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APPROACH FOR THE STUDY OF SOLITON IN OPTICAL FIBER*

TANG YI(唐 翌), YAN XIAO-HONG(颜晓红)
Department of Physics, Xiangtan University, Xiangtan 411105, China

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In the anomalous group velocity dispersion regime, under the influence of optical Kerr nonlinearity, the monomode optical fiber supports stable propagation of an initial soliton pulse. In this paper, we present a singular perturbation approach for the study of various perturbations acting on this initial soliton pulse, In contrast with the previous perturbation theory for this subject, this approach gets rid of dependence on any tool of the sophisticated inverse scattering transform.

Keywords: optical fiber, soliton, perturbation
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I. INTRODUCTION

Solitons in nonlinear monomode optical fibers have attracted considerable attention in the past decades not only for their potential application in repeaterless communication over transoceanic distances, but also for their value of fundamental research.[1−2] The propagation of an initial pulse in the optical fiber with anomalous group velocity dispersion (GVD) under the influence of the optical Kerr effect is one of the leading subjects in this area; it is governed by the following equation

$$iu_x + ik_1u_t - \frac{1}{2}k_2u_{tt} + \sigma|u|^2u = 0, \qquad (1)$$

where $u(x,t)$ is the complex field envelope, x is the propagation distance, t is the time, k_1 and k_2 are GVD parameters, and σ is the nonlinearity coefficient responsible for self-phase modulation. For the sake of mathematical convenience, Eq.(1) is usually studied by transforming it into a frame moving with the group velocity $v_g = k_1^{-1}$ (the so-called retarded time frame), namely, to replace the time by retarded time $T = t - x/v_g = t - k_1x$. Under this transformation, Eq.(1) reduced to[3]

$$iu_x - \frac{1}{2}k_2u_{TT} + \sigma|u|^2u = 0. \qquad (2)$$

In the case of anomalous GVD, i.e., $k_2 < 0$, one usually normalizes Eq.(2)as

$$iu_{x'} + \frac{1}{2}u_{TT'} + |u|^2u = 0. \qquad (3)$$

This form appears in the study of soliton in optical fiber, and is referred to as nonlinear Schrödinger equation customarily. From the sense of mathematics, Eq.(3) is equivalent to the standard nonlinear Schrödinger equation, analogously, it can support stable propagation of a single soliton in the form

$$u_{\rm sol}(x',T') = 2\eta\exp[-2i\xi T' - 2i(\xi^2-\eta^2)x']\,\mathrm{sech}2\eta(T'+2\xi x'). \qquad (4)$$

Actually, the propagation equation (1) must include some more effects in the optical fiber, such as transmission loss in fibers, higher-order dispersion, delayed nonlinear response, etc., to describe the propagation of a pulse in an optical fiber. If we just take into account the presence of the self-steepening effect in Eq.(1), we are able to derive an exact solution of solitary wave for the so-called modified nonlinear schrödinger equation.[4−5] Except for this case, other effects are extensively studied in the framework of perturbation theories.[6−9] Examining the existing theories for this study, we find that they are based either directly on the sophisticated inverse scattering transform, or indirectly on some knowledge of the inverse scattering transform. Another feature of these perturbation theories is that they are performed in the so-called retarded time frame, for the purpose of a direct application of established perturbation theories to the standard nonlinear Schödinger equation, but it really brings some confusion in physical understanding. Because of such drawbacks mentioned here, it is helpful to develop a new simple theory. This is the main purpose of the present paper.

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Ⅱ. THEORY

Now, in the anomalous GVD regime, we start with the normalized propagation equation (1) under the influence of perturbation

$$iu_x + iu_t + \frac{1}{2}u_{tt} + |u|^2 u = i\varepsilon R[u] \tag{5}$$

in the absence of perturbations. Equation(5) possesses a single soliton solution in the form of Eq.(4) with slightly different parameters, we write it as

$$u^{(0)} = 2\eta \exp(-i\theta)\mathrm{sech}z. \tag{6}$$

From the point of view of the classical perturbation theroy, under the action of perturbation, the parameters of soliton will undergo a variation pertaining to slow scales, and a linear dispersive wave will be generated as a correction to the soliton. Based on these concepts, we introduce a slow space variable $x_1 = \varepsilon x$, and assume that $z = 2\eta(t + \varepsilon^{-1}T_0 + T_1), \theta = (\xi\eta^{-1}z - \varepsilon^{-1}\theta_0 - \theta_1)$, they depend on x_1 through $\xi, \eta, T_0, T_1, \theta_0, \theta_1$. It is obvious that z has the sense of space variable in the reference frame co-moving with optical solition. In $x - z$ reference frame, we seek for the solution of Eq.(5) in the form

$$u(x, x_1, z) = [2\eta g(z) + \varepsilon v(x, z)]\exp[-i\theta(x_1, z)], \tag{7}$$

where $g(z)$=sech(z). Inserting Eq.(7) into Eq.(5), setting coefficients of ε^0 and ε^1 to zero, we obtain the zeroth-order equation and the first-order equation for $v(x, z)$. The zeroth-order equation requires that the parameters T_0, θ_0 and ξ, η satisfy the following relations

