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CRITICAL MAGNETIC SHEAR FOR THE TOKAMAK IDEAL INTERNAL KINK MODE IN THE ION KINETIC REGIME*

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It is shown that for the ideal internal kink mode of tokamak plasma, there exists a critical magnetic shear s_{0c} for its excitation when the finite ion Larmor radius effect is considered in the mode singular layer. This can be related to the sudden onset of the sawtooth and also to the stabilization of the sawtooth by the trapped hot ions observed experimentally. The alpha particle effect on this mode is revisited and some new results are reported.

Keywords: ideal internal kink, critical magnetic shear, sawtooth stabilization
PACC: 5235, 5255, 5225

I. INTRODUCTION

The tokamak internal kink mode has long been a subject of theoretical and experimental studies. It has been shown that the linear behavior of the ideal internal kink,[1] the resistive internal kink[2] and the fishbone[3,4] can be described by an unified dispersion relation[5−7] in the small Larmor radius limit where the singular layer physics is governed by the magnetohydrodynamic (MHD) equations (the fluid description). Using this unified dispersion relation, the sawtooth stabilization and the fishbone excitation can be attributed to the action of different parts of the energetic ion response. The linear stabilization of the ideal internal kink mode or the resistive internal kink mode which then leads to the sawtooth-free discharge with $q_0 < 1$, is due to the so-called zero-frequency response while the fishbone excitation is due to the non-zero frequency part of this response. So far, most of the analyses are restricted to this fluid description. However, it has already been noted that the fluid description is not adequate for high temperature plasmas where the ion thermal Larmor radius can be larger than the width of the mode singular layer. In many medium size and large size tokamak machines and in future tokamak reactors plasma parameters drop in the ion kinetic regime so that the finite ion Larmor radius effect should be taken into account in the layer physics. To include it, the ion gyro-kinetic equation should be used to describe the ion physics while for the electrons, an adiabatic response from the drift kinetic equation is an adequate one.[8,9]

Physically, the ion kinetic regime is characterized by the parameter criterion

$$\rho_{\rm i} > \delta, \tag{1}$$

where $\rho_{\rm i}$ is the thermal ion Larmor radius while δ is the width of the singular layer. More adequately, as can be seen from present analysis, the parameter characterizing the finite Larmor radius effect is

$$\xi = \left(\frac{\omega_{\rm A}}{\omega_{*\rm i}}\right) s_0 \left(\frac{\rho_{\rm i}}{r_1}\right), \tag{2}$$

where $\omega_{\rm A}, \omega_{*\rm i}, s_0$ and r_1 are the Alfven frequency, the ion diamagnetic frequency, the magnetic shear, and the radius of the $q = 1$ surface, respectively. All parameters are evaluated at the $q = 1$ surface. If ξ has a value of the order of unity, the finite Larmor radius effect will be very significant. The stability domain for the fishbone mode changes much in this case.[10] In the present paper, we indicate that the stability domain of the ideal internal kink mode both with and without energetic ions is quite different from that obtained by the fluid theory. A profound feature is the presence of some critical shear value, the mode becomes unstable only when the realistic shear surpasses this value. This means that only when the q_0 becomes lower than a critical one, q_{0c}, the ideal kink mode then suddenly onsets. If during the whole heating phase, the real q_0 cannot be lower than this value, the mode remains stable and the sawtooth is stabilized. This seems to be closely related with some recent experimental observations that during the whole sawtooth period, the central q value was always lower than unity (typically, 0.8–0.9).

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Ⅱ. DISPERSION RELATION, STABLE DOMAIN, AND MODE GROWTH RATE

To make easy comparison, the dispersion relation used in this paper will be written in the same form as in related literatures[9,11]

$$F(\nu)[\omega(\omega-\omega_{*\mathrm{i}})]^{1/2}=\mathrm{i}\omega_{\mathrm{A}}(\lambda_{\mathrm{h}}+\lambda_{\mathrm{K}}), \tag{3}$$

where

$$F(\nu)=4\frac{(2\nu-1)^{1/2}\Gamma^2(5/4+\nu/2)}{(2\nu+1)^{3/2}\Gamma^2(3/4+\nu/2)}, \tag{4}$$

$$\nu^2=\frac{1}{4}-\frac{(\omega-\omega_{*\mathrm{i}})(\omega-\omega_{*\mathrm{e}})}{(1+\tau)(\rho_{\mathrm{i}}/r_1)^2 s_0^2\omega_{\mathrm{A}}^2}, \tag{5}$$

with $\tau = T_{\mathrm{e}}/T_{\mathrm{i}}$. In the following we assume a parabolic q-profile within the $q=1$ surface as

$$q=q_0+(1-q_0)x^2,\quad x=r/r_1. \tag{6}$$

This leads to a simple relation between the magnetic shear and the q on the magnetic axis

$$s_0=\left(\frac{r}{q}\frac{\mathrm{d}q}{\mathrm{d}r}\right)_{r_1}=2(1-q_0). \tag{7}$$

