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STATISTICAL-MECHANICAL ENTROPY OF THE GENERAL STATIC BLACK HOLE DUE TO ELECTROMAGNETIC FIELD*

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Statistical-mechanical entropy arising from the electromagnetic field in the general four-dimensional static black hole spacetime is investigated by means of the "brick wall" model. An expression for the entropy is obtained and some examples are considered. The results show that the entropy arising from the electromagnetic field is exactly twice the one for a massless scalar field.

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I. INTRODUCTION

Much effort has been concentrated on the study of quantum entropy of black holes recently[1−23] since it is excepted that the study can shed light on the problem of obtaining a statistical meaning of the Bekenstein-Hawking entropy which is proportional to the area of the event horizon.[24−26] Especially, the thermodynamical and the statistical-mechanical entropies arising from scalar fields have been studied extensively.[1−16] Frolov and Fursaev[4] reviewed the studies of the two kinds of entropies for the static black holes. They showed that the covariant Euclidean free energy F^{E} and the statistical-mechanical free energy F^{C} are equivalent when one uses the ultraviolet regularization[4] for static black holes. Recently, we studied the quantum entropy of the rotating black holes.[18] We showed that one-loop correction to the thermodynamical entropy and the statistical-mechanical entropy due to the quantum scalar fields are equivalent for the Kerr-Newman and the Einstein-Maxwell dilaton-axion black holes.

Although much progress has been made for the scalar fields, the study of the quantum entropy due to an electromagnetic field has just begun. Kabat[21] studied the entropy for the electromagnetic field case in the Rindler space and found an unexpected surface term which corresponds to particle paths beginning and ending at the event horizon. This term gives a negative contribution to the entropy of the system and is large enough to make the total entropy negative at the equilibrium temperature. However, Iellici and Moretti[22] proved that the surface term is gauge dependent in the four-dimensional case and therefore can be discarded. Recently, Cognola and Lecca[23] used the "brick wall" model (BWM) to study the statistical-mechanical entropy for the electromagnetic fields in the Reissner-Nordström black hole spacetime. They showed that there is no such surface term in BWM, and they found that the leading term of the entropy for the electromagnetic fields is exactly twice the one for a massless scalar field. But, at the moment, the question whether or not the relation obtained by Cognola and Lecca is valid when we study the entropy (include both a leading and a sub-leading terms) and when we extend the investigation to the general static black holes is still open. The purpose of this paper is to settle the questions by deducing an expression of the statistical-mechanical entropy arising from the electromagnetic field in the general four-dimensional non-extreme static black holes.

We first show, following Cognola and Lecca[23], that the electromagnetic field in the four-dimensional static spacetime can be expressed in term of a couple of scalar fields satisfying Klein-Gordon-like equations. We then seek the total number of modes with energy less than E, and use it to calculate the free energy. The statistical-mechanical entropy is finally obtained by the variation of free energy with respect to inverse temperature.

II. STATISTICAL-MECHANICAL ENTROPY OF THE GENERAL STATIC BLACK HOLE DUE TO ELECTROMAGNETIC FIELD

In the four-dimensional spacetime, the metric for

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the general non-extremal static black hole can be expressed as

$$\begin{aligned} \mathrm{d}s^2 =& g_{tt}\mathrm{d}t^2 + g_{rr}\mathrm{d}r^2 \\ &+ R_1(\mathrm{d}\theta^2 + \alpha^2\sin^2\theta\mathrm{d}\varphi^2), \end{aligned} \tag{1}$$

where $g_{tt} = -g^{rr} = -G_1(r)(r - r_{\mathrm{H}})(G_1(r)$ is a regular function on the outermost event horizon and outside the horizon), R_1 is a function of the coordinate r only, and α is a constant. In order to include more general cases we set factor α in metric (1): For the static spherically symmetric black hole $\alpha = 1$; for the Schwarzschild black hole with a cosmic string[27] or the Reissner-Nordström black hole pierced by a cosmic string[28] $\alpha^2 = (1 - \mu^2)$,where μ is the mass per unit length of the string.

