Volume 9, Number 5
1009-1963/2000/09(05)/0353-06
CHINESE PHYSICS
May, 2000
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FLAKE STRUCTURE IN THE SPECKLE FIELD PRODUCED BY A FEW SCATTERERS*

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(Received 27 October 1999)

This paper reports a new phenomenon, namely, flake structure in the speckle field produced by a few scatterers. This structure appears in the off-axis region and its size is much larger than that of the normal speckles. The one-dimensional simulations show that as the roughness of the random surface sample increases or its lateral correlation length decreases, the flakes go farther away from the axis, their relative intensities increase, their sizes become bigger and the fluctuations in them decrease. A sub-scatterer model that we call isoclinic element is proposed to explain the formation and properties of flakes. Experimental evidence shows the existence of the flake structure.

PACC: 4220; 4230

I. INTRODUCTION

It is well known that the scattering of coherent light from a random surface that contains a large number of scatterers produces a Gaussian speckle field, in which the average size of the speckle grains is the same as that of the diffraction Airy spot of scattering aperture.[1] When a random surface contains only a few scatterers, the statistical properties of the scattered speckle field have not been well understood to our knowledge. The random walk model that takes a whole scatterer as the scattering element continues to be used in the theoretical study[2−5] just as in the study of Gaussian field, and it gives the result that the whole speckle field is non-Gaussian. However, our recent computational simulations[6] showed that the probability density of speckle intensities produced by a few scatterers is space-dependent, and the speckle field in the paraxial region on the observation plane obeys Gaussian statistics while in the off-axis region it is non-Gaussian. This paper reports a new phenomenon, i.e., flake structure in the speckle field produced by a few scatterers, studies its characteristics by computational simulations and proposes the sub-scatterer model of isoclinic element to give a preliminary explanation to the cause of its formation.

II. THE COMPUTATIONAL SIMULATION AND EXPERIMENTAL OBSERVATION OF SPECKLE FLAKE AND ITS CHARACTERISTICS

We now consider only the speckle field on the Fraunhofer or far field plane produced by the light scattering of Gaussian random surfaces with Gaussian correlation. The statistical properties of surfaces of this kind are depicted by the mean-square-root deviation roughness w and the lateral correlation length ξ. Assuming that a parallel coherent light beam with unity amplitude illuminates a random surface on $\boldsymbol{r}_0(x_0, y_0)$ plane, we have the complex amplitude of the speckle field on Fraunhofer plane or far-field plane $\boldsymbol{r}(x, y)$:

$$U(\boldsymbol{r}) = \int A(\boldsymbol{r}_0) \exp[-\mathrm{i}4\pi z(\boldsymbol{r}_0)/\lambda] \cdot \exp(-i2\pi \boldsymbol{r}_0 \cdot \boldsymbol{r}/\lambda f)\mathrm{d}^2\boldsymbol{r}_0, \quad (1)$$

where $z(\boldsymbol{r}_0)$ is the height function of the surface, $A(\boldsymbol{r}_0)$ is the aperture function, λ is the wavelength of the illuminating beam and f is the distance between the random surface and the observation plane. The intensity $I(\boldsymbol{r}) = U(\boldsymbol{r})U^*(\boldsymbol{r})$ of the speckle field is readily known from Eq.(1). In Refs.[6] and [7], We have described in detail the methods for the simulational generation of Gaussian random surfaces with Gaussian correlation and the speckle intensities $I(\boldsymbol{r})$ based on Eq.(1).

In the simulational generation of the speckle intensity in this paper, f and λ are set to be 20cm and 0.6238 μm respectively. Let us now start the description of the flake structure in speckle field from the one-dimensional simulation. In Fig.1(a), a generated random surface sample of Gaussian correlation with $w = 0.3\,\mu$m, $\xi = 3.0\,\mu$m and the aperture size $L = 24\,\mu$m is shown, and we refer to it as sample 1.

* Project supported by the National Natural Science Foundation of China (Grant No. 69978012).

