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CRITICAL SCALING THEORY OF GENERALIZED PHASE TRANSITION AND ITS UNIVERSALITY*

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Generalized phase transition (GPT) refers to the transition process of material systems from one steady-state to another. It includes equilibrium phase transition (EPT) and nonequilibrium phase transition (NPT), and phase transitions intermediate between them. In this paper some results on the study of critical scaling relations of the NPT and EPT are obtained. We developed the critical scaling theory of EPT and advanced a universal critical scaling theory of GPT. The critical scaling relations(scaling laws) has more universality. The critical exponents calculated from our theory are identical with the results of experiments and other theories about EPT and NPT systems. Because the basic model of the theory does not depend on the concrete material system, it has a certain unversality. Its results thus can be applied to generlized phase transition systems, such as the electrorheological fluid and magnetorheological fluid systems.

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I. INTRODUCTION

Under the control of external conditions, a certain self-organization orderly structure will appear spontaneously in some material systems with a certain background state. The orderly stucture is the result of phase transition. According to different background states, the phase transitions can be classified as equilibrium phase transition (EPT)(such as gas-liquid phase transition and ferromagnetic phase transition),nonequilibrium phase transition (NPT)(such as Benard thermal convection and laser phase transition) and synthetic phase transition(such as electrorheological fluid and magnetorheological fluid)etc. In a general sense,the phase transition process in fact refers to the transition process of a system from one steady state to another when some outer controlling parameter R reaches a centain critical value R_c. The critical point is the link point of phase transitions, and the critical phenomenon is the key of phase transition problem.

The EPT critical phenomena have been studied for a long time. Andrews discovered the EPT critical point in 1860s, but it was not until 1940s that Landau formulated the mean field theory of EPT critical phenomena.He not only preliminarily described the physical picture of EPT but also calculated part of the EPT critical exponents. In 1965, Widon[1] first put forward the scaling idea,from which the EPT critical scaling law and the relation formulas to fit the critical exponents could be derived. In 1966, Kadanoff[2] succeeded in putting scaling idea into use in the Ising model. On the basis of a large number of experimental facts, he advanced the concept of critical universality about EPT.[2]That is to say, all kinds of EPT systems can be divided into a number of universal categories according to their space dimensions and numbers of inherent degrees of freedom. And for systems of the same universal category, their critical exponents are completely identical. In 1971,Wilson[3] inspired by the scaling idea,applied the method of renormalization group to the EPT critical theory and gained an important breakthrough. They not only gave a correct description of EPT critical phenomena and worked out critical exponents precisely, but also proved the correctness of the scaling law and the universality concept. The critical theory of EPT has thus been tending to ripen on the basis of scaling idea and universality concept.

The study of NPT critical phenomena did not start until the theories of dissipative structure [4] and synergetics[5] appeared in 1970's.After a thorough res-

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earch, it was shown that although the background states and features of the critical phenomena accompanying the NPT and EPT are totally different, their critical behaviors may have many common features, such as critical moderation, correlation divergence, sudden mutation of order, symmetry breaking etc.[6−10] In addition,both NPT and EPT can be scaled in the vicinity of their critical points. Each of them has a corresponding scaling law and a potential function that describes the background state of the system.[11] The common features of the two critical phenomena occur not accidentally. There must be a common internal factor that controls them.Universality categories also exists in NPT systems. Many examples show that for systems belonging to the same category,no matter what substances and variables they may consist of,and no matter how different their apparent feature may be, their critical behaviours are similar. This shows that when these systems tend to the critical point their general character tends to dominate over their specific character. So they have the same critical exponents. All of this is very similar to the critical universality of EPT.[12]

Based on the standardized model of NPT system, we have inverstigated the critical scaling law and universality of NPT, and discussed the factors that must be considered when the NPT universality cstegories were divided, and worked out the critical exponents of three types of NPT.Compared with the results of experiments and other theories, the results we obtained are very satisfactory. Moreover, we also approached and compared the relationship between NPT and EPT critical phenomena from the aspects of critical scaling law and universality. The result shows that the equilibrium state can be regarded as a special case of nonequllibrium steady state, thus the common nature of them are revealed to a certain extent.[13] In addition, the studies of electrorheological and magnetorheological fluid show that there are also some critical points in them. We call them synthetic phase transition (SPT)critical phenomena. Here we give a joint name of EPT, NPT and SPT as generalized phase transition(GPT).