Introducing new coordinates

$$z = \frac{\sin\theta\mathrm{e}^{\mathrm{i}\alpha\varphi}}{1-\cos\theta}, \quad \bar{z} = \frac{\sin\theta\mathrm{e}^{-\mathrm{i}\alpha\varphi}}{1-\cos\theta}, \tag{2}$$

the metric (1) reads

$$\mathrm{d}s^2 = g_{tt}\mathrm{d}t^2 + g_{rr}\mathrm{d}r^2 + g_{z\bar{z}}\mathrm{d}z\mathrm{d}\bar{z}, \tag{3}$$

where

$$g_{z\bar{z}} = \frac{2R_1}{(1+z\bar{z})^2}. \tag{4}$$

In terms of the electromagnetic potentical A_μ, we can express the Maxwell equation $\nabla_\mu F^{\mu\nu} = 0$ in the following form[23]

$$LA_\mu = A_\nu\partial_\mu(g^{\rho\lambda}\Gamma^\nu_{\rho\lambda}) + 2g^{\rho\lambda}\Gamma^\nu_{\rho\mu}\partial_\lambda A_\nu + \partial_\mu\nabla_\nu A^\nu, \tag{5}$$

where $L = \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$. Fixing the gauge condition $A_t = 0$ and substituting Eq.(3) into Eq.(5), after some calculations we have

$$LA_r = -\frac{1}{R_1}\frac{\partial}{\partial r}(g^{rr}\frac{\mathrm{d}R_1}{\mathrm{d}r}A_r), \tag{6}$$

$$\left(L + 2\frac{\partial g^{z\bar{z}}}{\partial z}\frac{\partial}{\partial\bar{z}}\right)A_z = g^{rr}\frac{\mathrm{d}\ln R_1}{\mathrm{d}r}\frac{\partial A_z}{\partial r} + g^{rr}\frac{\mathrm{d}}{\mathrm{d}r}\left(\ln\frac{g_{tt}}{R_1}\right)\frac{\partial A_r}{\partial z}, \tag{7}$$

$$\left(L + 2\frac{\partial g^{z\bar{z}}}{\partial\bar{z}}\frac{\partial}{\partial z}\right)A_z = g^{rr}\frac{\mathrm{d}\ln R_1}{\mathrm{d}r}\frac{\partial A_{\bar{z}}}{\partial r} + g^{rr}\frac{\mathrm{d}}{\mathrm{d}r}\left(\ln\frac{g_{tt}}{R_1}\right)\frac{\partial A_r}{\partial\bar{z}}, \tag{8}$$

and a constrained condition

$$\nabla_\mu A^\mu + \frac{\mathrm{d}g_{tt}}{\mathrm{d}r}A_r = 0. \tag{9}$$

We obtain from Eqs.(6)–(9) two classes of independent electromagnetic potential

$$A^{\mathrm{I}}_\mu = \left(0, 0, \frac{1}{\sqrt{2l(l+1)\omega}}\frac{\partial\Phi_1}{\partial z}, \frac{-1}{\sqrt{2l(l+1)\omega}}\frac{\partial\Phi_1}{\partial\bar{z}}\right), \tag{10}$$

$$A^{\mathrm{II}}_\mu = \left(0, \sqrt{\frac{l(l+1)}{2\omega^3}}\frac{\Phi_2}{R_1}, \frac{g^{rr}}{\sqrt{2l(l+1)\omega^3}}\frac{\partial^2\Phi_2}{\partial z\partial r}, \frac{g^{rr}}{\sqrt{2l(l+1)\omega^3}}\frac{\partial^2\Phi_2}{\partial\bar{z}\partial r}\right). \tag{11}$$

It is shown that A^{I}_μ and A^{II}_μ depend on a couple of scalar functions Φ_1 and Φ_2, respectively. Since both the Φ_1 and Φ_2 satisfy the Klein-Gordon-like equation

$$\Box\Phi_i = g^{rr}\frac{\mathrm{d}\ln R_1}{\mathrm{d}r}\frac{\partial\Phi_i}{\partial r} \qquad (i = 1, 2), \tag{12}$$

where $\Phi_i(t, r, z, \bar{z}) = \mathrm{e}^{-\omega t}f(r)Y^m_l(z, \bar{z})$, hereafter we will drop the subscript of the Φ_i.

We impose the brick-wall boundary condition in which the wave function is cut off just outside the event horizon, i.e., $A_\mu = (0, 0, 0, 0)$ at $r = r_{\mathrm{H}} + h$ (h is a small positive quantity and signifies a ultraviolet cutoff). There is also an infrared cutoff $A_\mu = (0, 0, 0, 0)$ at $r = L(L \gg r_{\mathrm{H}})$. We know from Eqs.(10) and (11) that these boundary conditions change into $\Phi = 0$ and $\partial\Phi/\partial z = 0$ at $r = r_{\mathrm{H}} + h$ and $r = L$.