$$T_{0x_1} = (2\xi - 1), \tag{8}$$

$$\theta_{0x_1} = 2(\xi^+ + \eta^2). \tag{9}$$

By virtue of relations (8) and (9), the somewhat tedious first-order equation is simplified to

$$\begin{aligned} &iv_x + 2\eta^2 v_{zz} + 8\eta^2 \mathrm{sech}^2 z v \\ &\quad + 4\eta^2 \mathrm{sech}^2 z v^* - 2\eta^2 v \\ &\quad = i\mathrm{Re}^{i\theta} - i2\eta_{x_1 1}\varphi_1(z) + i4\eta^2 T_{1x_1}\varphi_2(z) \\ &\quad - 2\xi_{x_1}\phi_2(z) + 2\eta(\theta_{1x_1} - 2\xi T_{1x_1})\phi_1(z), \end{aligned} \tag{10}$$

where v^* denotes the complex conjugate of v, and we have defined $\phi_1(z) = \mathrm{sceh}z, \phi_2(z) = z\mathrm{sech}z$, $\varphi_1(z) = \mathrm{sech}z(1 - z\tanh z), \varphi_2(z) = \mathrm{sech}z\tanh z$. Separating the real and imaginary parts of Eq.(10) by setting $v = v_1 + iv_2$ yields

$$\begin{aligned} &v_{1x} + 2\eta^2 \hat{L}_1 v_2 = F_1(z) \\ &\quad \equiv \mathrm{Re}(\mathrm{Re}^{i\theta}) - 2\eta_{x_1}\varphi_1(z) + 4\eta^2 T_{1x_1}\varphi_2(z), \end{aligned} \tag{11}$$

$$\begin{aligned} &v_{2x} - 2\eta^2 \hat{L}_2 v_1 = F_2(z) \\ &\quad \equiv \mathrm{Im}(\mathrm{Re}^{i\theta}) - 2\eta(\theta_{1x_1} - 2\xi T_{1x_1})\phi_1(z) \\ &\quad + 2\xi_{x_1}\varphi_2(z), \end{aligned} \tag{12}$$

where we have defined two self-adjoint operators $\hat{L}_1 = \partial_{zz} + (2\mathrm{sech}^2 z - 1)$ and $\hat{L}_2 = \partial_{zz} + (6\mathrm{sech}^2 z - 1)$.

Befroe we set to solving the real simultaneous equations (11) and (12), we must handle their homogeneous counterpart, which yields the following generalized eigenvalue problem

$$\hat{L}_1\phi = \lambda\varphi, \tag{13}$$

$$\hat{L}_2\phi = \lambda\varphi. \tag{14}$$

The above eigenvalue problem is not Hermitian; using the operator $\hat{L}_2$ to act on both sides of Eq.(13) will lead to $\hat{L}_2\hat{L}_1\phi = \lambda^2\phi$, and the same action of $\hat{L}_1$ on Eq.(14) yields $\hat{L}_1\hat{L}_2\varphi = \lambda^2\varphi$. Since $\hat{L}_1$ and $\hat{L}_2$ are all self-adjoint, $\hat{L}_1\hat{L}_2$ and $\hat{L}_2\hat{L}_1$ are a pair of adjoint operators, thus, ϕ and φ are adjoint eigenstates of each other. The eigenstates of operator $\hat{L}_2\hat{L}_1$ consist of a continuous spectrum $\phi(z, k)$ and two degenerate discrete states $\phi_1(z)$ and $\phi_2(z)$ which are defined as above. It is the same for the operator $\hat{L}_1\hat{L}_2$, its eigenstates consist of $\varphi(z, k)$ and $\varphi_1(z)$ and $\varphi_2(z)$. Returning to the original eigenvalue problem Eqs.(13) and (14), we find that $\phi_2(z)$ and $\varphi_1(z)$ do not include in eigenstates, but they are necessary for the closure of these states, and there exist relations $\hat{L}_1\phi_2(z) = -2\varphi_2(z), \hat{L}_2\varphi_1(z) = 2\phi_1(z)$. Analytic expressions of continuous spectrum $\phi(z, k)$ and $\varphi(z, k)$ are

$$\begin{aligned} &\phi(z, k) = \frac{1}{\sqrt{2\pi}(k^2 + 1)}(1 - 2ik\tanh z - k^2)e^{ikz}, \\ &\text{for} \quad \lambda = -(k^2 + 1); \end{aligned} \tag{15}$$

$$\begin{aligned} \varphi(z, k) = &\frac{1}{\sqrt{2\pi}(k^2 + 1)}(1 - 2\mathrm{sech}^2 z \\ &- 2ik\tanh z - k^2)e^{ikz}, \\ &\text{for} \quad \lambda = -(k^2 + 1). \end{aligned} \tag{16}$$

We have strictly proved that $\phi = \{\phi(z, k), \phi_1(z), \phi_2(z)\}$ and $\varphi = \{\varphi(z, k), \varphi_1(z), \varphi_2(z)\}$ constitute a

complete set of orthogonal adjoint base, which guarantees the following expansions.

