(22), we obtain

$$\ddot{\xi}_1 - 2\dot{\xi}_0(2\ddot{q}_1 + 1) - \ddot{\xi}_0\dot{q}_1 = -\xi_0\frac{1 + \dot{q}_1}{(1 + t)^2} - \frac{\dot{\xi}_1 - \dot{q}_1\dot{\xi}_0}{1 + t}, \tag{42}$$

$$\ddot{\xi}_2 - 4\dot{\xi}_0\ddot{q}_2 - \ddot{\xi}_0\dot{q}_2 = \xi_0\frac{1 + \dot{q}_1}{(1 + t)^2} + \frac{\dot{\xi}_1 - \dot{q}_1\dot{\xi}_0}{1 + t}. \tag{43}$$

Taking notice of Eq.(41), we can take the infinitesimal generators ξ_0, ξ_s as follows:

$$\xi_0 = 0, \quad \xi_1 = \dot{q}_1 + \dot{q}_2 + t, \quad \xi_2 = 0. \tag{44}$$

By virtue of Eq.(23), we can obtain

$$\tilde{X}^{(1)}(f) = -\xi_0\dot{q}_1 - (\dot{\xi}_1 - \dot{q}_1\dot{\xi}_0)t + (\dot{\xi}_2 - \dot{q}_2\dot{\xi}_0) = 0. \tag{45}$$

In the same way, through Eq.(21), we can obtain

$$(\xi_1 - \dot{q}_1\xi_0) - (\xi_2 - \dot{q}_2\xi_0) = 0. \tag{46}$$

According to Propositions above, the infinitesimal generators in Eq.(44) admit Eq.(45), but do not satisfy Eq.(46), thus the system possesses Mei symmetry and loosely Mei symmetry.

In the end, we investigate Mei conserved quantity of the system. Through calculation we obtain

$$\tilde{X}^{(2)}(S) = \tilde{X}^{(1)}(\Lambda_1) = \tilde{X}^{(1)}(\Lambda_2) = 0. \tag{47}$$

From Eq.(24), if Eq.(41) is taken into account, we obtain

$$G_{\mathrm{M}} = \dot{q}_1 + \dot{q}_2 + t = 0, \tag{48}$$

then using Eq.(25), Mei conserved quantity is achieved

$$I_{\mathrm{M}} = \dot{q}_1 + \dot{q}_2 + t = \mathrm{const}. \tag{49}$$

7. Conclusions

In the following, we describe the steps for studying Mei symmetry and Mei conserved quantity of Appell equation for nonholonomic systems of non-Chetaev type. Firstly, we give a new expression Eq.(14) of the total derivative of function with respective to t along the curve of motion of the system. Secondly, we obtain three definitions and corresponding discriminant equations of Mei symmetry of Appell equation. Thirdly, we obtain structure equation (24) of Mei symmetry in Appell function. Finally, we obtain the expression Eq.(25) of Mei conserved quantity in Appell function. So it has solved the problem of Mei symmetry.

In addition, if non-potential generalized force $Q''_s = 0$ in this paper, the problem of Appell equation for nonholonomic system of non-Chetaev type will be one of Appell equation for nonholonomic conservative system of non-Chetaev type. If $f_{\beta s} = \partial f_\beta/\partial\dot{q}_s$, it will be the problem of Appell equation for nonholonomic system of Chetaev type. If $Q''_s = 0$, $f_\beta = f_\beta(t, \boldsymbol{q}) = 0$, it will be the problem of Appell equation of conservative system subjected to holonomic constraint. Obviously, the conclusions in this paper are still applicable. Thus these conclusions have universal significance.

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