Employing the Wentzel-Kramers-Brillouin approximation and inserting the metric (3) into Eq.(12), we find the following expression for the number of eigenstates with energy less that E

$$
\begin{aligned}
n(E) &= \sum_l (2l+1) n_r \\
&= \frac{1}{\pi}\sum_l (2l+1)\int_{r_{\rm H}+h}^{L} g_{rr}\sqrt{E^2 - \frac{g^{rr}l(l+1)}{R_1}}\,{\rm d}r \\
&= \frac{1}{\pi}\int (2l+1){\rm d}l \int_{r_{\rm H}+h}^{L} g_{rr}\sqrt{E^2 - \frac{g^{rr}l(l+1)}{R_1}}\,{\rm d}r \\
&= \frac{2}{3\pi}\int_{r_{\rm H}+h}^{L} g_{rr}^2 R_1 E^3 {\rm d}r \\
&= \frac{2E^3}{3\pi}\left(\frac{R_1}{G_1^2}\right)_{r_{\rm H}}\frac{1}{h} + \frac{2E^3}{3\pi}\left[\frac{R_1}{G_1^3}\left(\frac{G_1}{R_1}\frac{{\rm d}R_1}{{\rm d}r} - 2\frac{{\rm d}G_1}{{\rm d}r}\right)\right]_{r_{\rm H}} \ln\frac{L}{h}.
\end{aligned} \tag{13}
$$

Since there are two independent scalar fields both satisfying Eq.(12), the free energy at inverse temperature β can be expressed as

$$
\begin{aligned}
\beta F &= -\sum_n \ln(1-{\rm e}^{-\beta E}) = -\beta\int_0^\infty \frac{2n(E)}{{\rm e}^{\beta E}-1}{\rm d}E \\
&= -\frac{4\beta}{3\pi}\left\{\left(\frac{R_1}{G_1^2}\right)_{r_{\rm H}}\frac{1}{h} + \left[\frac{\rm d}{{\rm d}r}\left(\frac{R_1}{C_1^2}\right)\right]_{r_{\rm H}}\ln\frac{L}{h}\right\}\int_0^\infty \frac{E^3}{{\rm e}^{\beta E}-1}{\rm d}E \\
&= -\frac{4\pi^3}{45\beta^3}\left\{\left(\frac{R_1}{G_1^2}\right)_{r_{\rm H}}\frac{1}{h} + \left[\frac{R_1}{G_1^3}\left(\frac{G_1}{R_1}\frac{{\rm d}R_1}{{\rm d}r} - 2\frac{{\rm d}G_1}{{\rm d}r}\right)\right]_{r_{\rm H}}\ln\frac{L}{h}\right\}.
\end{aligned} \tag{14}
$$

It is obvious, as we expected, that the temperature of the free energy is Planckian, namely, it is proportional to β^{-4}.[29-31] If we set $\delta^2 = 2\epsilon^2/15$ and $\Lambda^2 = L\epsilon/h$ as we did in Refs.[14] and [16] (where $\delta = \int_{r_{\rm H}}^{r_{\rm H}+h}\sqrt{g_{rr}}{\rm d}r \approx 2\sqrt{\frac{h}{G_1(r_{\rm H})}}$ is the proper distance from the horizon $r_{\rm H}$ to $r_{\rm H}+h$, ϵ is the ultraviolet cutoff, and Λ is the infrared cutoff of the Solodukhin[32,33]) and note Hawking inverse temperature $\beta_{\rm H} = \left(\frac{4\pi}{G_1}\right)_{r_{\rm H}}$, Eq.(14) can be rewritten as

$$
\beta F = -\frac{1}{24}\frac{R_1}{\epsilon^2}\left(\frac{\beta_{\rm H}}{\beta}\right)^3 - \frac{R_1}{360}\left(\frac{\beta_{\rm H}}{\beta}\right)^3\left(\frac{G_1}{R_1}\frac{{\rm d}R_1}{{\rm d}r} - 2\frac{{\rm d}G_1}{{\rm d}r}\right)_{r_{\rm H}}\ln\frac{\Lambda}{\epsilon}. \tag{15}
$$

