Volume 9, Number 7
1009-1963/2000/09(07)/0490-04
CHINESE PHYSICS
July, 2000
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THE NONLINEAR QCD DYNAMICAL EQUATIONS WITH HIGHER ORDER EFFECTIVE COUPLING

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(Received 10 January 1999)

Through studying the nonlinear QCD dynamical equations with higher order effective coupling, we find that shadowing effect is substantially weaker than that neglecting higher order effective coupling. This conclusion is very important for explaining the data from DESY *ep* collider HERA.

Keywords: nonlinear QCD dynamical equations, HERA experiments
PACC: 1235

I. INTRODUCTION

The behavior of parton distribution at small-x region has recently been the important subject.[1−4] The nonlinear QCD dynamical equation was proposed to describe the shadowing effect in the small-x region.[5,6] The properties of the nonlinear QCD dynamical equation have been studied by many authors.[2,6−10] There are two ways to study the nonlinear QCD dynamical equation. One is numerical solving the nonlinear QCD dynamical equation by using the input distribution $xg(x,Q^2)$ at $Q^2 = Q_0^2$. The other is using the semi-classical approximation to study the properties of the nonlinear QCD dynamical equation.[2,6−10] In the previous studies[2,6−10] people used the leading order QCD effective coupling $\alpha_{LO}(Q^2)$ to study the nonlinear QCD dynamical equation, where

$$\alpha_{LO}(Q^2) = \frac{12\pi}{(33-2n_f)\log(Q^2/\Lambda^2)}. \qquad (1)$$

When Q^2 is large, using $\alpha_{LO}(Q^2)$ to study the nonlinear QCD dynamical equation is appropriate. But it is inappropriate in the case when Q^2 is low, such as Q^2=4GeV^2, because of the higher order notably affecting the results. The purpose of this paper is to study the nonlinear QCD dynamical equation in the small-x, comparatively low Q^2 region, but not lower than 4GeV^2 so that perturbative QCD can be applied. We find that some important conclusions of the nonlinear QCD dynamical equation should be rewritten.

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II. SOLUTIONS OF THE NONLINEAR QCD DYNAMICAL EQUATION IN HIGHER ORDER

The nonlinear QCD dynamical equation for the gluon distribution at small-x region in the DLLA is given by

$$\frac{\partial^2 xg(x,Q^2)}{\partial\ln(1/x)\partial\ln(Q^2)} = \frac{3\alpha_s(Q^2)}{\pi} xg(x,Q^2) - \frac{81\alpha_s^2(Q^2)}{16R^2Q^2}[xg(x,Q^2)]^2. \qquad (2)$$

If $\alpha_s(Q^2)$ is used by the leading order coupling formula (1), then Eq.(2) is as follows:

$$\partial_y\partial_t G(y,t) = c_1 G(y,t) - \lambda\exp[-t-\exp(t)]G^2(y,t), \qquad (3)$$

where y=ln(1/x), t=ln[ln(Q^2/Λ^2)], $G(y,t) = xg(x, Q^2)$ and c_1=12/(11-2n_f/3), λ=$9\pi^2c/16R^2\Lambda^2$. Using the semi-classical approximation which keeps only the first-order derivatives of the function $z(y,t)$, Eq.(3) is rewritten as[8]

$$\partial_y z(y,t)\partial_t z(y,t) = c_1 - \lambda\exp[-t-\exp(t)+z(y,t)], \qquad (4)$$

where $z(y,t)$=log[$G(y,t)$].Equations(2)–(4) have been studied by many authors.[2,6−10] Some authors applied Eqs.(2)–(4) to study the nuclear shadowing[11] at Q_0^2=4GeV^2. However, it will be shown that the shadowing in the nonlinear QCD dynamical equation in higher order $\alpha_s(Q^2)$ is weaker than that in Eqs.(2)–(4). So the pervious researches on both nucleon shadowing and nuclear shadowing based on the nonlinear

QCD dynamical equation should be restudied. Due to the complication in computing the nonlinear QCD dynamical equation in higher order, the equation has not been derived to higher order approximation in literatures. It should be noted that different order of running coupling constant has different scale dependence, and such difference in scale-dependence is minimized when coeffcient functions or evolution kernels at the same order in perturbative expansion are used, namely, any inconsistent use of the orders of running coupling constant and evolution kernels will break the perfect cancellation in perturbation theory. However, in this paper, in order to study the nonlinear QCD dynamical equation in higher order, we make approximation that difference in scale-dependence is not notable when evolution kernels at the different order in perturbative expansion, that is to say, the QCD effective coupling is calculated to higher order whereas the evolution kernels keep to leading order. Now let us consider the nonlinear QCD dynamical equation in higher order QCD effective coupling α_s. By renormalization technique, the effective coupling in QCD can be computed to order $\alpha_{LO}^2(Q^2)$[12]

$$\alpha_s(Q^2) = \alpha_{LO}(Q^2)[1 - \frac{1}{4\pi}\frac{306 - 38n_f}{33 - 2n_f} \cdot \alpha_{LO}(Q^2)\ln\ln(Q^2/\Lambda)]. \quad (5)$$