On the right hand side of Eq.(3), the quantity $\lambda_{\mathrm{h}} = \omega_{\mathrm{A}}\gamma_{\mathrm{mhd}}$ measures the energy variation of the interior part, where γ_{mhd} is defined as

$$\gamma_{\mathrm{mhd}}=\omega_{\mathrm{A}}\frac{3\pi}{2}\left(\frac{r_1}{R}\right)^2[\hat{\beta}_{\mathrm{p}}^2-\hat{\beta}_{\mathrm{pc}}^2], \tag{8}$$

with $\hat{\beta}_{\mathrm{pc}}^2 = 13/144$ for the choice of the q-profile of Eq.(6),[1] while

$$\hat{\beta}_{\mathrm{p}}=-\frac{8\pi}{B_{\mathrm{p}}^2(r_1)}\int_0^1 \mathrm{d}x x^2\frac{\mathrm{d}p}{\mathrm{d}x}. \tag{9}$$

Without loss of generality, we assume that the plasma pressure has a parabolic profile as well within the $q=1$ surface

$$p(x)=p_0(1-c_{\mathrm{p}}x^2), \tag{10}$$

then we have $\hat{\beta}_{\mathrm{p}} = \frac{c_{\mathrm{p}}}{2}\beta_0\left(\frac{R}{r}\right)^2$, where β_0 is the toroidal beta value of the plasma. According to the internal kink mode theory in the fluid limit, the ideal internal kink mode becomes unstable when

$$\gamma_{\mathrm{mhd}}>\frac{\omega_{*\mathrm{i}}}{2}, \tag{11}$$

while for $\gamma_{\mathrm{mhd}} < 0$, the resistive internal kink mode is the main candidate for the sawtooth oscillation. We now consider just the intermediate region that

$$\frac{\omega_{*\mathrm{i}}}{2}>\gamma_{\mathrm{mhd}}>0, \tag{12}$$

and indicate that the stability depends on the magnetic shear value.

A. Stable domain without energetic ions ($\lambda_{\mathrm{K}}=0$)

Consider the marginal stable state so that ω is real and so are the value ν and the function $F(\nu)$. The imaginary part of Eq.(3) gives

$$F(\nu)[\omega(\omega_{*\mathrm{i}}-\omega)]^{1/2}=\gamma_{\mathrm{mhd}}. \tag{13}$$

For $\omega = \omega_{*\mathrm{i}}/2$, we obtain the marginal state and we have

$$\gamma_{\mathrm{mhd}}=F(\nu_{\mathrm{c}})\frac{\omega_{*\mathrm{i}}}{2} \tag{14}$$

with

$$\nu_{\mathrm{c}}=\frac{1}{2}\sqrt{1+\frac{(2\tau+1)}{(1+\tau)\xi^2}}, \tag{15}$$

where ξ is given in Eq.(2). The result is plotted in Fig.1. A profound feature is that within the parameter window given by Eq.(12), there exists a critical

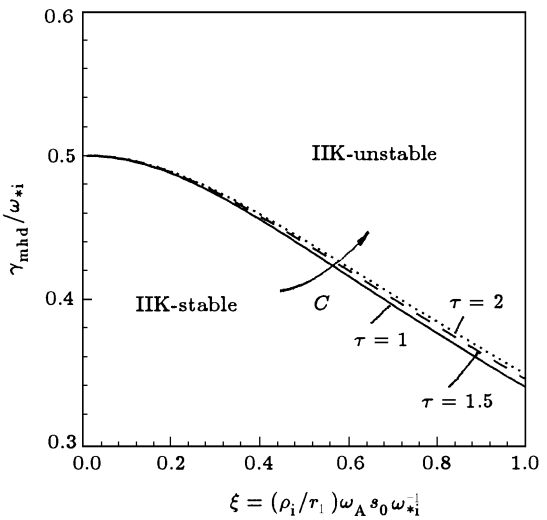


Fig.1. Marginal stability curve for the ideal internal kink mode (ⅡK) in the $(\gamma_{\mathrm{mhd}}, \xi)$ plane for different τ-values. Below the curves is the stable regime and above them is the unstable regime. To follow a sawtooth period, a curve C is drawn, corresponding to the evolution of relevant parameters.