Therefore, the statistical-mechanical entropy is given by

$$
\begin{aligned}
S_{em}^{\rm SM} &= \left(\beta^2\frac{\partial F}{\partial\beta}\right)_{\beta H} \\
&= \frac{A_\Sigma}{24\pi\epsilon^2} + \frac{A_\Sigma}{360\pi}\left(\frac{G_1}{R_1}\frac{{\rm d}R_1}{{\rm d}r} - 2\frac{{\rm d}G_1}{{\rm d}r}\right)_{r_{\rm H}}\ln\frac{\Lambda}{\epsilon} \\
&= \frac{A_\Sigma}{24\pi\epsilon^2} - \frac{A_\Sigma}{360\pi}\left(\frac{{\rm d}^2 g^{rr}}{{\rm d}r^2} - \frac{1}{R_1}\frac{{\rm d}g^{rr}}{{\rm d}r}\frac{{\rm d}R_1}{{\rm d}r}\right)_{r_{\rm H}}\ln\frac{\Lambda}{\epsilon}.
\end{aligned} \tag{16}
$$

We can easily get the Cognola and Lecca's result[23] when we substitute the metric of the Reissner-Nordström black hole into Eq.(16) and only consider the leading term $\frac{A_\Sigma}{24\pi\epsilon^2}$ (δ in this paper is equal to l in Ref.[23]).

Ⅲ. DISCUSSION AND SUMMARY

In this section, let us begin a discussion with some study of the statistical-mechanical entropy for some famous black holes by using formula (16).

A. The Schwarzschild black hole

Substituting the metric of the Schwarzschild black hole

$$\mathrm{d}s^2 = -\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\dfrac{2M}{r}} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2 \tag{17}$$

into Eq.(16), we find the leading behavior of the entropy

$$S_{\mathrm{Sch}} = \frac{A_h}{24\pi\epsilon^2} + \frac{2}{45}\ln\left(\frac{\Lambda}{\epsilon}\right), \tag{18}$$

where $A_h = 4\pi r_h^2$ is the area of the event horizon.

B. The Reissner-Nordström black hole

The metric of the Reissner-Nordström black hole is described by

$$\mathrm{d}s^2 = -\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\dfrac{2M}{r}+\dfrac{Q^2}{r^2}} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2. \tag{19}$$

Inserting metric (19) into Eq.(16) we get

$$S_{\mathrm{RN}} = \frac{A_h}{24\pi\epsilon^2} + \left(-\frac{1}{45}+\frac{1}{30}\frac{4\pi r_+}{\beta}\right)\ln\left(\frac{\Lambda}{\epsilon}\right), \tag{20}$$

where $A_h = 4\pi r_+^2$, and $\beta = 4\pi r_+^2/(r_+ - r_-)$.

C. The Garfinkle-Horowitz-Strominger dilatonic black hole

By using the metric of Garfinkle-Horowitz-Strominger dilatonic black hole[34]

$$\mathrm{d}s^2 = -\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\frac{2M}{r}} + r(r-a)(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2) \tag{21}$$

and Eq.(16), we get

$$S_{\mathrm{GHS}} = \frac{A_h}{24\pi\epsilon^2} + \left(\frac{1}{90}+\frac{1}{30}\frac{A_h}{r_+\beta}\right)\ln\left(\frac{\Lambda}{\epsilon}\right), \tag{22}$$

where $A_h = 4\pi r_+(r_+ - a), r_+ = 2M, \beta = 4\pi r_+$, and $a = Q^2/2M\mathrm{e}^{-2\Phi_0}$. For the extremal black hole, i.e., $r_+ = 2M = a$ and $\beta(r_+) = 8\pi M$, the first part in Eq.(22) vanishes and the logarithmical term becomes

$$S_{\mathrm{GHS}}^{\mathrm{ext}} = \frac{1}{90}\ln\left(\frac{\Lambda}{\epsilon}\right), \tag{23}$$

which is whole entropy of the black hole.

D. The static Gibbons-Maeda dilaton black hole

The metric of static Gibbons-Maeda dilaton black hole is described by[35]

$$\mathrm{d}s^2 = -\frac{(r-r_+)(r-r_-)}{R^2}\mathrm{d}t^2 + \frac{R^2\mathrm{d}r^2}{(r-r_+)(r-r_-)} + R^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2), \tag{24}$$

where $r_\pm = M \pm \sqrt{M^2+D^2-P^2-Q^2}, D = (P^2 - Q^2)/2M$ and $R^2 = r^2 - D^2$. The parameters Q and P represent electric charge and magnetic charge, respectively.