In the following we shall introduce the universal GPT standardized model we used to simulate and describe the critical activities of general nonequilibrium steady state phase transition.On its basis,we have approached the critical scaling character of GPT and derived a group of universal critical scaling relationships of GPT. Five groups of critical exponents have been calculated which coincide well with the experimantal and theoretical results of EPT and NPT in 3-dimensional space,known at present.This shows the rationality and correctness of our GPT standardized model.Because theGPT standardized model does not depend on the specific material system, it has high degree of universality and is important in guiding the study of SPT critical phenomena.

II. GPT STANDARDIZED MODEL AND ITS DESCRIPTION IN CRITICAL AREA

Generally speaking,GPT refers to the transition process of material systems from one steady state to another. Its critical reference state is steady state (including nonequilibrium stationary state and equilibrium state.The equilibrium state can be regarded as a special case of nonequilibrium stationary state). For the normal GPT system, because of the existence of dissipation, the reference state is restricted by the principles of detailed balance, circulative balance and dissipative balance. In the system, a great deal of microscopic unit processes and unit actions mutually compensate and counteracted each other, which guarantees the stability of the system's macroscopic state. The stability has to be preserved by consuming material, energy, information or entropy from the outside.

In order to make the description reasonable and research convenient the standardized method is adopted here.[14] This means that the whole GPT system can be regarded as a construction consisting of N_e micro volume elements of unit cubes with the same linear size e. The volume of every unit micro cube is $\Delta V_{r_i} = e^d$. Here, "d" is the space dimension of the system. The ith micro volume element can be marked by its center position vector $\boldsymbol{r_i}$. Suppose $q_1, q_2, \ldots, q_n$ are the physical variables necessary for the description of a certain nonequilibrium process of the system. They are functions of both time t and space $\boldsymbol{x}$.These physical variables can be regarded as n components of the system state vector $\boldsymbol{q}(\boldsymbol{x},t)$. Here, $\boldsymbol{q}$ is a random vector which changes as the space and time changes.Under steady state conditions, there must be a certain probability distribution for these physical variables in the micro volume element ΔV_{r_i} : $\boldsymbol{\rho}(\boldsymbol{x},t) = \{\rho_1(\boldsymbol{x},t), \rho_2(\boldsymbol{x},t), \cdots, \rho_n(\boldsymbol{x},t)\}$. So the ith micro volume element's state in the time t can be described by the average of the state vector $\boldsymbol{q}$ $(\boldsymbol{x},t)$ over space in the micro unit volume ΔV_{r_i},

$$q_i(\boldsymbol{r}_i,t)=C_i\iiint\limits_{\Delta V_{r_i}}[-\boldsymbol{\rho}(\boldsymbol{x}-\boldsymbol{r}_i,t)\cdot\boldsymbol{q}(\boldsymbol{x},t)]\mathrm{d}\boldsymbol{x}. \quad (1)$$

Thus the whole steady state system can be described by a group of state configurations

$$\{q_i(\boldsymbol{r}_i,t)|i=1,2,\cdots,N_e\}. \quad (2)$$

Different states of the system correspond to different configurations.and the changes of the configuraion reflect the changes of the system state.