$$v_1(x,z)=\int_{-\infty}^{+\infty}\tilde{v}_1(x,k)\varphi(z,k)\mathrm{d}k + \tilde{v}_{11}(x)\varphi_1(z)+\tilde{v}_{12}(x)\varphi_2(z), \quad (17)$$

$$v_2(x,z)=\int_{-\infty}^{+\infty}\tilde{v}_2(x,k)\phi(z,k)\mathrm{d}k + \tilde{v}_{21}(x)\phi_1(z)+\tilde{v}_{22}(x)\phi_2(z). \quad (18)$$

Inserting expansions (17) and (18) into Eqs.(11) and (12), and after some analysis, we find that the discrete states are directly related to the secular terms. In order to eliminate these terms, the source $F_1(z)$ must be orthogonal to the discrete states $\phi_1(z)$ and $\phi_2(z)$, and $F_2(z)$ orthogonal to $\varphi_1(z)$ and $\varphi_2(z)$ as well. These conditions will lead to four equations governing the parameters' slow variation

$$\eta_{x_1}=\frac{1}{2}\int_{-\infty}^{\infty}\mathrm{Re}(\mathrm{Re}^{i\theta})\mathrm{sech}z\mathrm{d}z, \quad (19)$$

$$T_{1x_1}=-\frac{1}{4\eta^2}\int_{-\infty}^{\infty}\mathrm{Re}(\mathrm{Re}^{i\theta})z\mathrm{sech}z\mathrm{d}z, \quad (20)$$

$$\xi_{x_1}=-\frac{1}{2}\int_{-\infty}^{\infty}\mathrm{Im}(\mathrm{Re}^{i\theta})\mathrm{tanh}z\mathrm{sech}z\mathrm{d}z, \quad (21)$$

$$\theta_{1x_1}=2\xi T_{1x_1}+\frac{1}{2\eta}\int_{-\infty}^{\infty}\mathrm{Im}(\mathrm{Re}^{i\theta})\mathrm{sech}z(1-z\mathrm{tanh}z)\mathrm{d}z. \quad (22)$$

Under the above conditions, the discrete states have no contribution to the first-order correction, thus, the firstorder correction turns out to be truly dispersive wave

$$\begin{aligned}\Delta u &= v(x,z)e^{-i\theta}\\ &= e^{-i\theta}\int_{-\infty}^{\infty}\frac{f_2(k)}{2\eta^2(k^2+1)}\{1-\alpha^+(k)\exp[i2\eta^2(k^2+1)x]\\ &-\alpha^-(k)\exp[-i2\eta^2(k^2+1)x]\}\varphi(z,k)\mathrm{d}k\\ &-ie^{-i\theta}\int_{-\infty}^{\infty}\frac{f_1(k)}{2\eta^2(k^2+1)}\{1-\beta^+(k)\exp[i2\eta^2(k^2+1)x]\\ &-\beta^-(k)\exp[-i2\eta^2(k^2+1)x]\}\phi(z,k)\mathrm{d}k, \quad (23)\end{aligned}$$

where

$$f_1(k)=\int_{-\infty}^{\infty}\mathrm{Im}(\mathrm{Re}^{i\theta})\varphi^*(z,k)\mathrm{d}z, \quad (24)$$

$$f_2(k)=\int_{-\infty}^{\infty}\mathrm{Re}(\mathrm{Re}^{i\theta})\phi^*(z,k)\mathrm{d}z, \quad (25)$$

and $\alpha^+(k),\alpha^-(k),\beta^+(k),\beta^-(k)$ are to be determined by appropriate solution-fixing conditions of the specific problem.

Ⅲ. CONCLUSION

In this paper, we have studied the propagation of soliton in the monomode optical fiber. The dynamic equation we considered is nonlinear Schrödinger equation in laboratory reference frame, but not in "retarded time" frame. Relations of parameters' slow variation of optical soliton and the dispersive wave radiating from it have been derived by a different approach from the existing theory,[6] in which the complicated construction of expansion space relies one the tools of inverse scattering transform (IST). As the approach presented here totally gets rid of any dependence on the sophisticated IST, we would rather introduce it as an alternative way for the study of this subject.

REFERENCES

[1] Yu. S. Kivshar and B. A. Malomed, *Rev. Mod. Phys.*, **61** (1989),763.

[2] H. A. Haus and W. S. Wong, *Rev. Mod. Phys.*, **68**(1996), 423.

[3] G. P. Agrawal, Nonlinear Fiber Optics, San Diego, Academic Press, INC.(1989)

[4] D. Anderson and M. Lisak, *Phys. Rev.*, **A27**(1983),1393.

[5] Z. Y. Chen and N. N. Huang, *Phys. Rev.*, **A41**(1990), 4066.

[6] D. J. Kaup, *Phys. Rev.*, **A42**(1990),5689.

[7] D. J. Kaup, *Phys. Rev.*, **A44**(1991),4582.

[8] S. Burtsev and D. J. Kaup, *J. Opt. Soc. Am.*, **B14**(1997), 627.

[9] J. N. Elgin, *Phys. Rev.*, **A47**(1993),4331.