Figure 1(b) gives the simulated intensity distribution produced by this sample. From this intensity distribution, we see that in the paraxial region the speckle grains are small and their average size is close to that of the normal speckle, which is equal to the size of the diffraction Airy spot of the aperture, while in the off-axial regions, there exist speckle grains whose sizes are much bigger than those of the normal speckle grains, and there are small intensity fluctuations in them. We have performed many simulations for random surface samples with different values of w, ξ and L, and found this kind of structure is universal in the speckle field produced by a few scatterers. The experimental observation and the simulations of the speckle field produced by the two-dimensional random surface samples show that this structure actually appears to be flake-like, so we give it the term "flake structure" of the speckle field.

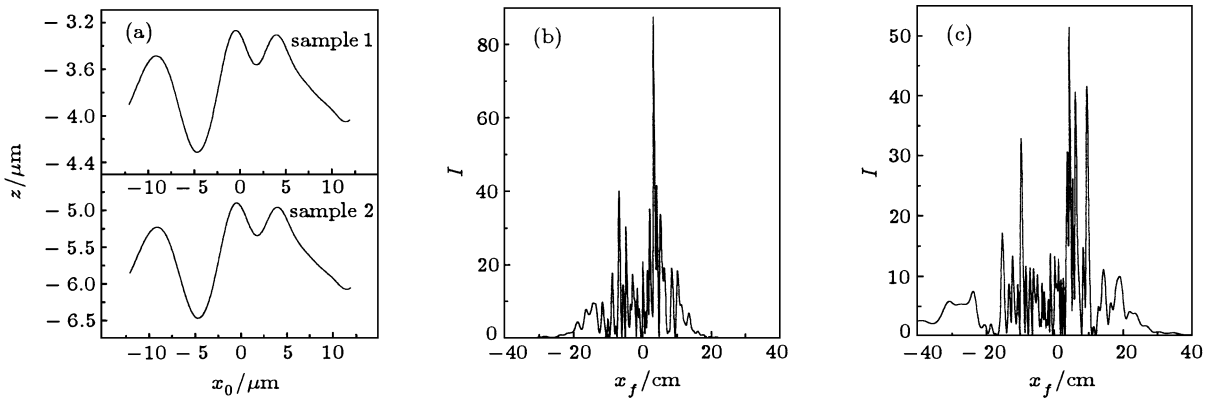


Fig.1. Plots showing effect of roughness on flake structure. (a) generated surface samples 1 and 2; (b) the intensity distribution of sample 1; (c) the intensity distribution of sample 2.

We now depict the properties of the flake structure by one-dimensional simulations.

(1) The effect of surface roughness on flake structure. In the case that all other parameters are the same, with the increase of surface roughness w, the flake structure goes farther away from the center of the observation plane, and becomes more obvious, i.e., the size of the flake becomes bigger, the relative maximum of its intensity increases, and the fluctuation decreases. These features are shown in Fig.1, where Fig.1(a) gives surface sample 2 which is generated with the same random number set as in the generation of sample 1, but w is set to be $0.45\,\mu$m, and we say surface samples 1 and 2 resemble each other. Figure 1(c) is the plot of the intensity distribution produced by surface sample 2.

(2) The influence of lateral correlation length ξ on flake structure. For surface samples with different values of ξ, when the average numbers of the scatterers included in the surface samples are the same, the sizes of the scattering appertures will be different. In this case, samples with bigger ξ produce speckle flakes that are closer to the center of the observation plane, with the sizes of the flakes being smaller. The pitches of the fluctuations in the flakes become smaller due to the bigger size of the aperture. Surface samples 3 and 4 plotted in Fig.2 (a) have the same w value($0.75\,\mu$m), while for sample 3, $\xi = 4.5\,\mu$m, and for sample 4, $\xi = 3.0\,\mu$m and they resemble each other. Figures 2(b) and 2(c) give the intensity distributions corresponding to sample 3 and 5 respectively. Surface sample 5 in Fig.2(a) has the same statistical parameters as sample 3, i.e., $\xi = 4.5\,\mu$m and $w = 0.75\,\mu$m, but its aperture size is the same as sample 4. This is for a comparision of the flake structures produced by samples with different ξ values but with the same average number of scatterers. Figure 2(d) is the intensity distribution produced by sample 5, and we can see that the sizes and positions of the flakes are close to those produced by sample 3, but different from those by sample 4. This means the influence of lateral correlation length ξ is far more obvious than that of the average number of scatterers on the flake structures.