shear s_{0c} and the mode becomes unstable only when the realistic shear surpasses the critical one. Consider the evolution of plasma parameters during heating, we find that of all parameters relevant to the internal kink mode, the most changeable one is the shear, other parameters involved also vary but in relatively slower way. A typical trajectory curve C in this figure is then drawn to follow a sawtooth period. The electron heating increases the electron temperature and correspondingly the central current density, this leads to the drop of q_0 and the increase in s_0 and ξ, the increase

in the plasma pressure gradient raises the value $\gamma_{\rm mhd}$, finally, the trajectory enters into the unstable region, and the ideal internal kink mode develops suddenly. This is also shown in Fig.2 where the mode frequency and the growth rate are plotted against $\gamma_{\rm mhd}$. Figure 3 shows the growth rate versus $\gamma_{\rm mhd}$ for different ξ.

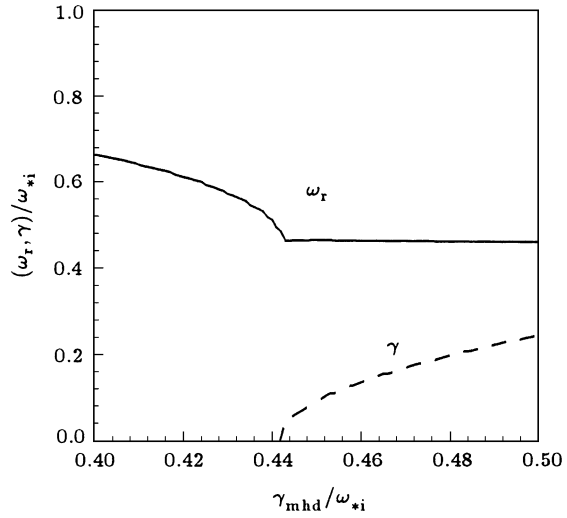


Fig.2. The mode frequency and the growth rate of IIIK versus $\gamma_{\rm mhd}$ for $\tau = 1.5, \xi = 0.5$. For the selected parameters, the mode becomes unstable when $\gamma_{\rm mhd}/\omega_{*\rm i} > 0.4403$.

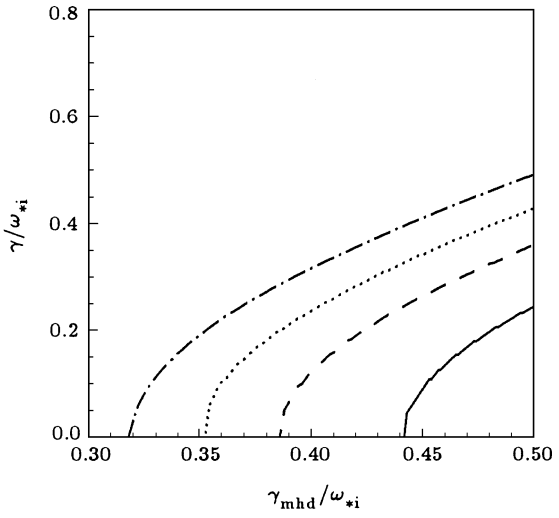


Fig.3. The growth rate versus $\gamma_{\rm mhd}$ for different ξ's. From the left to the right the ξ-values are 0.5, 0.8, 1.0, and 1.2, respectively, $\omega_{*\rm i} = 1.0; \tau = 1.5$.