To order α_{LO}^2 (Q^2), the nonlinear QCD dynamical equation is

$$\partial_y\partial_t G(y,t) = c_1(1 - c_1c_2\frac{t}{\mathrm{e}^t})G(y,t) - \lambda(1 - c_1c_2\frac{t}{\mathrm{e}^t})^2 \cdot \exp[-t - \exp(t)]G^2(y,t), \quad (6)$$

where c_2=(306–38n_f)/[12(33–2n_f)]. The semi-classical approximation of Eq. (6) is as follows:

$$\partial_y z(y,t)\partial_t z(y,t) = c_1(1 - c_1c_2\frac{t}{\mathrm{e}^t}) - \lambda(1 - c_1c_2\frac{t}{\mathrm{e}^t})^2\exp[-t - \exp(t) + z]. \quad (7)$$

We use the method of characteristic[2,8] to solve Eq.(7). For an equation of the following general form:

$$F(p, q, t, y, z) = 0, \quad (8)$$

where

$$p = \partial_t z(y,t), \qquad q = \partial_y z(y,t), \quad (9)$$

the characteristic differential equations have the following form:

$$\frac{\mathrm{d}t(\tau)}{\mathrm{d}\tau} = F_p, \quad \frac{\mathrm{d}y(\tau)}{\mathrm{d}\tau} = F_q,$$
$$\frac{\mathrm{d}z(\tau)}{\mathrm{d}\tau} = pF_p + qF_q,$$
$$\frac{\mathrm{d}p(\tau)}{\mathrm{d}\tau} = -(F_t + pF_z),$$
$$\frac{\mathrm{d}q(\tau)}{\mathrm{d}\tau} = -(F_y + pF_z), \quad (10)$$

where τ is the "inner" time. F_p, F_q, F_t, F_y, F_z, of Eq.7 are

$$F_p = q, \quad F_q = p,$$
$$F_z = \lambda(1 - c_1c_2t\mathrm{e}^{-t})^2\exp(-t - \mathrm{e}^t + z),$$
$$F_t = c_1^2c_2(\mathrm{e}^{-t} - t\mathrm{e}^{-t}) + 2\lambda(1 - c_1c_2t\mathrm{e}^{-t}) \cdot (-c_1c_2\mathrm{e}^{-t} + c_1c_2t\mathrm{e}^{-t})\exp(-t - \mathrm{e}^t + z) + \lambda(1 - c_1c_2t\mathrm{e}^{-t})^2\exp(-t - \mathrm{e}^t + z)(-\mathrm{e}^t - 1),$$
$$F_y = 0. \quad (11)$$

Like Ref.[8], we use the initial conditions

$$y_0 = \log(1/x_0) = \log(100),$$
$$t_0 = \log[\log(Q_0^2/\Lambda^2)] = \log[\log(4\mathrm{GeV}^2/\Lambda^2)],$$
$$z_0 = \log(3.38), \ \ p_0 = c_1(1 - c_1c_2t_0\mathrm{e}^{-t_0})/\delta_{\mathrm{bare}},$$
$$\Lambda = 0.2\mathrm{GeV}, \ \ \delta_{\mathrm{bare}} = 0.5. \quad (12)$$

Because of $F(p_0, q_0, t_0, y_0, z_0)$=0, when y_0, t_0, z_0, p_0 is fixed, q_0 is obtained. In Fig.1 we plot the characteristic curves of $t(y)$ obtained from solving the linear evolution equation and Eq.(4) by using their characteristic equations, of which the initial conditions are same as Eqs.(12), except $p_0 = c_1/\delta_{\mathrm{bare}}$. The solid curve is the solution of the linear evolution equation (λ=0). The dot-dashed curve and dashed curve are the results of Eq.(4) with R=5GeV^{-1} corresponding to gluons uniformly distributing within proton and R=2GeV^{-1} corresponding to gluons concentrating in "hot-spots" within proton, respectively. Figure 1 shows that shadowing effect is notable in the case R=2GeV^{-1}.[8] Comparing with Fig.1, we plot the characteristic curves of $t(y)$ obtained from solving the linear evolution and Eqs.(10) in Fig.2. Figure 2 shows clearly that shadowing effect is substantially weaker than that in Eq.(4), whether R=2GeV^{-1} or R=5GeV^{-1}. So the conclusions of Eqs.(10) with higher order QCD effect coupling order are different from the conclusions of Eq.(4) without it. By analysing the value of the leading order QCD effect