For the stable stationary-state system,as a result of "flow" and "dissipation" there must be a certain physical variable which is held constant in the ith unit micro volume element,such as the kind of material,energy, information etc.in unit volume (here called M_{con}). At this moment,an amount of the physical variable with the same attribute is imported into the unit volume element in unit time from outside (here called M_{imp}). In order to prevent "heaping up", a corresponding amount must be dissipated from the volume element (here called M_{dis}). In order to maintain the universality,we introduced two quantities (without dimension) M_{D} (dissipation parameter)and M_{T} (input parameter) to measure the dissipated intensity and input intensity of the system at this moment.Their definitions are

$$M_{\mathrm{D}}=\frac{M_{\mathrm{con}}+M_{\mathrm{dis}}}{M_{\mathrm{con}}} \quad \text{and} \quad M_{\mathrm{T}}=\frac{M_{\mathrm{con}}+M_{\mathrm{imp}}}{M_{\mathrm{con}}}. \quad (3)$$

According to the continuity principle, the stability of the system requires $M_{\mathrm{dis}}=M_{\mathrm{imp}}$.Consequently, the stationary state of the system is stable before the system reachs the critical point, and one of the key conditions of stability is

$$M_{\mathrm{D}}=M_{\mathrm{T}}. \quad (4)$$

In a GPT system,the input quantity M_{imp} usually can be controlled by the outside conditons. So the input parameter M_{T} usually acts as a controlling function. During GPT,the change of M_{T} is continuous and smooth. In the dissipative system, a stationary state corresponds to a dissipative pattern.For a certain specific dissipative pattern, its dissipation capability is limited.When the system tends to GPT critical point,there is

$$M_{\mathrm{D}}\neq M_{\mathrm{T}}. \quad (5)$$

At this moment the former dissipative pattern cannot satisfy the requirement that the dissipation and input must compensate perfectly so as to maintain the condition of steady state.The "heaping up" formed in the system makes the original state unstable. It forces the unsteady system to seek another new dissipative pattern in order to be steady again. At this time, correlativity strengthens and all kinds of fluctuations become extremely active, promoting the system to step into a new dissipative state and restore the relation $M_{\mathrm{D}}=M_{\mathrm{T}}$. The new dissipative pattern maintains the new stability of the system.Thus,based on the steady state,a system transfers from its old state to a new one and thereby, a generalized phase transition is accomplished.

When the system tends to the GPT critical point, some of the variables $\{q_1,q_2,\ldots,q_n\}$ become soft-mode,with relaxing slowing-down, and form the order parameter u. At this moment, other variables are dominated by u. The evolution of the whole system will be governed by the order parameter u. According to the critical dominance principle and using the adiabatic elimination method in Synergetics,[5] the local order parameter equation of unit micro volume element can be derived.[15] The ith micro unit's state at this time can be described by its local order paramenter $u_i(\boldsymbol{r}_i,t)$. And the state of the whole system can be described by a group of local order parameter configuration

$$\{u_i(\boldsymbol{r}_i,t)|i=1,2,\cdots,N_e\}. \quad (6)$$

The dissipative parameter M_{D} will also be dominated by the u in the neighbohood of the critical point, and we have[14]

$$M_{\mathrm{D}}\propto u^{\varphi}. \quad (7)$$

Here φ reflects the specific dissipative pattern of the system.

Critical phenomena are large scale cooperation phenomena in space, their universality shows that it does not depend on the specific details of the model. Thus, when the system tends to the critical point, we can use the coarse-graining method, and have the group local order parameter configuration instead of the unit local order parameter configuration to describe the evolution of the whole system.[12]

Ⅲ. CRITICAL SCALINGABILITY OF GENERALIZED PHASE TRANSITION

In GPT systems, order parameter u is a random varable. Its movement and variation obey the gener-

alized Langevin equation

$$\frac{\mathrm{d}u}{\mathrm{d}t} = K(u) + g \cdot \mathcal{F}(t), \tag{8}$$

where $K(u)$ is a nonlinear function, $\mathcal{F}(t)$ is a random fluctuation force, and it possesses the following property:[16]

$$< \mathcal{F}(t) >= 0, \quad < \mathcal{F}(t_1)\mathcal{F}(t_2) >= Q\delta(t_1 - t_2). \tag{9}$$