III. ENERGETIC ION EFFECT

The zero-frequency response to the internal mode displacement functions was analyzed for a highly anisotropic pressure case (most of the hot ions were assumed to be trapped) to describe the ICRF heating produced energetic ions[12] and results in the following expression for $\lambda_{\rm K}$:

$$\lambda_{\rm K}(0) = \frac{4\pi^3}{B_{\rm p}^2 s_0}\left(\frac{r_1}{R}\right)\left(\frac{2}{m_{\rm h}}\right)^{3/2}\int_0^1 {\rm d}x x \cdot \int_{1-\varepsilon}^{1+\varepsilon} {\rm d}\Lambda \Lambda I_{\rm b}\frac{I_{\rm c}I_{\rm d}-I_{\rm q}^2}{I_{\rm d}}\int_0^\infty {\rm d}E E^{3/2}\frac{\partial F_{\rm h}}{\partial x}. \quad (16)$$

The definitions of all relevant quantities are referred to Refs.[12,13]. We only indicate that the dependence of this term on the shear s_0 is mainly due to the explicit part, i.e., $\lambda_{\rm K}(0) \propto \dfrac{1}{s_0}$, other part appearing implicitly in the integral gives a weak relation with the shear. If we write

$$\lambda_{\rm K}(0) = \frac{\beta_{\rm ph}}{s_0}\left(\frac{r_1}{R}\right)\Lambda(0), \quad (17)$$

then we can approximately write

$$\Lambda_{\rm K}(0) = b_0 + b_1 s_0, \quad (18)$$

with b_0 and b_1 being functions of $r_{\rm h}/r_1$, the scale length of the heating region to that of the q=1 surface, and the anisotropy parameter $p_{//\rm h}(0)/p_{\perp\rm h}(0)$. Accordingly, the marginal stability condition is determined by

$$F(\nu_{\rm c})\left(\frac{\omega_{*\rm i}}{2\omega_{\rm A}}\right) = \bar{\lambda} = \lambda_{\rm h} + \lambda_{\rm K}(0). \quad (19)$$

The function $\bar{\lambda}$ versus s_0 is then similar with that of $\gamma_{\rm mhd}$ versus ξ shown in Fig.1. Keep this in mind, the stable regime for plasma beta value is determined by

$$\lambda_{\rm h} = \bar{\lambda} - \lambda_{\rm K}(0). \quad (20)$$

For typical experimental parameters, for example, of the JET, $\lambda_{\rm h}$ has a value of 10^{-2}, while the value of function $\Lambda_{\rm K}(0)$, being a function of q_0 or s_0, is ussually negative and depends on the heating deposition and the degree of anisotropy. It is seen from Eq.(17) that for small s_0, the $\lambda_{\rm K}(0)$ term can be very large so that a small fraction of trapped hot ions can make the mode stable. For medium s_0, say $s_0 = 0.5$ i.e., $q_0 = 0.75$, for $r_{\rm h}/r_1 = 0.6$ to 0.7, and highly anisotropy (cases d, f in Fig.7 of Ref.[13]), $\Lambda_{\rm K}(0)$ would be around 0.4–0.5; if the hot ion's poloidal beta arrives a value of the order of 10^{-1}, the stabilizing effect would be very significant.

IV. SAWTOOTH-FREE OPERATION OF D,T BURNING PLASMA

The fusion alpha effect on the internal kink mode has been analyzed by many authors.[6,13,14] The alpha

particles have an isotropic distribution so that their response to the internal kink mode is somewhat different from the ICRF heating produced energetic ions.[6] One of these differences is that the adiabatic part of the response function is negligible and the stabilization part of this response (the zero-frequency response) can be written as follows[6]:

$$\lambda_{\rm K}(0) = -\frac{\sqrt{2}}{2s_0}\left(\frac{r_1}{R}\right)^{3/2}\hat{\beta}_{p\alpha}, \tag{21}$$

with

$$\begin{aligned}\hat{\beta}_{p\alpha} &= -\frac{8\pi}{B_{\rm p}^2(r_1)}\int_0^1 {\rm d}x x^{3/2} I_\beta(x,q_0)\frac{{\rm d}p_\alpha}{{\rm d}x},\\ I_\beta(x,q_0) &= \frac{3\pi R}{8r_1 x}\int_{1-r_1/R}^{1+r_1/R}{\rm d}\Lambda\frac{I_{\rm q}^2 I_0}{I_{\rm c}+sI_{\rm d}}.\end{aligned} \tag{22}$$

All symbols used here are the same as in Refs.[6,8]. In the following, only the simplest tokamak configuration of concentric magnetic surfaces is considered though an extension to the non-circular cross-section calculation was available.[14]