The leading behavior of the entropy of a scalar field in the background of the black hole can be obtained by using Eqs.(6) and (24), which is explicitly given by

$$S_{\mathrm{GM}} = \frac{A_h}{24\pi\epsilon^2} + \left(-\frac{1}{45}+\frac{1}{15}\frac{4\pi r_+}{\beta}\right)\ln\left(\frac{\Lambda}{\epsilon}\right), \tag{25}$$

where $A_h = 4\pi(r_+^2 - D^2)$, and $\beta = 4\pi(r_+^2 - D^2)/(r_+ - r_-)$.

E. The Garfinkle-Horne dilaton black hole

The Garfinkle-Horne dilaton black hole metric in the Einstein-Maxwell-dilation theory can be expressed as[34,36]

$$\mathrm{d}s^2 = -\left(1-\frac{r_+}{r}\right)\left(1-\frac{r_-}{r}\right)^{\frac{1-a^2}{1+a^2}}\mathrm{d}t^2 + \left(1-\frac{r_+}{r}\right)^{-1}\left(1-\frac{r_-}{r}\right)^{\frac{a^2-1}{1+a^2}}\mathrm{d}r^2 + r^2\left(1-\frac{r_-}{r}\right)^{\frac{2a^2}{1+a^2}}(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2), \tag{26}$$

with dilaton field $\mathrm{e}^{2\Phi} = \left(1-\dfrac{r_-}{r}\right)^{\frac{2a}{1+a^2}}\mathrm{e}^{-2\Phi_0}$ and Maxwell field $F = \dfrac{Q}{r^2}\mathrm{d}t\bigwedge\mathrm{d}r$. And a is a coupling constant, $r = r_+$ is the location of the event horizon. For $a = 0, r = r_-$ is the location of the inner Cauchy horizon; however, for $a > 0$ the surface $r = r_-$ is singular. The mass M and charge Q of the black hole are related

to parameters r_+ and r_- by $2M = r_+ + \left(\frac{1-a^2}{1+a^2}\right) r_-$ and $Q^2 = \frac{r+r_-}{1+a^2} \mathrm{e}^{2a\Phi_0}$.

Substitution of Eq.(26) into Eq.(16) yields

$$S_{\mathrm{GH}} = \frac{A_h}{24\pi\epsilon^2} + \frac{1}{45}\left(\frac{(2a^2-1)}{1+a^2} + \frac{3}{1+a^2}\frac{A_h}{r_+\beta}\right)\ln\frac{\Lambda}{\epsilon}, \tag{27}$$

where $\beta = \frac{2\pi}{k} = \frac{4\pi r_+}{\left(1-\frac{r_-}{r_+}\right)^{\frac{1+a^2}{1+a^2}}}$, and $A_h = 4\pi r_+^2 \left(1-\frac{r_-}{r_+}\right)^{\frac{2a^2}{1+a^2}}$.

Comparing Eqs.(18),(20),(22),(25) and (27) with the results (11), (14), (18), (22) and (26) in Ref.[15], we know that, for the general four-dimensional non-extreme static black holes, the statistical-mechanical entropy arising from electromagnetic fields is exactly twice the one due to a massless scalar field if we ignore the contribution from the vacuum surrounding the system.

To summary, by employing the gauge $A_t = 0$, we showed that the electromagnetic potential A_μ in the general four-dimensional static black hole spacetime can be expressed in terms of two independent scalar fields. Then, by means of the scalar fields and't Hooft's BWM the statistical-mechanical entropy arising from the electromagnetic field is studied. Using formula (16) we have investigated the entropy for some famous black holes, such as the Schwarzschild black hole, the Reissner-Nordström black hole, the Gibbons-Maeda dilaton black hole, the Garfinkle-Horne dilatonic black hole and the Garfinkle-Horowitz-Strominger black hole. The results show that the entropy contains both a quadratic and a logarithmic divergent term as usual. By comparing the entropies with the results obtained in Ref.[15], we know the entropy arising from electromagnetic field is exactly twice the one for a massless scalar field. As our discussions in Refs.[15] and [18], the entropy can be renormalized by the standard scheme.

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