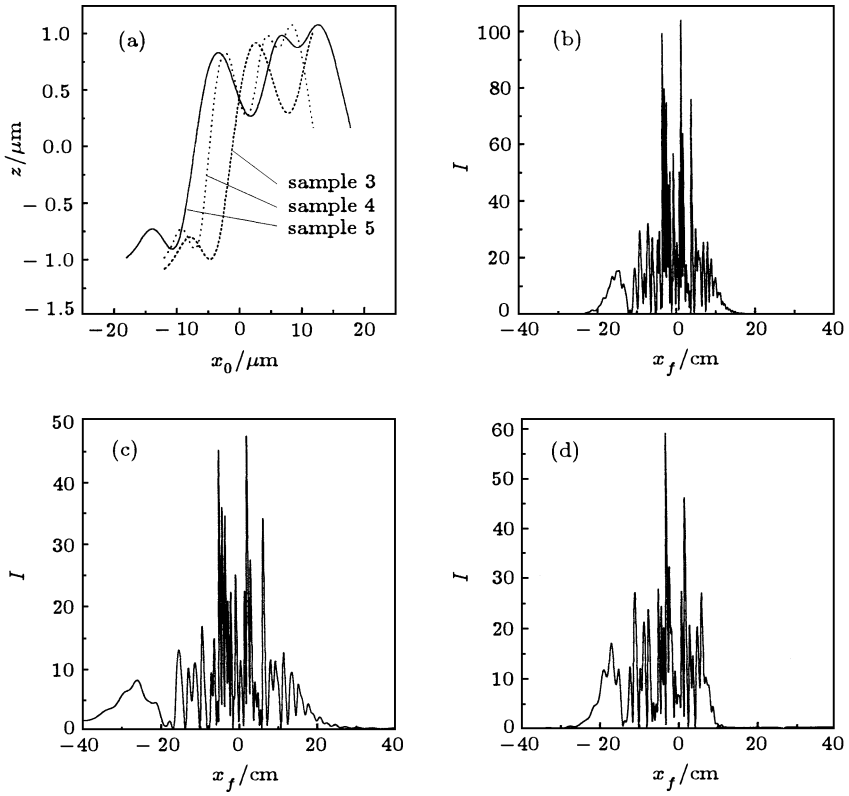


Fig.2. Plots showing the effect of lateral correlation length on flake structure. (a) The generated surface samples 3,4 and 5. Figures (b), (c) and (d) are, respectively, the intensity distributions of sample 3,4 and 5.

(3) The evolution of flake structure with the increase of average number of scatterers. Both surface samples 6 and 7 given in Fig.3(a) resemble samples 3 and 5, and they possess the same parameters: $w = 0.75\,\mu$m and $\xi = 4.5\,\mu$m, but their sizes of scattering area L are 60 and $80\,\mu$m respectively. Figures 3(b) and 3(c) give their intensity distributions respectively. Comparing the intensities produced by samples 3,5,6 and 7, we see that in the case of w and ξ being fixed, with the increase of the number of scatterers, the modulations of the flake structure become more and more severe, and the pitches of the modulations become smaller. It can be concluded that continued increase of scatterers will lead an evolution of the speckle field to a normal one whose average grain size is the same as the size of the diffraction Airy spot of the scattering area.