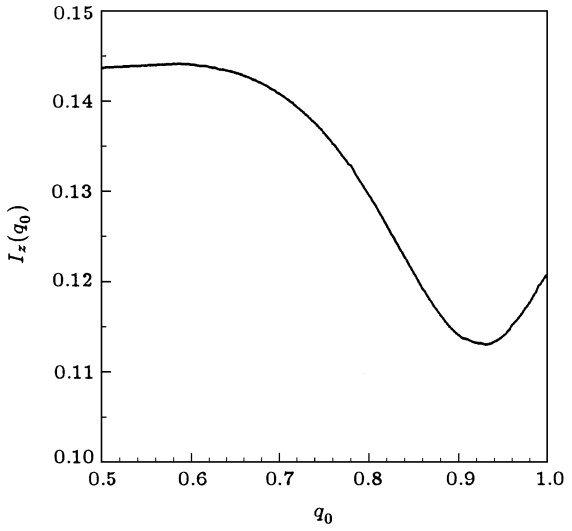


Fig.4. Function $I_Z(q_0)$ in Eq.(24) versus q_0.

We assume the alpha particle pressure has also a parabolic form

$$p_\alpha(x) = p_{\alpha 0}(1-b_\alpha x^2), \qquad x \le 1. \tag{23}$$

The parameter b_α is relevant with alpha particle transport and the radius of the $q=1$ surface. In this case, we have

$$\hat{\beta}_{\rm p\alpha} = \frac{3\pi}{2}b_\alpha\beta_{\rm p\alpha}(0)I_z(q_0) \tag{24}$$

and the function $I_z(q_0)$ is shown in Fig.4. Furthermore, the alpha particle parameters should be determined from its transport processes that are rather involved so that we carry out only the simplest analysis here to estimate the relevant quantities. We assume local deposition of the alphas particle and its pressure is determined only by the thermalization process. In this case we have

$$\begin{aligned}\beta_{\rm p\alpha}(0) =& \frac{16\pi nT}{B^2}\left(\frac{R}{r_1}\right)^2 C_\alpha(T),\\ C_\alpha(T) =& 4.51\times 10^{14} < \sigma\nu >_{DT}\\ & \cdot\{1 - T/5 + 0.37(T/5)^{7/4}\},\end{aligned} \tag{25}$$

where the electron and ion temperatures were assumed to be equal and measured in 10 keV. $C_\alpha(T)$ will increase with T abruptly and is plotted in Fig.5.

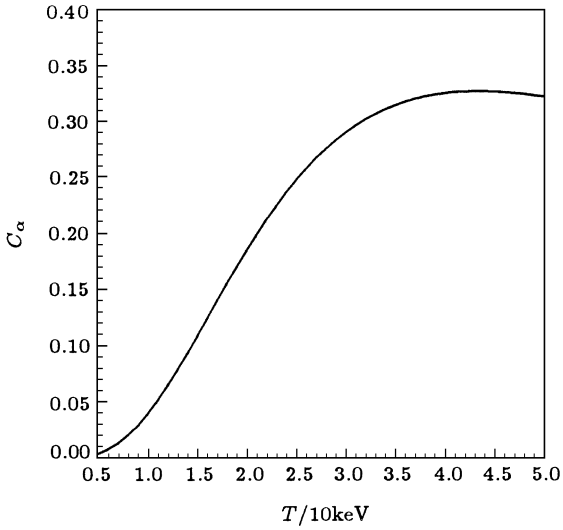


Fig.5. Function $C_\alpha(T)$ versus T. This D, T fusion reaction rate related quantity appears in Eq.(25) in the text.

Substitute all relevant quantities into Eq.(21), we finally obtain

$$\lambda_{\rm K}(0) = -\frac{3\sqrt{2}}{4}\pi b_\alpha\beta_{\rm th}(0)\left(\frac{R}{r_1}\right)^{1/2}(I_z(q_0)/s_0), \tag{26}$$

and

$$b_\alpha = 1 - \frac{(1-b_{\rm th})C_\alpha(r_1)}{C_\alpha(0)}, \tag{27}$$

$$b_{\rm th} = 1 - p(r_1)/p(0). \tag{28}$$

A similar discussion as the above can be carried out for the stabilization condition of the ideal internal kink mode with the alpha particle effect. Firstly, we can define the parameter region

$$\frac{\omega_{*\rm i}}{2\omega_{\rm A}} \ge (\lambda_{\rm h} + \lambda_{\rm K}(0)) \ge 0 \tag{29}$$

in the (T_0, s_0) plane. Note that the ratio of the ion diamagnetic drift frequency to the Alfven frequency is also a function of the ion temperature

$$\frac{\omega_{\rm A}}{\omega_{*\rm i}} = 40\left(\frac{r_1}{R}\right)\frac{r_1 B^2}{b_{\rm th}\sqrt{n_{\rm e0}T}}. \tag{30}$$