In formula (8), g is a function which has connection with the quality of the random fluctuation force. For systems with additive noise, g=1; for those with multiplicative noise, $g = g(u)$. Under the condition of formula (9) there is the Fokker-Plank equation which is corresponding to formula (8)

$$\frac{\partial \mathcal{P}(u,t)}{\partial t} = \frac{\partial}{\partial u}[K(u) - Qg'g]\mathcal{P}(u,t) + Q\frac{\partial^2}{\partial u^2}g^2\mathcal{P}(u,t). \tag{10}$$

The stationary solution of the formula, under natural boundary conditions, has the following structural form:[17]

$$\mathcal{P}(u) = N\frac{1}{|g|}\exp\left[-\frac{1}{Q}\int\frac{K(u)}{g^2}\mathrm{d}u\right]. \tag{11}$$

Here N is the normalized constant. It can be proved that, for a stationary system, the local order parameter u always has a definite distribution $\mathcal{P}(u)$.[15] So we can define a generalized random potential $V(u)$ corresponding to the stationary state.

$$V(u) \triangleq -\frac{Q}{2}\ln\left[\frac{\mathcal{P}(u)}{N}\right]. \tag{12}$$

This generalized potential reflects the probability distribution and stability of the range of local order parameter values when the system is in a steady state. For a normal GPT steady state system, because of nonlinearity, nonhomogeneity and strong coupling, $V(u)$ is usually a nonlinear function of the order parameter u and its gradient $\bigtriangledown u$. Considering a stationary system that is not only controlled by the parameter R but also affected by outside field $\tilde{F}$, $V(u)$ always can be expressed in the following general form:[14]

$$\begin{aligned} V(u,R,\tilde{F}) =& f(u) + a(R)g(u) \\ &+ b(\tilde{F})h(u) + I(\bigtriangledown u). \end{aligned} \tag{13}$$

Here $f(u), g(u), h(u)$ and I$(\bigtriangledown u)$are all nonlinear function, and $f(u)$ reflects the degree of critical degeneration of the system, $g(u)$ and $h(u)$ reflect the coupling intensities of the system with R and $\tilde{F}$, respectively, and $I(\bigtriangledown u)$ embodies the non-uniformity of the system. In the space of $(u, R, \tilde{F})$, the point $(0,R_c,0)$is the critical point of $V(u, R, \tilde{F})$. In general, the specific structure of the generalized random potential is too complicated to be used to discuss a GPT process directly. However,when a system tends to the critical point, if we pay attention to the general character in different GPT systems, and use the idea of synergetics and catastrophe theory and the analytical mothod of function as well as the topological transformation of differentiation, we can get the critical asymptotic form of the generalized potential $V(u)$ in the infinitesimal vicinity of the critical point for a common GPT system. In order to have universality, two reduced parameters, the reduced controlling parameter ε and the reduced outside field F, are introduced,i.e.,

$$\begin{aligned} &\varepsilon = (R - R_c)/R_c, \\ &F = \tilde{F}/(\text{dimension of } \tilde{F}). \end{aligned} \tag{14}$$

Then V can be simplified in the vicinity of the critical point, and takes the simple asymptotic form:[13]

$$\begin{aligned} V(u,\varepsilon,F) =& B_1u^x + B_2\varepsilon^n u^y \\ &+ B_3F^m u^z + B_4(J \bigtriangledown u)^\omega. \end{aligned} \tag{15}$$

Here J is the generalized flow caused by system's nonuniformity generalized force $(X = \bigtriangledown u)$; B_1, B_2, B_3 and B_4 are constant coefficients for the corresponding items; x reflects the degree of the critical degeneration of the system; y and z respectively reflect the coupling intensities of controlling parameters and outside-fields; n and m are related with their corresponding coupling methods; ω reflects the non-uniformity of the system. In the space of (u, ε, F), the point (0,0,0) is the (x–2) time degenerate critical point of the generalized potential $V(u, \varepsilon, F)$.