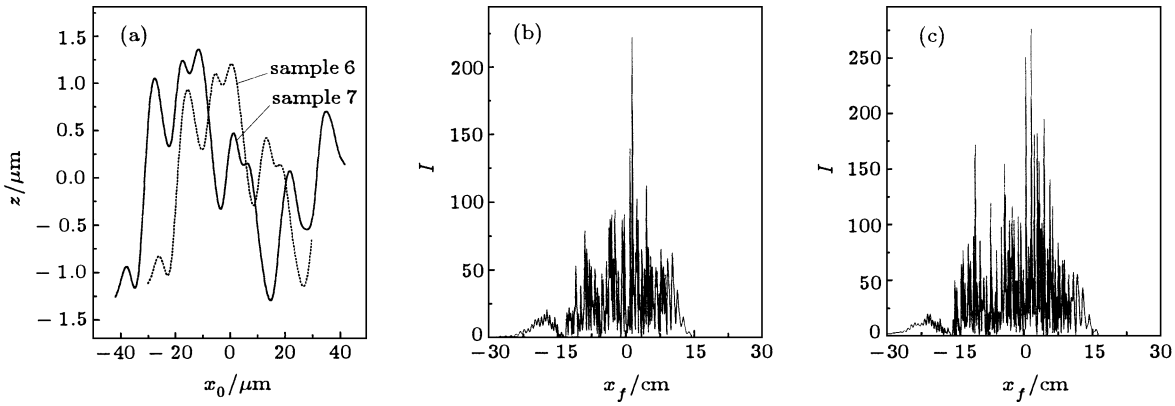


Fig.3. Plots showing the evolution of flake structure with the increase of the average number of scatterers. (a) Samples 6 and 7, (b) the intensity distribution of sample 6, (c) the intensity distribution of sample 7.

In order to show the real shape of the flake structure, we have performed the computational simulation and experimental observation of speckle fields of two-dimensional random surfaces. Figure 4(a) gives the height distribution of a generated surface sample with the parameters $w = 1.0\,\mu$m, $\xi = 3.0\,\mu$m and the scattering area 12.0 μm $\times$ 12.0 μm, and Fig.4(b)gives the simulated speckle field of this sample. In the experimental observation, the rough backside of a silicon wafer has been taken as the surface sample. The diameter of the scattering area is 20 μm. The illuminating light source is a He-Ne laser and the wavelength is 0.6328 μm. The morphology of the sample is inspected with atomic force microscope (AFM, Park Autoprobe, Non-contact mode, UL20 tip) and the height distribution of a part of the sample with 30.0 μm $\times$ 30.0 μm in size is given in Fig.5(a). Data analysis from other measurements of bigger range (100 μm $\times$ 100 μm) of the sample gives the sample parameters $w = 0.277\,\mu$m and $\xi = 7.02\,\mu$m. Figures 5(b) and 5(c) are the photographs of the speckle field produced by this silicon wafer. However, it must be noted that the part of the surface sample given in Fig.5(a) is neither of the two producing the speckle fields of Figs.5(b) and 5(c), for it is impossible to find it again in the optical setup after the AFM measurement is taken.

The above simulation and observation show the existence of flake structures in speckle field.

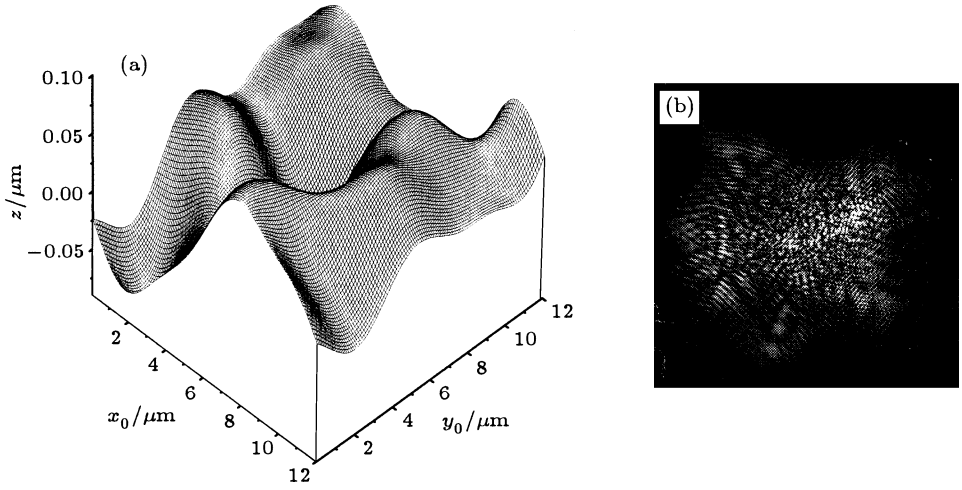


Fig.4. (a) A generated two-dimensional surface sample. (b) The generated speckle pattern.