For given plasma density profile and device parameters $(R, B, r_1,)$ as well as the pressure profile ($b_{\rm th}$ and then b_α), the solution of the dispersion relation will depend on the two parameters: s_0 and T. Marginal stabilization region on the s_0, T_0 plane can be determined from the solution of the followin equation:

$$F(\nu_{\rm c}) = \frac{2\omega_{\rm A}}{\omega_{*\rm i}}(\lambda_{\rm h} + \lambda_{\rm K}(0)). \qquad (31)$$

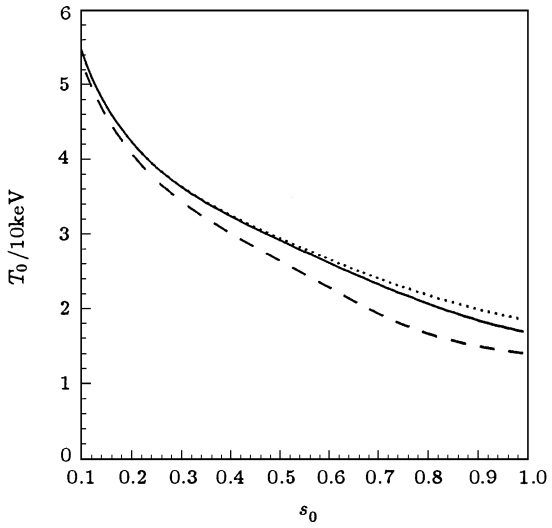


Fig.6. The marginal stability curve for IIK for the D, T burning plasma with the alpha particle stabilization (the solid curve). Also shown are the curve δ of $\bar{\lambda} = 0$ (dashed) and $\bar{\lambda} = \omega_{*\rm i}/2\omega_{\rm A}$ (dotted).

Either Eq.(30) or Eq.(31) is a function of the ion temperature T and the shear s_0 (or the central safety factor q_0). We can then derive the sawtooth free regime in the (T_0, s_0) plane for the ignited D,T plasma. These are shown in Figs.6 and 7, respectively.

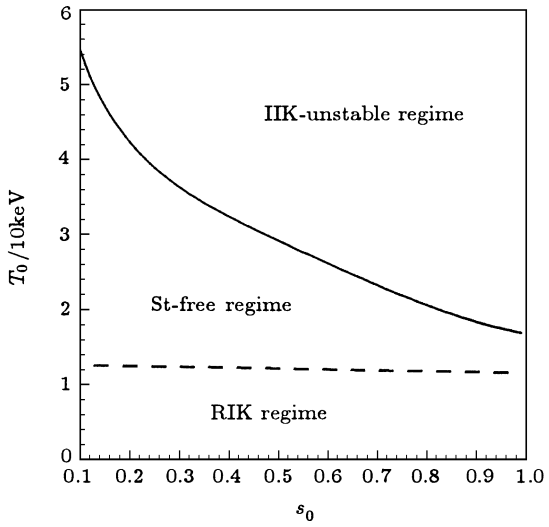


Fig.7. Sawtooth-free (St-free) window for the D, T burning tokamak plasma in the (T_0, s_0) plane with and without the alpha particle stabilization. The solid curve is for the case with the alpha effect and the dashed one is for the case without such effect. Below the dashed curve is the regime for the resistive internal kink mode (RIK) that we have not analyzed in this paper. The regime above the solid curve is the ideal internal kink mode (IIK) unstable regime. $\beta_{\rm th} = 0.2$.

V. CONCLUSION

For large and reactor tokamaks, the main unstable mode responsible for the sawtooth process should be the ideal internal kink mode. A suitable description for this mode should rely on the FLR theory. The dispersion relation is analyzed and is found that there exists some critical shear, and only when the realistic shear surpasses it the ideal internal kink mode becomes unstable. Due to the zero-frequency stabilization of fusion alpha particle, the D,T burning plasma can be in some sawtooth quiescent regime if the central temperature is not too high.

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