In the GPT standardized model, the space-time correlation function of its local order parameters can be defined as

$$\begin{aligned} G(\boldsymbol{r},t) &= G(\boldsymbol{r}_1,t_1,\boldsymbol{r}_2,t_2) \\ &=< \delta u(\boldsymbol{r}_1,t_1)\delta u(\boldsymbol{r}_2,t_2) > . \end{aligned} \tag{16}$$

Its physical singnificance is the extent of the connection and influence which the fluctuation of local order parameter at time t_1 and placer $\boldsymbol{r}_1$ produce to the fluctuation of order parameter at time t_2 and place $\boldsymbol{r}_2$ in a certain local order parameter configuration. For the smooth stochastic process in a system with the unchangeability of translation, we have

$$\boldsymbol{r} = \boldsymbol{r}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \boldsymbol{r}_1 - \boldsymbol{r}_2,$$
$$t = t(t_1, t_2) = t_1 - t_2. \tag{17}$$

The Fouriers component of $G(r,t)$ in d-dimensional space may be defined as

$$G(k,t) = \int \mathrm{d}^d r G(r,t) \mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{r}},$$
$$G(r,t) = (2\pi)^{-d} \int \mathrm{d}^d k G(k,t) \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{r}}. \tag{18}$$

Using formulas (8)–(11) and formulas (15)–(18), the critical asymptotic form of $G(k,t)$ in the infinitesimal vicinity of the critical point can be derived as[15]

$$G(k,t) = \frac{Q'\xi^{\omega}}{1+\xi^{\omega}k^{\omega}} \mathrm{e}^{-t/\tau_k}, \tag{19}$$

where $Q' = Q/[2A_4\omega(\omega-1)U^{\omega-2}]$, and the correlative length ξ and the correlative relaxation time τ_k have the following forms:

$$\xi = [A_4\omega(\omega-1)U^{\omega-2}/S]^{1/\omega},$$
$$\tau_k = [S + A_4\omega(\omega-1)U^{\omega-2}k^{\omega}]^{-1}. \tag{20}$$

Here U is the most probable value of u, that is, the local order parmeter configuration when the system is in a steady state, and S has the following form:

$$S = A_1 x(x-1)U^{x-2} + A_2\epsilon^n y(y-1)U^{y-2} + A_3 F^m z(z-1)U^{z-2}, \tag{21}$$

where $A_i (i = 1,2,3,4)$ are constants. Further analysis shows that, before getting near the critical point, ξ and τ_k will keep finite. Formula (19) does not show the singularities, and the whole system will keep its stability as usual. But when the system comes near the GPT critical point, namely when $\varepsilon \to 0$, $F \to 0$, we will find $U \to 0$. It will happen that

$$\xi \xrightarrow{\epsilon\to 0, F\to 0} \infty, \quad \tau_k|_{k\to 0} \xrightarrow{\epsilon\to 0, F\to 0} \infty. \tag{22}$$

At this time, there are some singularities occurring in $G(k,t)$,

$$G(k,t)|_{k\to 0} \xrightarrow{\epsilon\to 0, F\to 0} \infty. \tag{23}$$

It will cause unstability and "large fluctuation" in the system, and the GPT will result.

In the GPT standardized model, the random generalized potential formula (14) and the space-time correlation function (19) are two important functions reflecting the critical feature of GPT. It can be proved that in the critical point vicinity of GPT can be scaled with the aid of the properties of generalized homegeneous function and the idea of critical scaling.[13,18] This means that in the infinitiesimal vicinity of GPT critical point, the generalized potential V and the correlation function G are generalized homogeneous functions of parameters and independent variables in GPT,

$$V(b^p\epsilon, b^q F, b^{\rho} J) = bV(\epsilon, F, J),$$
$$G(\lambda^p k, \lambda^q t, \lambda^r \xi, \lambda^s \tau_k) = \lambda G(k, t, \xi, \tau_k). \tag{24}$$

It indicates that there really exist scalingable nature in the field of GPT. So there must exist scaling laws which correspond with the GPT system.