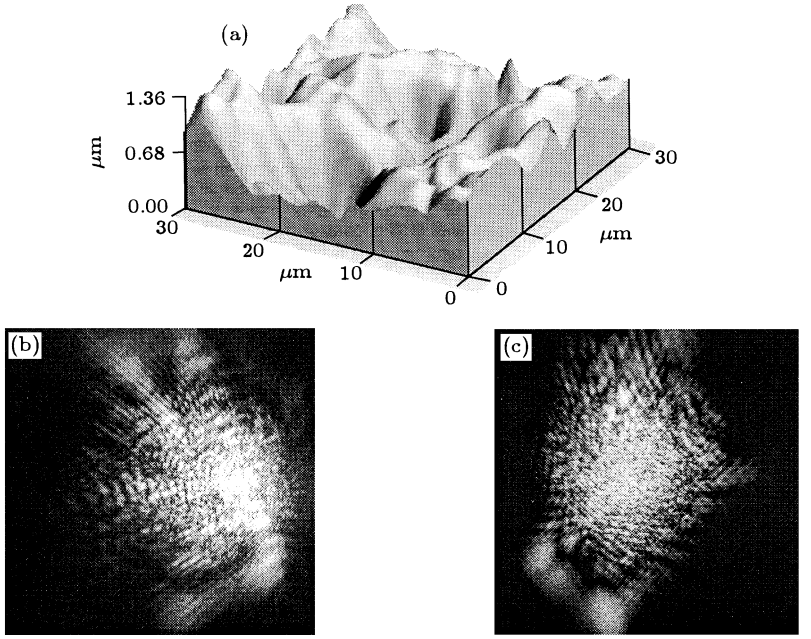


Fig.5. Experimental observation of flake structure. (a) The height distribution of the silicon backside sample obtained with AFM, (b) and (c) are photographs of the speckle patterns.

Ⅲ. THE ISOCLINIC MODEL FOR THE EXPLANATION OF THE FORMATION OF FLAKE STRUCTURES

In order to explain how the flake structure is formed, let us look into the generated one-dimensional surface sample 8 as is shown in Fig.6(a), whose w and ξ are 0.85 and 3 μm respectively. Figure 6(b) gives its speckle intensity distribution. We notice that segments 1 to 7 of the sample 8 labeled in Fig.6(a) are all comparatively straight, so each of them has its own slope, and we refer to these segments as isoclinic scattering elements. When a light wave illuminates the sample, the contribution of each isoclinic element to the speckle field is the whole effect of its reflection and diffraction as a mirror. When the element is big enough, the effect of its geometrical reflection far surpasses that of its diffraction, and then the light wave produced by this element is centered in the neighboring area corresponding to the direction of reflection, where apparently large intensity distribution appears. Then the flake structure of the speckle field is formed.

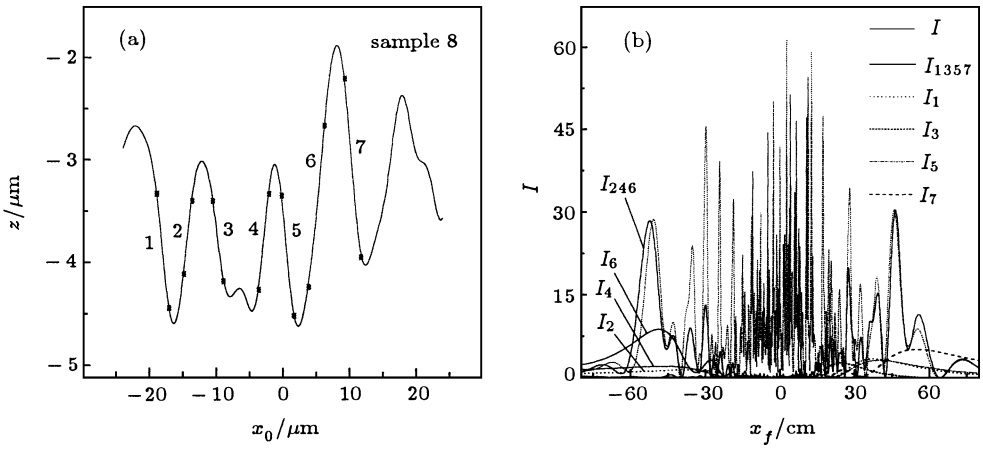


Fig.6. (a) Surface sample 8 with its 7 isoclinic elements labeled. (b) Intensity distributions produced by sample 8 and each of its 7 elements. I—the intensity of sample 8, I_i—the intensity produced by element i, I_{ijk}—the superimposed intensity of elements i, j and k.