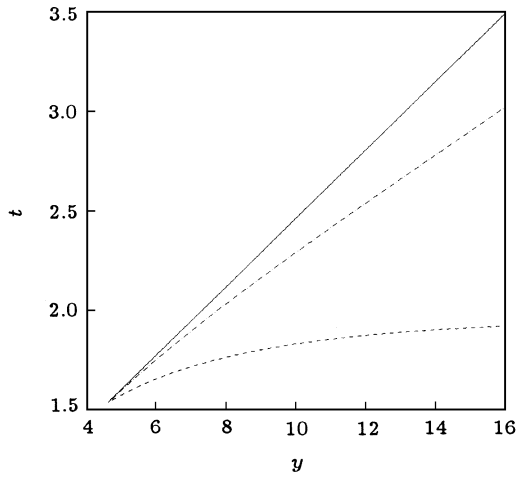


Fig.1. The characteristic curves of $t(y)$ are obtained by solving Eq.(4) with the parameters given in Eqs.(12), except for $p_0 = c_1/\delta_{\rm bare}$. The solid curve corresponds to linear evolution equation without shadowing term (λ=0), the dotted-dashed curve to the result of Eq.(4) when the hardon radius parameter R is 5GeV^{-1}, and the dashed curve to the result of Eq.(4) when the hardon radius parameter R is 2GeV^{-1}.

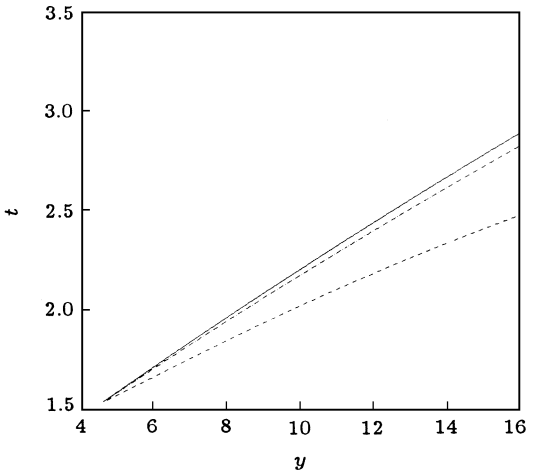


Fig.2. As for Fig.1, except for considering the higher order QCD effective coupling.

coupling, $\alpha_{LO}(Q^2)$, and the order $\alpha^2_{LO}(Q^2)$ effective coupling, we can explain the conclusions. When setting Q^2=4GeV^2, $\alpha_{LO}(Q^2)$=0.327 and $\alpha_s(Q^2)$=0.247,[12] so $\alpha_s(Q^2) < \alpha_{LO}(Q^2)$. Linear term in nonlinear QCD dynamical equation is proportional to $\alpha_s(Q^2)$, but nonlinear term in that is proportional to $\alpha^2_s(Q^2)$, so correspondingly shadowing in Eq.(6) is weaker than that in Eq.(4)

Ⅲ. CONCLUSIONS

Recently, one of the important questions is whether the shadowing effect can be observed by the current experiments at HERA. Some people say "can", such as Shabelski and Treleani,[13] while other people say "cannot", such as Golec-Biernat, Krasny and Riess.[14] Ayala, Gay Ducati and Levin[15] argued that the shadowing effect is large in the gluon distribution but small in $F_2(x, Q^2)$. In this paper, by considering the nonlinear QCD dynamical equation with higher order effective coupling, we have shown that shadowing effect in nonlinear QCD dynamical equation even in the case when gluons concentrate in "hot-spots" within proton (R=2GeV^{-1}) is not so notable as what has been studied. Although the data from DESY *ep* collider HERA can be fitted by using the linear evolution equation,[16] it can also be fitted by using the nonlinear QCD dynamical equation because of very weak shadowing in nonlinear QCD dynamical equation by considering the higher order effective coupling. However, shadowing effect in nuclear is very clear.[17] Some people[11] studied the phenomenon by using Eqs.(2)–(4) at Q^2_0=4GeV^2 without considering higher order QCD effective coupling. It may be incorrect because of the higher order QCD effective coupling considerably affecting the results of the nonlinear QCD dynamical equation. So shadowing effect in nuclear should be restudied by applying Eq.(6) instead of Eqs.(2)–(4). At present, we work on this line. Another reason to study shadowing in nuclei is to try to determine the important parameter R. In this paper, we do not research nuclear shadowing in detail. The conclusions of this paper can be verified by the HERA experiments.

ACKNOWLEDGEMENTS

One of us (He Bo) is grateful to Dr. Wang Chao for his helpful discussions.

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