IV. CRITICAL EXPONENTS AND SCALING RELATIONS OF GPT SYSTEMS

Results of the research show that some physical quantities show strange properties when a system comes near a GPT critical point. In the theory of phase transition, it is the best way to use critical exponents to describe the tendency of the critical behavior of the system.[19] In GPT standardized model, we introduce the following physical variables to show the critical properties of the GPT system:

1) u(order parameter), 2) F(applied coupled field), 3) ϵ(reduced control parameter),

4) $G(k,t)$(correlation function), 5) ξ(coherent length), 6) τ_k(relaxation time),

7) M_{D}(dissipative parameter), 8) $X = (\dfrac{\partial u}{\partial F})$, 9) $Y = (\dfrac{\partial u}{\partial R})$,

10) $B = (\dfrac{\partial V}{\partial R})$, 11) $C = (\dfrac{\partial^2 V}{\partial R^2})$, 12) $E = (\dfrac{\partial V}{\partial F})$. (25)

Table 1. Definition of GPT critical exponents: $Q \propto A^{ce}$ in D.

No.	ce	Q	A	in$D$
1	α	C	$1/\|\epsilon\|$	$F=0, \epsilon \to 0$
2	β	u	$\|\epsilon\|$	$F=0, \epsilon \to 0$
3	γ	X	$1/\|\epsilon\|$	$F=0, \epsilon \to 0$
4	δ	F	u	$\epsilon=0, F \to 0$
5	λ	Y	$1/F$	$\epsilon=0, F \to 0$
6	μ	B	$\|\epsilon\|$	$F=0, \epsilon \to 0$
7	σ	E	F	$\epsilon=0, F \to 0$
8	θ	M	$1/\|\epsilon\|$	$F=0, \epsilon \to 0$
9	ν	ξ	$1/\|\epsilon\|$	$F=0, \epsilon \to 0$
10	κ	ξ	$1/F$	$\epsilon=0, F \to 0$
11	η	$G(k,t)k^2$	k	$F=0, \epsilon \to 0$
12	τ	τ_k	$1/\|\epsilon\|$	$F=0, \epsilon \to 0$

ce:critical exponent.

Table 2. Expressions of GPT critical exponents.

Exponent	Expression	Factors concerned
α	$2-\frac{nx}{x-y}$	x, y, n
β	$\frac{n}{x-y}$	x, y, n
γ	$\frac{n(x-z-m)}{m(x-y)}$	x, y, z, n, m
δ	$\frac{x-z}{m}$	x, z, m
λ	$\frac{m(x-y-n)}{n(x-z)}$	x, y, z, n, m
μ	$\frac{nx-x+y}{x-y}$	x, z, m
σ	$\frac{mx-x+z}{x-z}$	x, z, m
θ	$\frac{\varphi m}{x-y}$	x, y, φ, m
ν	$\frac{n(x-\omega-\omega x\rho)}{\omega(x-y)}$	x, y, ω, ρ, n
κ	$\frac{m(x-\omega-\rho x\omega)}{\omega(x-2)}$	x, z, ω, ρ, m
η	$\frac{2\omega}{x-\omega-\rho x\omega}+2-d$	d, x, ω, ρ
τ	$\frac{x-2}{x-y}n$	x, y, n

Using the same method as in the equilibrium phase transition critical theory, we define a set of GPT critical exponents to describe and show the tendency of critical change of these physical quantities in a GPT system. They are listed in Table 1.