In Fig.6(a), we see that isoclinic elements 2,4 and 6 have positive slopes, and the element 6 has the biggest size and slope. Its slope is 0.6487, which determines that the angle of incidence is 32.97°. The corresponding position of the reflected beam on Fraunhofer plane is $x_f = 44.8$cm. From the intensity distributions of the individual isoclinic elements shown in Fig.6(b),it is seen that the maximum of the intensity distribution of element 6 is just in the vicinity of $x_f = 44.8$cm, where lies the biggest flake to the left of the center $x_f = 0$. Though the slopes of isoclinic elements 2,4 and 6 are somewhat different, the diffraction effect causes their light fields to overlap. And the phase differences of these fields are randomly distributed because of the randomness of both the positions x in the scattering area and their relative height $z(x_0)$. As a result, their superimposed intensity distribution is what is shown in Fig.6(b) by curve I_{246}. We see that this superimposed intensity makes the main contribution to the flakes in speckle field on the left side. Similarly, the intensity distributions of isoclinic elements 1,3,5 and 7 and the superimposition are also shown in Fig.6(b), to which the flakes on the right side are attributed.

The above depictions and the results in Fig.6 indicate that, flake structure in speckle field arises from the directional reflection and the diffraction of isoclinic elements acting as small mirrors. Compared with the conventional random walk model which takes the scatterers as the smallest elements, the isoclinic element model proposed here in this paper takes the straight part within a scatterer as the smallest scattering unit. Therefore, it can be regarded as a sub-scatterer model.

Since the light field in the off-axis area is produced mainly by the isoclinic elements with larger slopes, and in the scattering area containing a few scatterers, there is very small probability for these elements to possess slopes of close values, the probability for light fields produced by these elements to overlap is also very small. Therefore, flakes can be easily formed in off-axis regions.

In the case that other parameters remain con-

stant, either the increase of roughness w or the decrease of lateral correlation length ξ causes the increase of the slopes of the isoclinic elements, so the flakes go farther away from the center of Fraunhofer plane. The small modulations in the flakes result from two factors. One of them is that the slope of the isoclinic element producing a flake fluctuates a little, and the other is the small disturbance from the diffraction of other elements. With the increase of the number of scatterers, the elements with closely valued slopes increase, and the disturbances from other elements with different slopes become more intensive, so both the modulations and randomness of the modulations that the flakes undergo become more severe, and the flakes evolve to the normal granule structure and disappear finally.

The light field in the paraxial region is the superimposition of the light waves that are from the reflection and diffraction of the isoclinic elements with smaller slopes and from the diffraction of other elements with larger slope values. Since there is higher probability for isoclinic elements of smaller slopes to appear in the random surface, and on the average, the contribution of the diffraction from the elements of the larger slopes to light field in the paraxial region is much more obvious than that in the off-axis regions, the fluctuation of the intensity in the paraxial region is much more serious than that of the flakes in the off-axis regions. Therefore, the scattering of a few scatterers produces Gaussian speckle field in paraxial region whose average grain size is the same as the Airy spot and non-Gaussian speckle in off-axis regions. This is the conclusion we have obtained in Ref.[6], and we have not been able to give the explanations to it there.

IV. CONCLUSIONS

Flake structure is a phenomenon induced by the scattering of a few scatterers and it is a typical non-Gaussian stochastic process in speckle statistical optics. The isoclinic model proposed in this paper provides a proper explanation to the formation and characteristics of flake structures. We believe that this model would also be the key to the theoretical accomplishment of the probability distribution and other statistical properties of the speckle field produced by a small number of scatterers.

REFERENCES

[1] J. C. Dainty, Laser Speckle and Related Phenomena, 2nd ed. (Spinger, Berlin, 1984), ch.2.
[2] B. M. Levine, J. C. Dainty,*Opt. Commun.*, **45**(1983), 252.
[3] E. Jakeman, *Opt. Eng.*, **23**(1984), 453.
[4] E. Jakeman, R. A. Tough., *J. Opt. Soc. Am.*, **A4**(1987), 1746.
[5] E. Jakeman, *Advances in Phys.*, **37**(1988),471.
[6] Cheng Chuan-fu, Qi Dong-ping, Liu De-li *et al.*, *Acta Physica Sinica*, **48**(1999), 1635(in Chinese).
[7] Cheng Chuan-fu, Liu De-li, Qi Dong-ping, *Chinese Physics Letters*, **16**(1999), 397.