According to the critical scalingable properties of GPT systems, using the scaling transform comparison method and the scaling dimension analysis method, we get the scaling parameter expressions of these critical exponents in the vicinity of GPT critical point from formulae (24),(25) and Table 1, as shown in Table 2.[13] Here d is the space dimension of the system, and φ and ρ are parameter factors concerned with the dissipation intensity and dissipation current of the system. Eliminating factors, such as x, y, z, ω, of these expressions in Table 2, we obtain 8 independent GPT critical scaling relationships with universal significance. They are shown in Table 3.

Table 3. GPT critical scaling low.

No.	Scaling relationships
1	$(\lambda\delta+1)(\mu+1)\beta+(1-\delta)\beta=2-\alpha-mr$
2	$\beta(\delta-1)=\gamma$
3	$\gamma\sigma\lambda=\delta-\gamma-1$
4	$n\theta\kappa\delta=m\varphi\nu$
5	$\nu(d+\eta-2)=2\beta$
6	$\lambda(\lambda\delta+1)(\mu-m+1)=(\lambda-m)\delta(\alpha+1)$
7	$\varphi(\mu+1)(\alpha+1-m)=(\lambda\delta+1)(\mu+1)(\sigma+1)\theta-m\delta(\alpha+1)\theta$
8	$(\tau-2\beta)(m\varphi\nu+n\theta)=n\delta\theta(\alpha+1)(\nu+\beta+2\rho+\alpha\rho)-\rho(\tau-2\beta)$

From Table 3, we can see that the critical scaling laws of GPT are a family of functional relations of the critical exponents that are related to the parameters,such as n, m, φ and ρ. According to various concrete phase transition systems, it is possible that there are different scaling relations because of different parameters. Thus it can be seen that the critical situation of GPT is more complicated than that of EPT.

V. THEORETICAL VALUES OF GPT CRITICAL EXPONENTS AND THEIR COMPARISON WITH EXPERIMENTS

From Table 2, we can see that concrete values of GPT critical exponents depend on the factors, such as d, x, y, z, φ, ρ, n, m etc. So we can classify the GPT systems by $\{d, x, y, z, \varphi, \rho, n, m\}$ factors. The systems that have the same universal factors belong to the same universal category. These GPT systems

have the same critical activity, scaling relations, and critical exponents. In order to compare the results with those of known experiments, we calculated five groups of critical exponent values corresponding to five kinds of universal factors shown in Table 4 in three-dimensional space. The calculated results are shown in Table 5.

Table 4. Universal factors of five universal categories in GPT systems.

Critical universal factors of GPT systems									
Category	d	x	y	z	ω	φ	ρ	n	m
I	3	4	2	1	2	1	0	1	1
II	3	6	2	2	2	2	1/6	1	1
III	3	6	2	1	2	3	1/6	1	1
IV	3	4	2	1	2	0	0	1	1
V	3	6	1	2	2	0	0	3/2	4/5

Table 5. Theoretical values of critical exponents of GPT systems.

Critical exponents												
Category	α	β	γ	δ	λ	μ	σ	θ	ν	κ	η	τ
I	0	1/2	1	3	1/3	1	1/3	1/2	1/2	1/3	1	1
II	1/2	1/4	3/4	4	3/4	1/2	1/2	1/2	1/4	1/4	1	1
III	1/2	1/4	1	5	3/5	1/2	1/5	3/4	1/4	1/5	1	1
IV	0	1/2	1	3	1/3	1	1/3	0	1/2	1/3	1	1
V	1/5	3/10	6/5	5	1/15	4/5	1/5	0	3/5	2/15	0	6/5

Through accurately measuring the NPT critical exponents in Benard heat convection selforganization phenomenon, some authors obtained: β=1/2, ν=1/2, γ=1, τ=1.[20] According to the research of asingle-mode one-photon laser system driven by the outside coherent field, Giorgio and Scally[21] obtained the critical exponents as: α=0, β=1/2, γ=1, δ=3. Through precise experiment, Corti and Giorgio[22] got two critical exponents in the single-mode one-photon laser system driven by a coherent field, they are β=1/2, γ=1. Besides, there are critical exponents α=0, β=1/2, γ=1, δ=3, λ=1/3, μ=1, θ= 1/2 in Schologl phase transition system of the chemical reaction.[12] The results mentioned above are identified with the GPT critical exponents of the universality category I in Table 5. That is to say, they have the same critical exponents and show the same critical behaviors.

According to the result for the single-mode one-photon laser system with saturable absorber, Walgraef *et al.* [23] claimed that there are critical exponents α=1/2, β=1/4, in that kind of a NPT system. Jianmin[24] studied single-mode one-photon traveling wave laser system with saturable absorber, and got the critical exponents as α=1/2, β=1/4, γ=3/4, δ=4. In the single-mode one-photon standing wave laser system with saturable absorber, there are also the same critical exponents α=1/2, β=1/4, γ=3/4, δ=4.[15] All these results are identical with the critical universality category II of Table 5.

In addition, in nonequilibrium systems such as the laser phase transition, there also exists the critical point which is consistent with the universality category III of Table 5. It is found that its critical exponents are α=1/2, β=1/4, γ=1, δ=5.[25]

For the EPT systems, according to the Landau mean field theory, we have the EPT critical exponents deduced in three-dimensional space as follows: α=0, β=1/2, γ=1, δ=3, ν=1/2, η=1, τ=1.[20] For the EPT systems such as the superfluid and superconductor, the experimental results are very close to the values[19,26] listed in Table 5. The results agrees with the universality category IV.

Through rigorous expriment, it has been found that there exist large differences between the theoretical values of the mean field theory and the experimental values of the EPT system such as liquid-gas, ferromagetic and ferroelectic, binaryalloys and so on. The spread of the actual values is as follows: [27]

$$\alpha = (0 - 1/5), \beta = (3/10 - 2/5), \gamma = (6/5 - 7/5),$$
$$\delta = (4 - 5), u = (3/5 - 7/10), \eta = (0 - 1/10). \quad (26)$$

The results of higher degree calculation in the frame work of renormalization group theory of critical phenomena by Wilson are also in this range. These results are identified with the universality category V in Table 5.

IV. DISCUSSIONS

The above-mentioned results claimed that the critical scaling theory of GPT includes the information of both EPT and NPT critical scaling theories. Although the GPT standardized model is a simplified solvable model, the results derived with this model agree with the experimental results. Therefore, the simple hypothesis of the model is rational, and reflects the universality of the model.

If equilibrium state is regarded as a limiting case of nonequilibrium stationary state when its "flow" and "dissipation" approach zero (zero-flow and zero-dissipation limit), and taking the universal factors of category Ⅳ in Table 4, we can get the following expressions from the GPT critical scaling laws:

$$\alpha + 2\beta + \gamma = 2, \quad \alpha + \beta(\delta + 1) = 2,$$
$$\gamma = \nu(2 - \eta), \quad a = 2 - d\nu. \quad (27)$$

These results are just the critical scaling laws of EPT.[19,20] So EPT and NPT critical scaling relationships are all subgroups in GPT critical scaling relationships. Thus EPT and NPT critical universality categories are the subsets of GPT critical universality categories. Through further research, we can see that NPT and EPT have a unification nature in the aspects of reference state, potential function, correlation function, scalingablity, critical exponents, scaling law, universality and so on. [10]

Because the GPT standardized model does not depend on any specific material system and possess a considerable universality, the analysis and results based on this model are important for the study of critical nature in SPT systems such as electromagnetorheological fluid systems. Preliminary research has shown that this kind of critical behavior of SPT systems conforms with the laws of GPT critical scaling theory. Now, we are doing further research. Besides, the critical exponent values of item V in Table 5 are identical with the EPT critical exponents obtained in real space by experiment. But why should the universal factors have the values of item V in Table 4? What are the relations between the factors and the EPT critical point? Work along these lines is in progress.

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